THEOREM 4.4. *This* SO(4) *action on* S^7 *satisfies:*

(1) *It is a hyperpolar isometric action of cohomogeneity 1, with space of orbits the interval* $[0, \frac{\pi}{2}]$.

(2) *The two exceptional orbits are both diffeomorphic to* $P^2_{\mathbf{R}} \times S^3$ *and both are minimally embedded in* S^7.

(3) *The principal orbits are diffeomorphic to* $F^3(2,1) \times S^3$.

(4) *The square of the distance functions to the exceptional orbits are both Bott-Morse functions.*

(5) *The union of the two exceptional orbits, both copies of* $P^2_{\mathbf{R}} \times S^3$, *is the Spanier-Whitehead dual of one principal orbit* $F^3(2,1) \times S^3$.

We notice that the action of $\mathrm{SO}(n+1)$ on $\mathbf{C}^{n+1}$ considered in Section 2 also provides, when $n = 3$, an isometric action of cohomogeneity 1 of SO(4) on S^7. However, in this case the two special orbits are the inverse images of the quadric Q and the real projective space $\Pi \cong P^3_{\mathbf{R}}$ under the projection $S^7 \to P^3_{\mathbf{C}}$. So this action is not equivalent to the "twistorial" one given by Theorem 4.4.

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(Reçu le 15 avril 2001; version révisée reçue le 21 février 2003)

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