2.1 Concepts and notions

2. Framework for the Study

Documenting the state-of-the-art in a field and identifying deficiencies and needed research requires a *structuring framework*. This is particularly important in an area as complex and difficult to survey as the teaching and learning of mathematical modelling and applications, for this topic not only deals with most of the essential aspects of the teaching and learning of mathematics at large, but it also touches upon a wide variety of versions of the real world outside mathematics that one seeks to model.

2.1 Concepts and Notions

It is recognised that within the field variations exist with respect to the meaning and interpretation of some technical terms. Hence we give some working definitions that will indicate meanings intended within this document.

By *real world* we mean everything that has to do with nature, society or culture, including everyday life as well as school and university subjects or scientific and scholarly disciplines different from mathematics. Starting with a certain problematic *situation* in the real world, simplification and structuring leads to the formulation of a *problem* and thence to a *mathematical model* of the problem. Here we use the term problem to encompass both basic practical problems, and problems of an intellectual nature aimed at describing, explaining, understanding or even designing parts of the world. The situation – still a part of the real world in our sense – is then *mathematised* – that is the relevant objects, data, relations and conditions involved in it are translated into mathematics, resulting in a *mathematical model* of the original situation.

Now mathematical methods are used to derive *mathematical results*, which when re-translated into the real world, can be interpreted in relation to the original situation. At this point the problem solver *evaluates* the model by checking whether the problem solution is appropriate and reasonable for his or her purposes. If need be, as often occurs with 'genuine' real-world problems, the whole process has to be repeated with a modified or a totally different model. Finally the ultimate solution of the original real world problem is stated and communicated. It has become common practice to use the term *mathematical modelling* for the entire process consisting of structuring, mathematising, working mathematically and interpreting, validating, revisiting and reporting the model.

Sometimes the given problem situation is already pre-structured or is essentially a 'dressing up' of a purely mathematical problem in the words of a segment of the real world. This is often the case with classical school *word problems*. In this case mathematising means merely 'undressing' the problem, and the 'modelling process' extends no further than some use of mathematics and a simple interpretation.

An *application* of mathematics is sometimes used to describe any kind of linking of the real world and mathematics, for example beginning with a standard piece of analysis and showing how it can be applied to address a problem in another discipline. This is sometimes described as the *use of standard models*, as distinct from *mathematical modelling* that represents a complete process in the sense described above. During the last decade the term '*applications and modelling*' has been increasingly used to encompass all kinds of relationships between the real world and mathematics.