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Multiple Meson and γ-ray Production in Cosmic Ray Stars

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Abstract.

A shower of some 56 singly charged, relativistic particles, produced by a primary alpha particle of energy 10^{12}—10^{13} eV, which has been observed in a stack of photographic plates exposed to the cosmic radiation at an altitude of 30 km, is analyzed in detail. This shower presents evidence for the generation of both high energy mesons and gamma-rays with large multiplicity. Almost one half of the mesons of this shower are concentrated in a narrow core of halfwidth 1.5^0; the others are spread out over an angular interval of $\pm 60^0$.

Multiplication of charged particles (production of pairs) in the shower core indicates the presence of a large number of high energy gamme-ray quanta. The energy of the gamma-rays is estimated from the angle of divergence of the observed pairs. Assuming that the gamma-rays result from the decay of neutral mesons, the lifetime of these mesons is estimated to be $\lesssim 10^{-13}$ sec.

The structure of the shower can be explained either by the assumption of multiple meson production in primary, secondary and tertiary encounters in the target nucleus (plural production) and/or by the assumption of a considerable degree of anisotropy of the meson production in the center of mass system of the incident and target nucleons in the primary encounter.

These conclusions are based on the transformation relations connecting the angular distribution of the mesons in the laboratory system with the angular and the energy distribution of the mesons in the center of mass system, discussed in section II.

Besides pair production by high energy gamma-rays, a case of pair production by an energetic electron has been observed (section III).

I. Introduction.

During the past year nuclear photographic emulsions have been developed which are sufficiently sensitive to record the tracks of charged particles whatever their charge and energy. Photographic emulsions have therefore become a most useful tool for the study of high energy phenomena in nuclear physics. In particular the investigation of the production of mesons in high energy collisions of cosmic ray particles and the multiplication processes of the soft component have become accessible to the photographic emulsion technique.

*) Die Autoren möchten Herrn Professor SCHERRER zu seinem 60. Geburtstag die herzlichsten Glückwünsche aussprechen. Einer von uns möchte dem Jubilar, seinem verehrten Lehrer, seine besondere Dankbarkeit bezeugen.

Because of the relatively high frequency of occurrence of such events near the top of the atmosphere, it was found desirable to expose such plates in balloon flights at very great altitudes (~ 30 km).

Since the particles produced in very energetic collisions usually possess great penetrating power, it was found useful to expose large stacks of carefully aligned plates, such that the trajectory of a given particle can frequently be traced through many emulsion layers and therefore through a considerable amount of stopping material (20—50 gr/cm^2).

The photographic plate possesses two important advantages over most other particle detectors:

a) The specific energy loss of a particle can be determined with great accuracy by the method of grain counting. Fig. 1 shows a plot of the range of different particles in microns of emulsion versus their specific energy loss. The same graph contains also 3 curves (I, II, III) which show the number of developed silver grains per 100 μ (right hand scale) plotted versus specific energy loss for three emulsions of different sensitivity. Emulsions can now be obtained which are so uniform in sensitivity that the only appreciable error in determining the specific energy loss comes from the random statistical fluctuations in the number of grains in a track of finite length; an error which for tracks of several hundred microns amounts only to a few percent.

b) The angle between tracks can be measured with great accuracy. Angles as small as 2×10^{-4} radians can be measured in favourable cases. This high resolution makes it possible (as shown in section III) to detect and to estimate the energy of electron-positron pairs produced by γ-rays up to energies of ~ 50 Bev.

The investigations discussed below are mainly based on the distinctive features of the photographic emulsions listed under a) and b).

In section II we discuss the relations between the angular distribution of a meson shower as observed in the laboratory system and some parameters which characterize the process of meson production in the center of mass system of the nucleons, in particular:

the energy of the incident nucleon,

the multiplicity of meson production,

the energy spectrum and the angular distribution of mesons in the center of mass system and

the degree of inelasticity of the nucleon-nucleon collision giving rise to the meson shower.

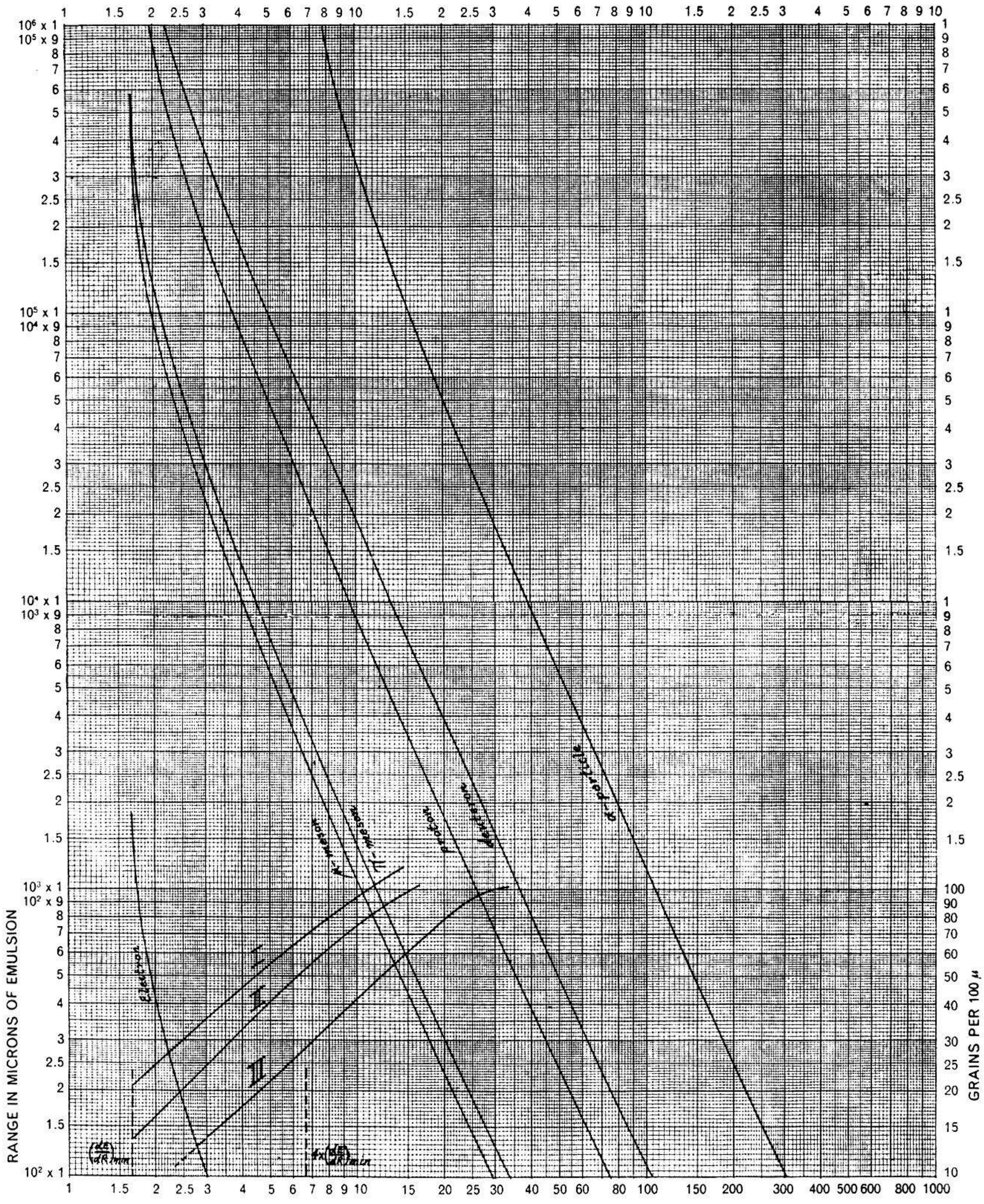

Fig. 1.

Range, specific energy loss and grain density of tracks.

The range of different particles in microns is plotted versus the specific energy loss in MeV/(gr/cm^2). The grain density (grains per 100 μ) (right hand scale) is plotted versus specific energy loss (curves I, II and III) for 3 emulsions of different sensitivity.

In section III we discuss the production of pairs of relativistic singly charged particles which have been observed to be associated with large meson showers. On the assumption that these pairs are electron-positron pairs produced by γ-rays, a method for the energy determination of such γ-rays is discussed.

Section IV contains the description of a particular meson shower (R-shower). This shower is of special interest because of the following features:

a) The high energy involved in the process (10^{12}—10^{13} eV).
b) The large number of mesons produced (~ 56 charged mesons).
c) The clear separation of the shower into two parts:
   a narrow core of width $\pm 1.5^0$,
   and a wide shower of comparable intensity spread over an angular range of $\pm 60^0$.
d) A considerable multiplication of charged particles in the core of the shower, presumably due to very energetic γ-rays whose number is comparable to the number of mesons.

Section V contains a discussion of the R-shower based on the relations obtained in Section II. It is shown:

a) That the narrow shower core is produced in primary collisions of the nucleons of the incident α-particle of energy 10^{12}—10^{13} eV and that at least a fraction of the wide shower is probably produced by secondary and tertiary collisions of the nucleons emerging from the primary encounters. The shower, therefore, is considered to represent an example of both multiple and plural meson production.

b) If one assumes that the greater part of the available energy is transferred to the meson component, an appreciable degree of anisotropy of meson emission in the center of mass system is required in order to account for the observed angular distribution of the shower particles.

c) The observed multiplication of charged particles in the shower core is consistent with the hypothesis that neutral mesons are produced in numbers comparable to the number of charged mesons and that the neutral mesons decay into two γ-rays in a time not longer than 10^{-13} sec.

II. Showers of Relativistic Mesons

In sensitive photographic emulsions exposed to cosmic radiation, especially at high altitudes, an appreciable fraction of the nuclear explosions are observed to be accompanied by the emission of singly charged relativistic particles[1][2][3]. These particles are mostly emitted in the forward direction with respect to the initiating primary.

Such showers are produced by relativistic neutral, singly (Fig. 2) and multiply (Fig. 3) charged particles. As many as 60 relativistic particles have been found to originate in a single nuclear explosion. The photographic plate in general does not permit to identify these particles as protons, mesons or electrons. Only in those cases where the multiplicity is very high one can conclude with certainty from consideration of charge conservation that not all the observed minimum ionization tracks can be due to protons.

However, investigations with cloud chambers and counters have made it very probable that the majority of the relativistic singly

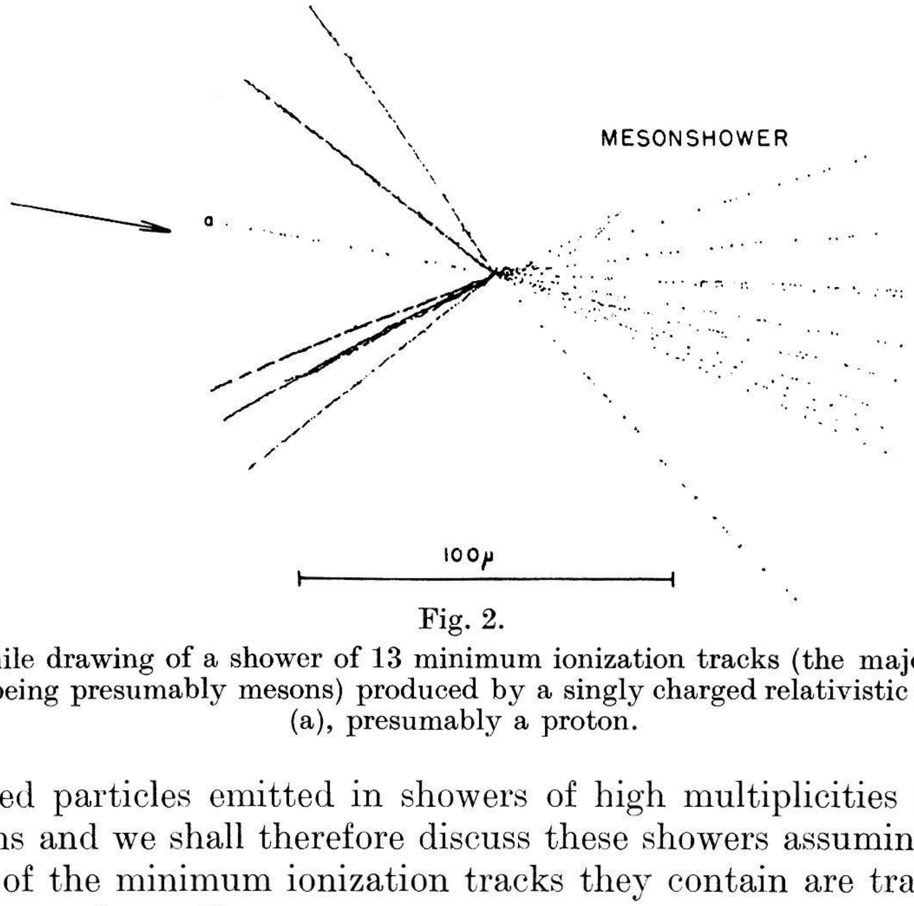

Fig. 2.

Facsimile drawing of a shower of 13 minimum ionization tracks (the majority of them being presumably mesons) produced by a singly charged relativistic particle (a), presumably a proton.

charged particles emitted in showers of high multiplicities are π-mesons and we shall therefore discuss these showers assuming that most of the minimum ionization tracks they contain are tracks of positive and negative π-mesons.

Meson showers produced in photographic emulsions admit in most cases an accurate measurement of the angular distribution of the mesons in the laboratory system (*L. system*). In the more energetic events the meson intensity will show a maximum in the forward direction. Using as polar axis the direction of the incident nucleus we shall describe the angular distribution mainly by three parameters:

Θ_m the angle at which the intensity reaches its maximum value.

$\Theta_{(\frac{1}{2})}$ the angle at which the intensity has fallen to one-half its maximum value, hereafter referred to as the halfwidth of the shower.

Θ_T, the half opening of the cone which contains at least 90% of all mesons in the shower, hereafter referred to as the width of the shower.

In photographic plates it is not possible to measure directly the energy (E_0) of the mesons if $E_0 \gtrsim 2\mu c^2$. But on the assumptions stated below, the order of magnitude of the energy can be deduced from an analysis of pairs of relativistic singly charged particles produced in the neighborhood of the shower and clearly associated with it. These assumptions are:

a) that these pairs are electron-positron pairs produced by γ-rays; we can then estimate the energy of the γ-rays from the angular divergence of the pairs, as shown in section III.

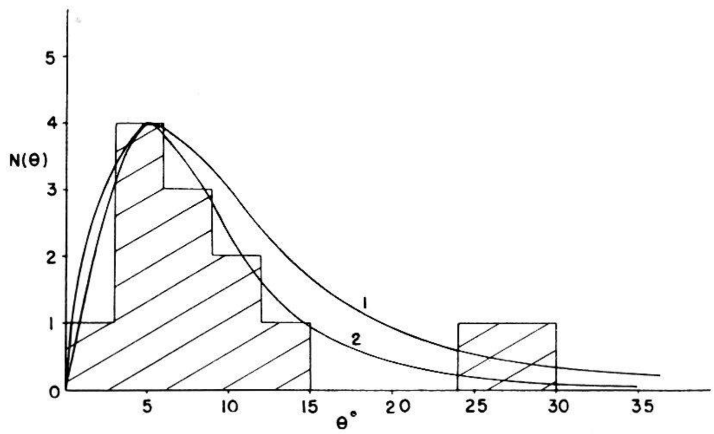

Fig. 2a.

Angular distribution of minimum ionization tracks in the shower of fig. 2. The histogram represents the polar angles with respect to the direction of the incident particle (a). The curves 1 and 2 represent the distribution calculated according to eq. 11 and 15, on the assumption of isotropic meson emission in the C. system and with an energy of the nucleon in the C. system of $\overline{\gamma} = 6$ corresponding to an energy of 70 BeV for the incident nucleons. Curve 1 is obtained on the assumption of equal energy for all mesons, curve 2 for an energy spectrum proportional to

$$\overline{p}_0^2\, d\,p_0/\overline{E}_0^4.$$

b) that these γ-rays are the decay products of neutral mesons, as is suggested by observations at Berkeley on high energy γ-rays associated with meson production[4]). The average energy of the neutral mesons, of the order of twice the γ-ray energy, may reasonably be assumed to be equal to the average energy $\langle E_0 \rangle$ of the actually observed charged mesons.

We shall assume throughout that the mesons are created in a nucleon-nucleon interaction between a nucleon of the incoming and a nucleon of the target nucleus. The center of mass system *(C. system)* is the reference system in which the total momentum of those two nucleons is zero.

In this chapter we shall discuss the relations between the observable quantities:
a) the multiplicity N;
b) the angular distribution function $N(\Theta)$ characterized by
  1) halfwidth $\Theta_{(\frac{1}{2})}$
  2) width Θ_T
  3) the meson energy $E_0 = \gamma_0 \mu c^2$
and the following quantities describing the process of meson production *in the C. system*:

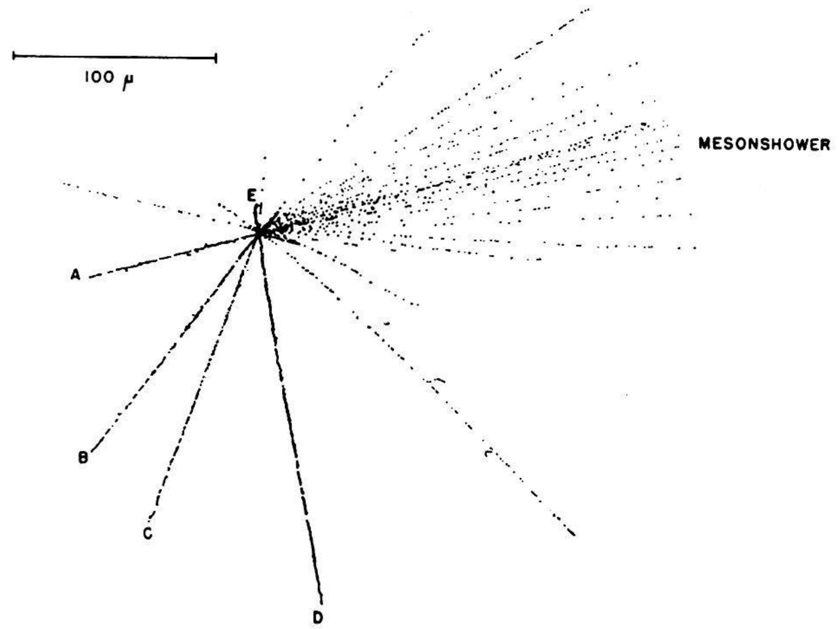

Fig. 3.
Facsimile drawing (incomplete) of a shower of 30 minimum ionization tracks (presumably mesons), produced by particle A which appears to be a carbon nucleus of the primary cosmic radiation.

a) the angular distribution of mesons,
b) the energy spectrum of the mesons,
c) the energy of the incoming nucleon,
d) the degree of inelasticity in the nucleon-nucleon collision.

Throughout the discussion we shall confine ourselves to relatively energetic collisions (energy of incoming nucleon $\sim$ 50 BeV) since we want to discuss large meson showers for which the observed angular distribution becomes statistically significant.

Definition of Symbols

$$\left.\begin{array}{l} \beta = v/c \text{ is the velocity of the nucleon in terms of the velocity of light} \\ \gamma = 1/\sqrt{1-\beta^2} \text{ is the energy in units of the rest energy} \\ p = \sqrt{\gamma^2 - 1} \text{ is the corresponding momentum} \end{array}\right\} \text{in the L. system}$$

$\bar{\beta}$, $\bar{\gamma}$, $\bar{p}$ are the corresponding quantities in the C. system.

Without subscripts these symbols refer to the incoming nucleon. The subscript zero is used for mesons.

$m = \overline{\beta}/\overline{\beta}_0$ is the ratio of the velocity of the nucleon and the velocity of the meson in the C. system.

ϑ is the polar angle with respect to the direction of the incoming nucleus in the C. system.

Θ is the corresponding angle in the L. system.

$\mu = 1/6.5$ is the mass of the π-meson in units of the proton mass.

$\alpha = \overline{\gamma} \tan \Theta$ is a convenient parameter characterizing the angular distribution in the L. system.

N is the number of mesons produced.

K $(0 < K < 1)$ is the fraction of the energy available in the C. system going into meson production, defined by

$$\sum_{i=1}^{N} \mu \overline{\gamma}_{0i} = 2(\overline{\gamma}-1)\, K;$$

K is the degree of inelasticity of the collision, if, as will be assumed, all the energy lost by the nucleons is transferred to mesons.

$N_m = \frac{2(\overline{\gamma}-1)\, K}{\mu}$ is the largest number of mesons, which could be produced in a given nucleon-nucleon collision (if all mesons were produced with zero kinetic energy in the C. system). N_m is the total energy transferred to the mesons in the C. system in units of their rest mass.

$\overline{\Gamma}_0 = N_m - N + 1$ is the largest energy (in units of its rest mass) with which a meson can be produced in the C. system.

The relation between the energy γ of the incoming nucleon in the L. system and its energy $\overline{\gamma}$ in the C. system is given by:

$$\gamma = 2\overline{\gamma}^2 - 1 \tag{1}$$

The velocity and angle of emission for mesons in the L. system and in the C. system are related by:

$$\alpha = \overline{\gamma} \tan \Theta = \sin \vartheta / (\cos \vartheta + m) \tag{2a}$$

$$\overline{\gamma} \tan \vartheta = \pm \sin \Theta / (\cos \Theta - \overline{\beta}/\beta_0) \tag{2b}$$

$$\overline{p}_0 \sin \vartheta = p_0 \sin \Theta \tag{2c}$$

The energy of the mesons in the L. system is obtained with the help of (2a, b, c)

$$\overline{\gamma}_0 = \gamma_0 \overline{\gamma} \pm \sqrt{\gamma_0^2 - 1}\sqrt{\overline{\gamma}^2 - 1} \cos \Theta \tag{3a}$$

where either (+) or (—) sign applies if $\overline{\gamma}_0 \leqslant \gamma \; (m \geqslant 1)$
and the (—) sign applies if $\overline{\gamma}_0 \geqslant \gamma \; (m \leqslant 1)$

$$\gamma_0 = \overline{\gamma}_0 \overline{\gamma} + \sqrt{\overline{\gamma}_0^2 - 1} \sqrt{\overline{\gamma}^2 - 1} \cos \vartheta \tag{3b}$$

$$\gamma_0 = \frac{(\overline{\gamma}_0/\overline{\gamma})(\overline{\gamma}^2 + \alpha^2) \pm \sqrt{\overline{\gamma}_0^2 - 1} \sqrt{\overline{\gamma}^2 - 1} \sqrt{1 - (m^2 - 1)\alpha^2}}{\alpha^2 + 1} \tag{3c}$$

If $m > 1$ the (+) and (—) signs give for a given angle in the L. system the two different energies corresponding to the two possible angles in the C. system.

If $m < 1$ the (+) sign applies if $\Theta < \pi/2$
the (—) sign applies if $\Theta > \pi/2$

From (3b) we see that since the meson distribution in the C. system must be symmetrical with respect to the plane perpendicular to the

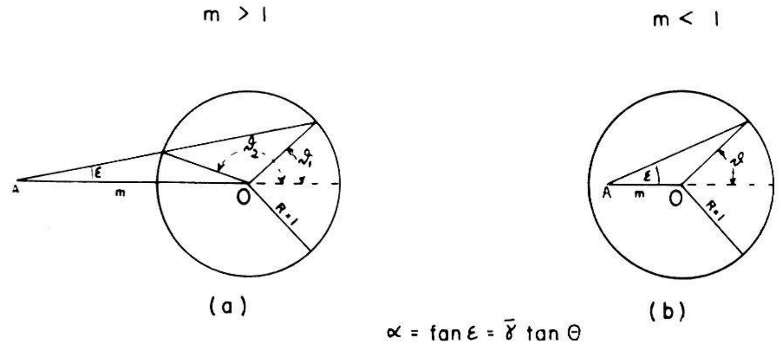

Fig. 4.

Graphical representation of the relation between angles in the laboratory system and the center of mass system. 4a refers to the case $m > 1$. 4b refers to the case $m < 1$. (See text.)

direction of the incoming nucleon, the average energy in the L. system $\langle\gamma_0\rangle$ is related to the average energy in the C. system $\langle\overline{\gamma}_0\rangle$ by

$$\langle\gamma_0\rangle = \overline{\gamma}\,\langle\overline{\gamma}_0\rangle \tag{4}$$

Equation (2a) is graphically represented in Fig. 4. Here the angle ε is defined by $\tan\varepsilon = \overline{\gamma}\tan\Theta = \alpha$, the distance OA is given by $m = \overline{\beta}/\overline{\beta}_0$ and the radius of the circle is chosen equal to unity.

As can be seen from the diagram:

If $m > 1$ (A lies outside the unit circle) there are two angles in the C. system (ϑ_1 and ϑ_2) which correspond to a single angle Θ in the L. system. The largest angle (Θ_C) which Θ can assume is given by

$$\sin \varepsilon = 1/m \text{ or} \tag{5a}$$

$$\sin \Theta_c = \overline{p}_0/\overline{p} \text{ or} \tag{5b}$$

$$\alpha_c = \overline{\gamma} \tan \Theta_c = 1/\sqrt{m^2 - 1} \tag{5c}$$

Hence at a given angle Θ only mesons emitted with energies:

$$\overline{\gamma}_0 \geqslant \cos\Theta \sqrt{\alpha^2+1} \tag{6}$$

in the C. system can be present.

The angle corresponding to Θ_C in the C. system is given by

$$\cos\vartheta_m = -1/m \tag{7}$$

If $m < 1$ (A lies inside the unit circle) a single value of ϑ corresponds to a single value of Θ. In this case Θ may assume any value $0 \leqslant \Theta \leqslant \pi$.

From 2a we obtain the additional relation

$$\cos\vartheta = \frac{-m\alpha^2 \pm \sqrt{1-(m^2-1)\alpha^2}}{\alpha^2+1} \tag{8}$$

here the (+) sign applies

a) if $m \leqslant 1$ and

b) if $m \geqslant 1$

and $0 \leqslant \vartheta \leqslant \vartheta_m$ (corresponding to $\vartheta = \vartheta_1$ in Fig. 4a). The (—) sign applies if $m \geqslant 1$ and $\vartheta_m \leqslant \vartheta \leqslant \pi$ (corresponding to $\vartheta = \vartheta_2$ in Fig. 4a).

If $F(\vartheta, \overline{\gamma}_0)\, d\overline{\Omega}\, d\overline{\gamma}_0$ is the number of mesons which in the C. system are emitted into the angular interval $d\overline{\Omega}$ with energies lying between $\overline{\gamma}_0$ and $\overline{\gamma}_0 + d\overline{\gamma}_0$ then the angular distribution function in the L. system (number of mesons between angles Θ and $\Theta + d\Theta$) is given by

$$N(\Theta)\,\mathrm{d}\Theta = 2\pi \sin\Theta\, d\Theta \int d\overline{\gamma}_0 F[\vartheta(\Theta, \overline{\gamma}_0); \overline{\gamma}_0] \left(\frac{\partial\cos\vartheta}{\partial\cos\Theta}\right)_{\overline{\gamma}_0} \tag{9}$$

For all values $\overline{\gamma}_0 \leqslant \overline{\gamma}$ ($m \geqslant 1$) both F and $J = \left(\frac{\partial\cos\vartheta}{\partial\cos\Theta}\right)_{\overline{\gamma}_0}$ will be double valued functions corresponding to the angles ϑ_1 and ϑ_2 in Fig. 4a. Hence the lower limit of integration is given (eq. 6) by $\overline{\gamma}_0 = \cos\Theta\sqrt{\alpha^2+1}$ and the upper limit of integration is $\overline{\gamma}_0 = \overline{\gamma}$.

For all values $\overline{\gamma}_0 \geqslant \overline{\gamma}$ ($m \leqslant 1$) both F and J are single valued; the lower limit of integration is $\overline{\gamma}_0 = \overline{\gamma}$ and the upper limit $\overline{\gamma}_0 = \overline{\Gamma}_0$ is equal to the highest energy with which mesons can be produced in the C. system.

From (8) we obtain

$$J = \left(\frac{\partial\cos\vartheta}{\partial\cos\Theta}\right)_{\overline{\gamma}_0} = \frac{\pm\overline{\gamma}^2\left[m \pm \sqrt{1-(m^2-1)\alpha^2}\right]^2}{\cos^3\Theta\,(\alpha^2+1)^2\sqrt{1-(m^2-1)\alpha^2}} \tag{10}$$

The $\pm$ signs apply as in eq. (8).

In order to discuss these relations further we shall now calculate the angular distributions of the mesons in the L. system which result from different assumptions about their angular and energy distribution in the C. system.

Case I. We assume that the mesons are produced in the C. system isotropic in angle and all with the same energy $\overline{\gamma}_0'$.

Then $F(\vartheta, \overline{\gamma}_0) = \frac{N}{4\pi}\,\delta(\overline{\gamma}_0' - \overline{\gamma}_0)$ for all values of ϑ and $\overline{\gamma}_0$. Dropping the prime symbol we obtain from (9) the angular distribution in the L. system:

$$N(\Theta) = \frac{N\overline{\gamma}}{\cos^2\Theta}\,\frac{\alpha}{(\alpha^2+1)^2}\,\frac{m^2+1-(m^2-1)\,\alpha^2}{\sqrt{1-(m^2-1)\,\alpha^2}} \text{ for } m \geqslant 1 \tag{11a}$$

and

$$N(\Theta) = \frac{N\overline{\gamma}}{2\cos^2\Theta}\,\frac{\alpha}{(\alpha^2+1)^2}\,\frac{\left[m+\sqrt{1-(m^2-1)\,\alpha^2}\right]^2}{\sqrt{1-(m^2-1)\,\alpha^2}} \text{ for } m \leqslant 1 \tag{11b}$$

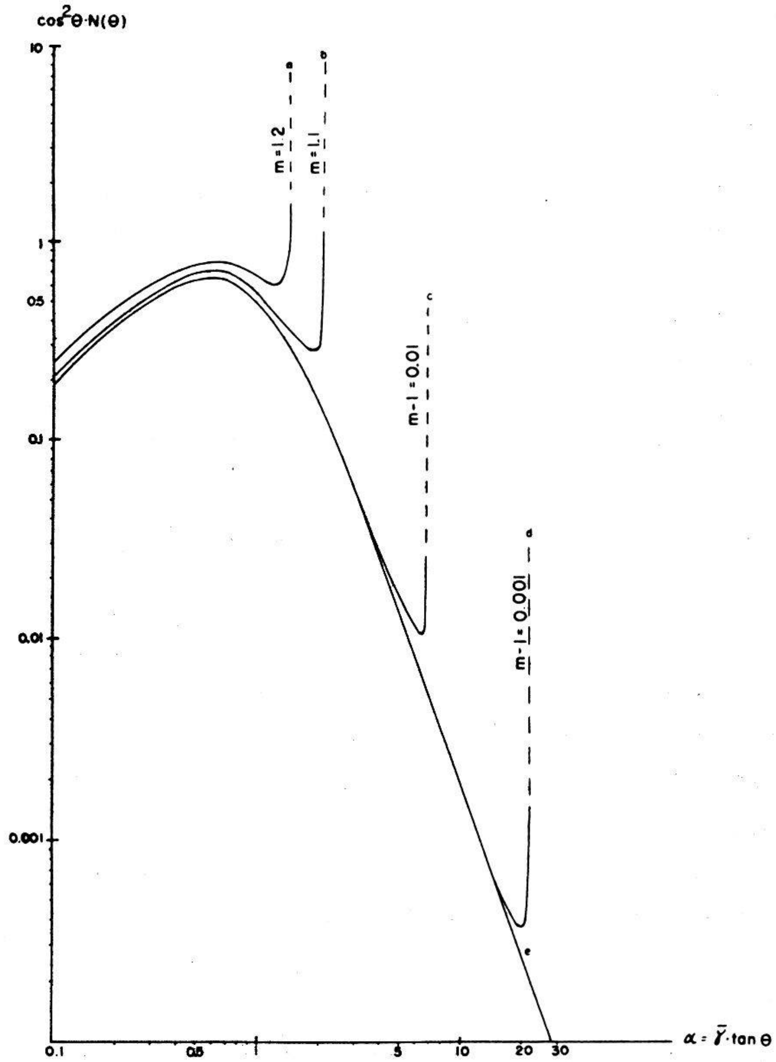

Fig. 5.

Angular distribution functions for Case I. (Isotropy and uniform energy of mesons in the C. system). $N(\Theta)\cos^2\Theta$ is plotted versus $\alpha = \overline{\gamma}\tan\Theta$ for various values of $m = \overline{\beta}/\overline{\beta}_0$. Curve e refers to the limiting case $m = 1$.

For $m \geqslant 1$ $N(\Theta)$ is the sum of two terms corresponding to the two possible energy values of eq. 3c.

In Fig. 5 $\cos^2 \Theta \cdot N(\Theta)$ is plotted against the parameter $\alpha = \bar{\gamma} \tan \Theta$ for various values of $m \geqslant 1$.

If $\bar{\gamma} \gg 1$ the width of this distribution is given by $\alpha \leqslant 3$ or

$$\Theta_T \lesssim 3/\bar{\gamma}$$

as shown below. A cone of angular half width $\Theta_T = 3/\bar{\gamma}$ will contain *all* the shower particles if $m \geqslant \sqrt{10}/3$ as follows directly from the cut-off angle in eq. (5c).

If $m \leqslant \sqrt{10}/3$ then we see from fig. 4a, that the point A must lie on or slightly outside the unit circle. The angle $\alpha = \tan \varepsilon \geqslant 3$ then corresponds to an angle ϑ in the C. system given by $\vartheta \geqslant 2\,\varepsilon$ or $\cos \vartheta \leqslant -0.8$. For an isotropic distribution in the C. system the contribution from the angles $2\,\varepsilon \leqslant \vartheta \leqslant \pi$ is given by

$$\int_{-1}^{-0.8} d\,(\cos \Theta) \Big/ \int_{-1}^{1} d\,(\cos \Theta) = {}^{1}\!/_{10}$$

Thus the angle $\Theta_T = 3/\bar{\gamma}$ will contain at least 90% of all shower particles *whatever the energy* $\bar{\gamma}_0$ with which the mesons are produced in the C. system.

If we further assume that the mesons are produced in the C. system with kinetic energies equal to or larger than their rest mass, such that

$$\bar{\gamma}_0^2 \gg 1 \text{ or } |\, m^2 - 1 \,| \approx \left| \frac{-1}{2\,\bar{\gamma}^2} + \frac{1}{2\,\bar{\gamma}_0^2} \right| \ll 1$$

we see from (11a) and (11b) that the angular distribution in the L. system is, for not too large α, given by

$$\alpha/(\alpha^2 + 1)^2$$

(see Fig. 5). Thus $N(\Theta)$ reaches a maximum value at

$$\Theta_m = \frac{1}{\sqrt{3}\,\bar{\gamma}} = \frac{0.58}{\bar{\gamma}}$$

and drops to one half its maximum value at

$$\Theta_{(\frac{1}{2})} = \frac{1.34}{\bar{\gamma}}$$

The energy of the incoming nucleon can therefore be obtained directly from the maximum or the half width of the meson distribution in the forward direction provided that both the incoming nucleon and the mesons in the C. system have relativistic velocities and the mesons have an isotropic angular distribution. Under these assumptions the total width of the shower can never be more than about twice the half width of the peak in the forward direction

$$\Theta_T \leqslant 2.2\, \Theta_{(\frac{1}{2})}$$

Since the total energy available for meson production is $2(\overline{\gamma}-1)K$ where K is the degree of inelasticity of the collision, the multiplicity of meson production is given by

$$N = \frac{2(\overline{\gamma}-1)K}{\mu\, \overline{\gamma}_0} \approx 9.2\, \gamma^{1/2} \frac{K}{\overline{\gamma}_0} \tag{12}$$

where γ is the energy of the incoming nucleon in the L. system in units of Mc^2.

Case II. The angular distribution obtained in Case I is not appreciably altered if we assume that the mesons are created in the C. system with a smoothly varying and not too extreme distribution in energy. For the discussion we have chosen as an example a spectrum proportional to $\overline{p}_0^2/\overline{E}_0^4\, d\overline{p}_0$ which is proportional to the phase space element at low energies and falls off as $\overline{E}_0^{-2}$ at high energies. Still assuming angular isotropy of meson production in the C. system, we have for the distribution function $F(\vartheta, \overline{\gamma}_0)$ of equation (9):

$$F = \frac{A}{4\pi} \frac{\sqrt{\overline{\gamma}_0^2-1}}{\overline{\gamma}_0^3}$$

independent of ϑ. The constant A and the multiplicity N are related by

$$A \int_1^{\overline{\Gamma}_0} \frac{d\overline{\gamma}_0 \sqrt{\overline{\gamma}_0^2-1}}{\overline{\gamma}_0^3} = N$$

and

$$A \int_1^{\overline{\Gamma}_0} \frac{d\overline{\gamma}_0 \sqrt{\overline{\gamma}_0^2-1}}{\overline{\gamma}_0^2} = \frac{2(\overline{\gamma}-1)K}{\mu} = N_m$$

where the upper limit of the integrals is given by the highest energy any meson can obtain i. e. $\overline{\Gamma}_0 = N_m - N + 1$

Solving for $\overline{\Gamma}_0$ one obtains

$$\overline{\Gamma}_0 - 1 = \frac{N_m}{2} \left[\frac{\arctan \sqrt{\overline{\Gamma}_0^2-1} - \sqrt{\overline{\Gamma}_0^2-1}/\overline{\Gamma}_0^2}{\operatorname{arcosh} \overline{\Gamma}_0 - \sqrt{\overline{\Gamma}_0^2-1}/\overline{\Gamma}_0} \right] \tag{13a}$$

which for large values of $\overline{\Gamma}_0$ reduces to

$$\overline{\Gamma}_0 = N_m \left[1 + \frac{\pi/4}{1 - 2\log \overline{\Gamma}_0} \right] \tag{13b}$$

In Fig. 6 we have plotted the relation between the multiplicity N and the energy of the incoming nucleon γ as obtained from 1 and 13a. The curve 2 is well represented by

$$N = 3.7\,(K^2\,\gamma)^{0.32} \qquad N \lesssim 10$$
$$N = 2.9\,(K^2\,\gamma)^{0.42} \qquad N \geqslant 10 \tag{14}$$

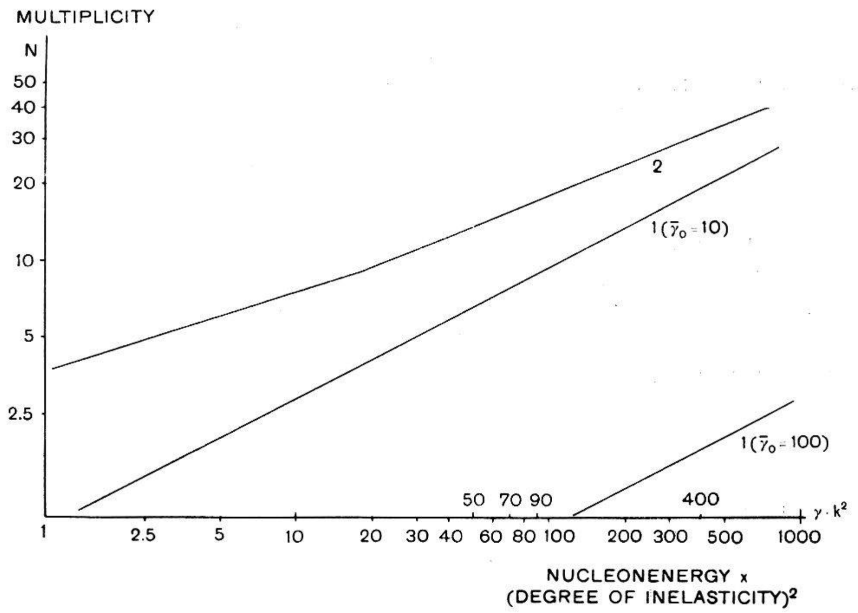

Fig. 6.
Multiplicity of meson production.

The multiplicity N is plotted as function of the product of the energy of the incident nucleon γ times the square of the degree of inelasticity K. Curve 1 gives the multiplicity for unique meson energies $\bar\gamma_0 = 10$ and $\bar\gamma_0 = 100$ in the C. system (case I). Curve 2 gives the multiplicity for the energy spectrum of case II.

Using eq. 9 and 10 one obtains for the angular distribution (with a $= \cos\Theta\,\sqrt{\alpha^2+1}$)

$$N(\Theta) = \frac{A\,\bar\gamma}{\cos^3\Theta}\,\frac{\alpha}{(\alpha^2+1)^2}\left[\int_a^{\bar\gamma}\left\{\bar\beta^2\cos^2\Theta + 1 - \frac{a^2}{\bar\gamma_0^2}\right\}\frac{d\bar\gamma_0}{\bar\gamma_0\sqrt{\bar\gamma_0^2-a^2}}\right.$$
$$\left.+\frac{1}{2}\int_{\bar\gamma}^{\bar\Gamma_0}\left\{\bar\beta^2\cos^2\Theta + 1 - \frac{a^2}{\bar\gamma_0^2}\right\}\frac{d\bar\gamma_0}{\bar\gamma_0\sqrt{\bar\gamma_0^2-a^2}} + \int_{\bar\gamma}^{\bar\Gamma_0}\frac{\bar\beta\cos\Theta}{\bar\gamma_0^2}\,d\bar\gamma_0\right]$$

which on integrating becomes

$$N(\Theta) = \frac{A\,\bar\gamma\,(2\,\bar\beta^2\cos^2\Theta + 1)}{4\cos^4\Theta}\,\frac{\alpha}{(\alpha^2+1)^{5/2}}\left[\operatorname{arcos}\frac{\bar\gamma}{a} + \operatorname{arcos}\frac{\bar\Gamma_0}{a}\right]$$
$$+\frac{A\,\bar\gamma}{2\cos^3\Theta}\,\frac{\alpha}{(\alpha^2+1)^2}\left[\frac{\bar\Gamma_0-\bar\gamma}{\bar\Gamma_0\,\bar\gamma}\,\bar\beta\cos\Theta - \frac{\sqrt{\bar\Gamma_0^2-a^2}}{\bar\Gamma_0^2}\right] \tag{15a}$$

The second term in $N(\Theta)$ is smaller than the first by a factor of order $1/\overline{\gamma}$ or $1/\overline{T}_0$. We therefore may neglect it in discussing the angular distribution. If we furthermore confine ourselves to values of $\alpha \leqslant 3$ which as shown above account for at least 90% of all mesons, then (since $\overline{\gamma} \gg 1$ and $\overline{T}_0 \gg 1$) we can replace each arcos by $\pi/2$. Thus

$$N(\Theta) = \frac{A\pi}{4}\overline{\gamma}\,\frac{2\cos^2\Theta+1}{\cos^4\Theta}\,\frac{\alpha}{(\alpha^2+1)^{5/2}} \tag{15b}$$

This distribution has a maximum at

$$\Theta_m = \frac{0.5}{\overline{\gamma}} \tag{15c}$$

and drops to half maximum intensity at

$$\Theta_{(\frac{1}{2})} = \frac{1.8}{\overline{\gamma}} \tag{15c}$$

The total width is again

$$\Theta_T \leqslant \frac{3}{\overline{\gamma}}$$

Thus both the assumption of a single meson energy $\overline{\gamma}_0 > 2$ and the assumption of a meson energy spectrum proportional to $\overline{p}_0^2\,d\overline{p}_0/\overline{E}_0^4$ lead to very similar angular distributions in the L. system. The primary energy can be uniquely determined from the shape of the angular distribution in the L. system in either case. In Fig. 12 a shower is illustrated (which will be discussed in detail in Section IV and V) which exhibits a narrow core confined to $\pm\,1.5^0$ and a wide shower of comparable intensity. The results derived above prove that such a combination of a narrow and wide meson shower cannot be produced in a single nucleon-nucleon collision, if the mesons are produced isotropically in the C. system (Fig. 5). The high multiplicity and the small width of the core show that $\overline{\gamma} \gg 1$ (actually $\overline{\gamma} = 20$ to 30). Since the wide shower has an angular spread much larger than $3/\overline{\gamma}$, it cannot represent mesons produced isotropically in the C. system in a single nucleon-nucleon collision.

Case III. We shall now investigate the effect of anisotropic meson production in the C. system on the angular distribution in the L. system. In order to obtain in a single nucleon-nucleon collision a narrow core and a wide shower of comparable intensity an angular anisotropy of meson production in the C. system is actually required.

Let us assume an angular distribution proportional to $\cos^{2n}\vartheta$ $(d\cos\vartheta)$. Particles which lie outside a narrow cone defined by $\tan\varepsilon = \overline{\gamma}\tan\Theta = \alpha > 3$ can only come from C. system angles $\cos\vartheta < \cos 2\varepsilon < -0.8$ or $\vartheta > 143^0$; while those outside the cone $\overline{\gamma}\tan\Theta > 6$

can only come from C. system angles $\cos\vartheta < -0.9397$ or $\vartheta > 160°$. For example we need $n > 2$ in order to have $\bar{\gamma}\tan\Theta > 3$, and $n > 9$ in order to have $\bar{\gamma}\tan\Theta > 6$ for 1/3 of all the mesons.

If we assume that all mesons are produced with equal energies

$$F(\vartheta, \bar{\gamma}_0) = \frac{2n+1}{4\pi} N \cos^{2n}\vartheta\, \delta(\bar{\gamma}_0' - \bar{\gamma}_0).$$

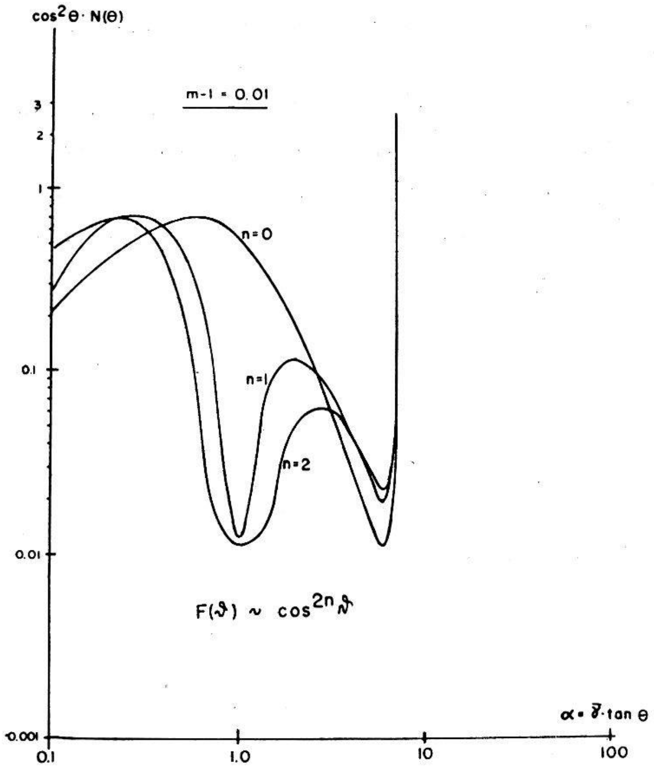

Fig. 7.

Angular distribution functions for case III. (Uniform meson energy and angular distribution proportional to $\cos^{2n}\vartheta\, d\cos\vartheta$ in the C. system). $\cos^2\Theta\, N(\Theta)$ is plotted for $m - 1 = \bar{\beta}/\bar{\beta}_0 - 1 = 0.01$ and various degrees of anisotropy ($n = 0, 1, 2$) versus $\alpha = \bar{\gamma}\tan\Theta$.

Making use of (8), inserting this expression into eq. (9) and dropping the prime symbol one obtains

$$N(\Theta) = \frac{2n+1}{2}\frac{N\bar{\gamma}}{\cos^2\Theta}\frac{\alpha}{(\alpha^2+1)^{2n+2}}\left[\frac{\left(m+\sqrt{1-(m^2-1)\alpha^2}\right)^2\left(-m\alpha^2+\sqrt{1-(m^2-1)\alpha^2}\right)^{2n}}{\sqrt{1-(m^2-1)\alpha^2}}\right.$$

$$\left.+\frac{\left(m-\sqrt{1-(m^2-1)\alpha^2}\right)^2\left(-m\alpha^2-\sqrt{1-(m^2-1)\alpha^2}\right)^{2n}}{\sqrt{1-(m^2-1)\alpha^2}}\right] \text{ for } m \geqslant 1 \quad (16a)$$

$$N(\Theta) = \frac{2n+1}{2}\frac{N\bar{\gamma}}{\cos^2\Theta}\frac{\alpha}{(\alpha^2+1)^{2n+2}}\times$$

$$\times\left[\frac{\left(m+\sqrt{1-(m^2-1)\alpha^2}\right)^2\left(-m\alpha^2+\sqrt{1-(m^2-1)\alpha^2}\right)^{2n}}{\sqrt{1-(m^2-1)\alpha^2}}\right] \text{ for } m \leqslant 1 \quad (16b)$$

If $\overline{\gamma}_0 \gg 1$ $(m \approx 1)$ one obtains for small values of Θ

$$N(\Theta) = 2(2n+1) \frac{N\overline{\gamma}}{\cos^2 \Theta} \frac{\alpha(\alpha^2-1)^{2n}}{(\alpha^2+1)^{2n+2}} \tag{16c}$$

In Fig. 7 the angular distribution of eq. (16) has been plotted for $m = 1.01$ and $n = 0$, 1 and 2. A strong intensity peak in the forward exists for all values of $m < 1.1$ $(\overline{\gamma}_0 > 3)$. Fig. 7 shows that a $\cos^{2n}\vartheta$ — angular distribution in the C. system reduces appreciably the width of the forward peak of the angular distribution function in the L. system plotted against $\alpha = \overline{\gamma} \tan \Theta$ as compared to an isotropic distribution. For a given half width $\Theta_{(\frac{1}{2})}$ of the shower, the corresponding value of $\overline{\gamma}$ is the smaller, the larger the exponent n. For mesons created with relativistic energies in the C. system but with different degrees of anisotropy the angles Θ_m and $\Theta_{(\frac{1}{2})}$ are given with good accuracy for $n \geqslant 1$, by

$$\Theta_m = \frac{0.5}{\overline{\gamma}\sqrt{2n+1}} \tag{17a}$$

and

$$\Theta_{(\frac{1}{2})} = \frac{0.96}{\overline{\gamma}\sqrt{2n+1}} \tag{18a}$$

while for $n = 0$ the values are

$$\Theta_m = 0.58/\overline{\gamma} \tag{17b}$$

and

$$\Theta_{(\frac{1}{2})} = 1.34/\overline{\gamma} \tag{18b}$$

For instance for a narrow shower of given width a $\cos^4 \vartheta$ angular distribution in the C. system will lead to a value of $\overline{\gamma}$ which is only 1/3 of that obtained for isotropic distribution. The estimate of the energy of the incoming nucleon is therefore reduced by a factor 9 compared with that obtained for an isotropic distribution.

However in an actual shower where the meson energy in the L. system can be estimated as outlined at the beginning of this section, it is not possible to assume an arbitrary high anisotropy.

Let $\langle\gamma_0\rangle$ be the estimated average energy of mesons in the shower core. According to (4) their average energy $\langle\overline{\gamma}_0\rangle$ in the C. system is

$$\langle \overline{\gamma}_0 \rangle = \langle \gamma_0 \rangle / \overline{\gamma}$$

Assuming an anisotropy of the form $\cos^{2n}\vartheta d \cos \vartheta$ we have from (18a) the energy of the incoming nucleus in the C. system

$$\overline{\gamma} = \frac{0.96}{\Theta_{(\frac{1}{2})}\sqrt{2n+1}} \quad (n \geqslant 1)$$

The multiplicity becomes

$$N^{(n)} = \frac{2\,K}{\mu}\frac{(\overline{\gamma}-1)}{\langle\overline{\gamma}_0\rangle} \approx \frac{2\,K\,\overline{\gamma}^2}{\mu\,\langle\gamma_0\rangle} = \frac{12\,K}{\langle\gamma_0\rangle\,\Theta^2_{(\frac{1}{2})}\,(2\,n+1)} \tag{19}$$

Thus for a given multiplicity and energy of the observed mesons and for a given half width of the shower core, an increase in the assumed anisotropy requires an increasing degree of inelasticity. The degree of anisotropy which can be assumed is therefore limited by the condition $K \leqq 1$. An example is discussed in detail in Section V.

Angular distribution of decay γ-rays from neutral mesons.

The angular distribution of the two decay γ-rays emitted isotropically in its rest system by a neutral meson of energy γ_0 can be obtained directly from eq. 11b

$$N_\gamma(\Theta) = \frac{N\,\gamma_0}{2\,|\cos^3\Theta|}\,\frac{\alpha}{(1+\alpha^2)^2}\,(1+\beta_0\cos\Theta)^2 \tag{20}$$

The average energy of the γ-rays in the L. system in units of μc^2 is given by

$$\langle E\rangle_\gamma = E_0\,\gamma_0 = \frac{\gamma_0}{2} \tag{21}$$

and, as in case 1, 90% of all γ-rays are contained within a cone of half angle

$$\Theta_T = 3/\gamma_0 \tag{22}$$

III. Electron Pair Production by γ-Rays and Fast Electrons.

In electronsensitive photographic emulsion exposed at high altitudes we observe not infrequently pairs of minimum ionization tracks diverging under a small angle from the point of origin in the emulsion (Fig. 8). The angle of divergence is sometimes so small that the two tracks are not resolved near the point of origin and for the first few 100—1000 μ the pair appears as a single track originating in the emulsion with a grain density corresponding to a specific energy lose $K = 2\,K_{\text{min}}$.

We consider these paired tracks to be tracks of electron pairs produced in the emulsion by high energy γ-rays or possibly by other neutral particles. This interpretation is supported by the fact that multiplication associated with paired tracks has been observed. Since the radiation unit in emulsion ($\lambda_0 = 3.0$ cm) is very large as compared to the average track length, we cannot expect to observe in the emulsion, as one can with a cloud chamber, an appreciable cascade multiplication initiated by a single particle or quantum. Nevertheless, two events showing multiplication have been observed. One energetic pair produced by a $\sim$ 10 Bev γ-ray of the shower

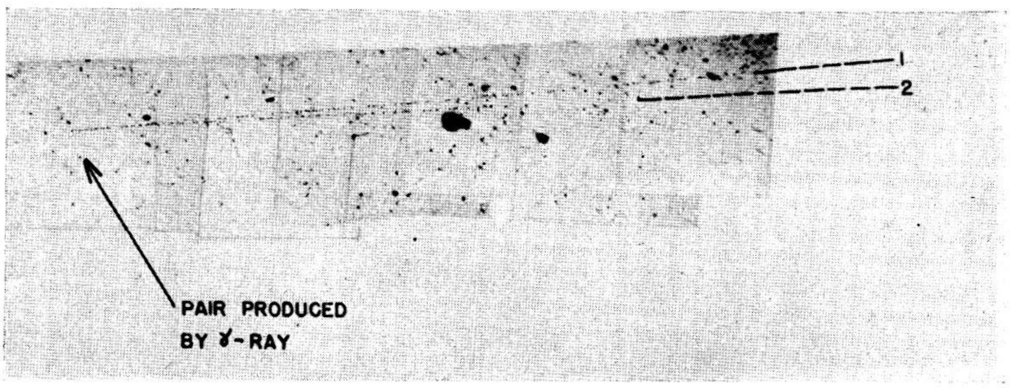

Fig. 8.
A pair of singly charged relativistic particles (presumably electron-positron pair created by a γ-ray) originating in the emulsion.

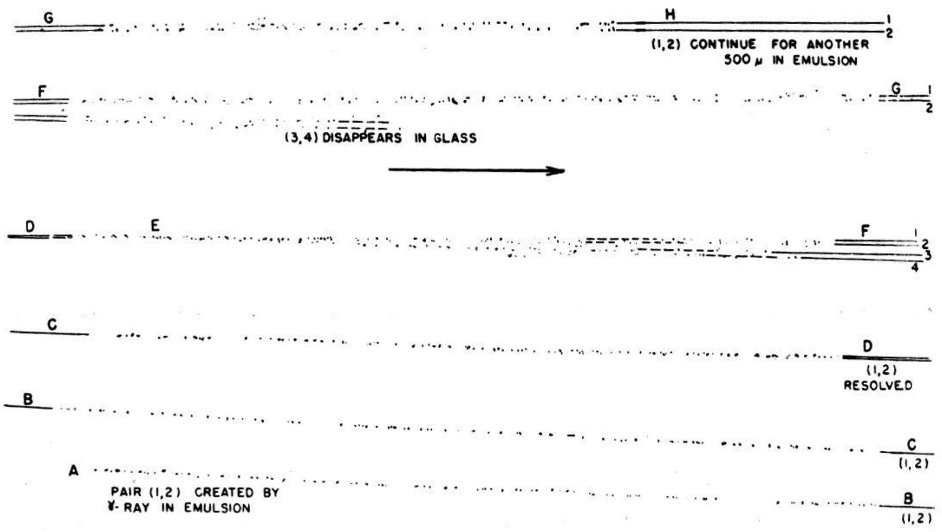

Fig. 9.
Multiplication of charged particles in the emulsion. An electron pair (1.2) originating at point A, the tracks of which are resolved after about 800 μ at point D, gives rise to another pair 3.4. All four tracks are resolved at F. The energy of the pair 1.2 is estimated from the angular divergence as 10 Bev.

to be discussed in section IV gives rise to another pair (Fig. 9) at a distance of 800 μ from the point of origin of the first pair in the emulsion. This second pair may be the result of pair production by a γ-ray which either was accompanying the pair or was created by one member of the pair as brems-radiation in the usual multiplication process of the soft component.

The direct production of an electron pair by a fast electron, a process which also provides a possible explanation of this event, seems to be the most probable interpretation of the pair creation shown in fig. 10.

Here an incident relativistic particle, presumable an electron, produces a pair (2,3) at A after having traversed 105 μ of emulsion. At A the grain density changes abruptly from that corresponding to

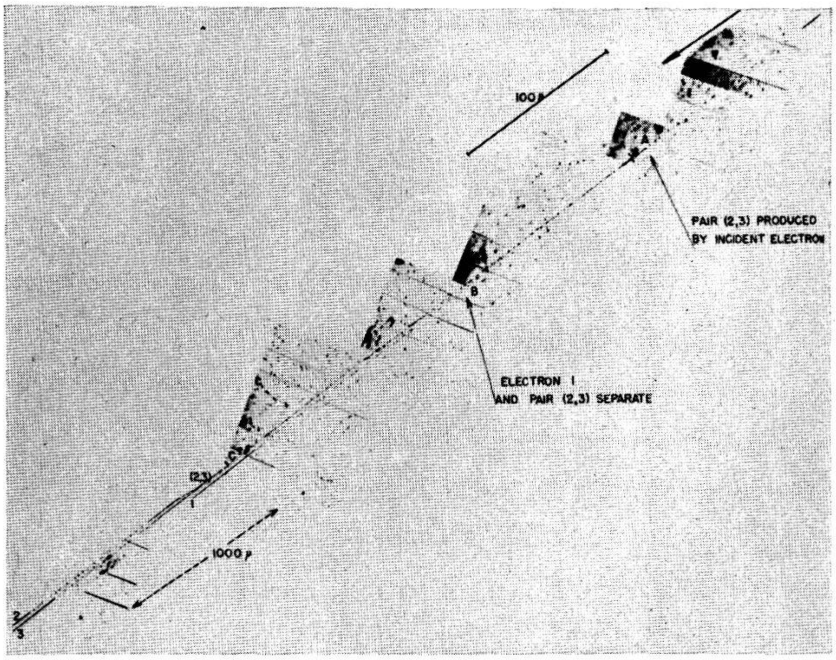

Fig. 10.
Pair Production by a Fast Electron.
An incident relativistic particle 1, presumably an electron produced a pair (2,3) at A after having traversed 105 μ in the emulsion. At A the grain density changes abruptly from that corresponding to minimum ionization $K_{\min}$ to that corresponding to $K = 3\ K_{\min}$. At point B the track with $K = 3\ K_{\min}$ has split into a minimum ionization track 1 and a track with $K = 2\ K_{\min}$. After point C this latter can be resolved into two minimum ionization tracks 2 and 3. Tracks 2 and 3 are separated by 2 μ when they leave the emulsion, 1250 μ from point A, the origin of the pair. The energy of pair (2,3) is estimated to be $E_{\text{pair}} \sim 5$ Bev.

minimum ionization $K_{\min}$ to that corresponding to three times minimum ionization $K = 3\ K_{\min}$. At point B the track has split into a minimum ionization track (1) and a track with $K = 2\ K_{\min}$. After point C this latter can be resolved into two minimum ionization tracks (2) and (3). Tracks (2) and (3) are separated by 2 μ when they leave the emulsion, 1250 μ from point A, the origin of the pair, a separation which corresponds to an energy of 5 Bev (see below).

If we assume that the pair (2, 3) has not been directly created by electron (1), then the fact that the point of origin of the pair coincides as closely as one can judge (that is within 0.5 μ) with the track (1) would be purely accidental.

The probability for such a chance coincidence to occur is quite small since, assuming the angle between the brems-radiation quantum and the electron (1) to be not smaller than $^1/_5$ of the angle of divergence of the pair, such a coincidence would imply that the quantum was emitted not more than 1,5 mm before it was converted into an electron pair. This distance corresponds to $\sim 0{\cdot}01$ radiation units, since the electron entered the emulsion coming from glass 105μ before point A (radiation unit in glass $\lambda_0 = 12.2$ cm).

At low energies ($E \geqslant 2\ \mathrm{mc}^2$) the ratio of pair production cross sections by electrons and γ-rays respectively is theoretically expected to be quite small (of the order of 1/137), in agreement with experiment (J. R. Feldmeier and G. B. Collins[5]); H. L. Bradt[6]); K. Siegbahn[7]).

The theoretical cross section for pair production by relativistic electrons (Heitler[8]), p. 203) increases more rapidly with energy than the cross section for pair production by γ-rays, and at energies of the order of a few Bev the theoretical probability of pair production by an electron is only about one order of magnitude less than the probability of pair production by a photon. We consider it therefore as very probable that Fig. 10 illustrates the creation of an electron pair by a high energy (~ 5 Bev) electron.

Estimate of γ-ray energies.

From the separation d of the two tracks of a pair created by a γ-ray at a distance L from the point of origin (Fig. 11) we can obtain an estimate of the energy $k = h\nu$ of the γ-ray.

This separation of the tracks is determined by two factors:

a) the *initial divergence* of the members of the pair;
b) *multiple scattering* of the electrons over the distance L.

a) *Initial divergence of the members of the pair.*

M. Stearns[9]) has calculated the root mean square average angle Θ_- between the direction of the electron of energy E_- and the direction of the photon of energy k which creates the pair in the field of a nucleus of charge Z:

$$\Theta_- = \frac{m\,c^2}{k} \log \frac{k}{m\,c^2} \cdot f_z\left(\frac{E_-}{k}\right) \tag{23}$$

The function $f_z\left(\frac{E_-}{k}\right)$, plotted in Fig. 5 of Stearns paper does not strongly depend on Z and, for $Z \geqslant 30$ is very nearly equal to unity for $E_- = E_+ = \frac{k}{2}$. Curve 1 of Fig. 11 shows the average angle $\Theta \approx \Theta_+ + \Theta_- \approx 2\,\Theta_-$ between the members of the pair plotted versus k for equipartition of energy between electron and positron. If the positron and electron of the pair of Fig. 10 for instance have equal energies and if scattering is neglected, the most probable energy of the pair is derived from the angle of divergence $\Theta = \frac{2\,\mu}{1250\,\mu} = 1.6 \cdot 10^{-3}$ to $k = 5.5$ Bev. The average angle $\langle \Theta_- \rangle$ between the

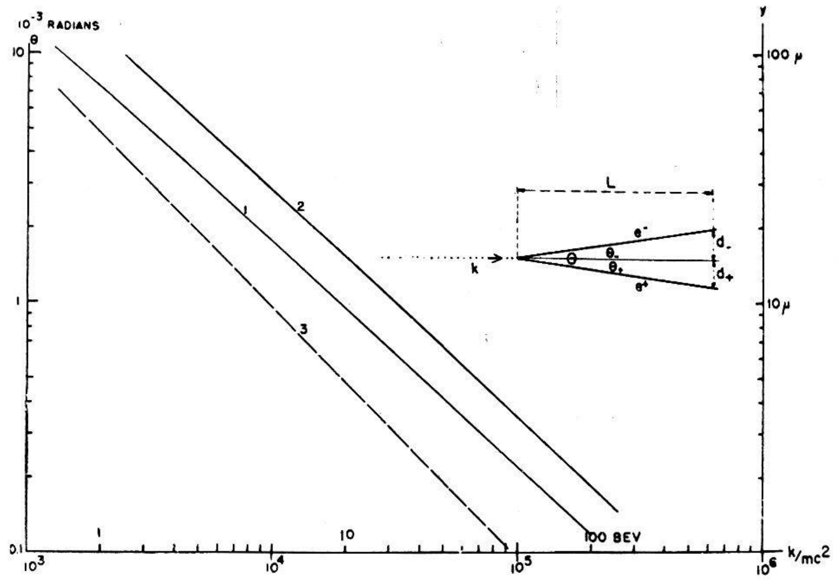

Fig. 11.

Average angle Θ of divergence between electron and positron created by a γ-ray as function of the γ-ray energy k/mc^2.
Curve 1 represents the divergence for equipartition of energy between the members of the pair.
Curve 2 represents the average divergence to be expected for a given energy k/mc^2.
Curve 3 (right hand scale) represents the average separation due to multiple scattering of the tracks of pairs produced in 2 cm glass after emergence from the glass.

direction of the electron and the direction of the photon averaged over the energy distribution of the electron (Heitler[8]) p. 199), is ~ 1.6 times greater than the value corresponding to equipartition of energy between electron and positron. $\langle \Theta \rangle = 2\,\langle \Theta_- \rangle$ is plotted as a function of k/mc^2 in curve 2 of Fig. 11.

b) *Multiple scattering.*

The evaluation of the track separation due to multiple Coulomb scattering is based on a recent paper by Snyder and Scott[10]), to which the following notations refer.

The projected scattering angle ϑ is measured in units of

$$\eta_0 = \frac{Z^{(\frac{1}{3})}}{137\sqrt{E^2-1}} = \begin{cases} 1.25\cdot 10^{-5}/E \text{ for emulsion} \\ 0.8 \ \cdot 10^{-5}/E \text{ for glass} \\ (E\ (\gg 1) \text{ in Bev}) \end{cases}$$

Distances are measured in units of the mean free path for scattering λ, defined by $1/\lambda = 1.85\cdot 10^{-20}\cdot \Sigma n_i Z_i^{(4/3)}$ (n_i, Z_i = number per cm^3 and charge of atoms of kind i is respectively)

$$\lambda = \begin{cases} 0.157\ \mu \text{ for emulsion} \\ 0.335\ \mu \text{ for glass} \end{cases}$$

With these units the average projected scattering angle $\eta = \vartheta/\eta_0$ is given by

$$\frac{\langle\eta\rangle}{\sqrt{Z}} \approx 2.5$$

The track separation (y) at a distance (z) due to multiple scattering is given by

$$\langle y^2\rangle = \frac{2}{3} z^2 \langle \eta^2\rangle$$

or

$$y = \sqrt{\langle y^2\rangle} = \sqrt{\frac{2}{3}}\cdot 2.5\, z^{3/2}\,\eta_0 = 2\,\eta_0\, z^{3/2}$$

Hence

$$y_{\text{scatt.}} = \begin{cases} 2.5 \times 10^{-5}\, z^{3/2}/E & \text{for emulsion (in units } \lambda e) \\ 1.6 \times 10^{-5}\, z^{3/2}/E & \text{for glass (in units } \lambda_g) \end{cases} \qquad (24)$$

As an example Fig. 11 (curve 3) shows the separation of the tracks of an average electron pair produced by γ-rays in a layer of 2 cm glass due to multiple scattering after traversing the glass.

An energy estimate of the γ-ray should then be based on the separation of the pair tracks at a given distance from their origin, using the angular divergence or the separation by scattering which ever is larger. For the examples to be discussed in section V, the initial divergence is more important than multiple scattering.

IV. Description of a Shower Showing Both Multiple Meson and γ-Ray Production (R-Shower[11]).

In a survey of Kodak NTB3 plates flown at a geomagnetic latitude of 51^0 at an altitude of 100000 feet we have observed a collision of an extremely energetic primary cosmic ray alpha particle with a heavy nucleus of the emulsion (Ag or Br): the collision gives rise

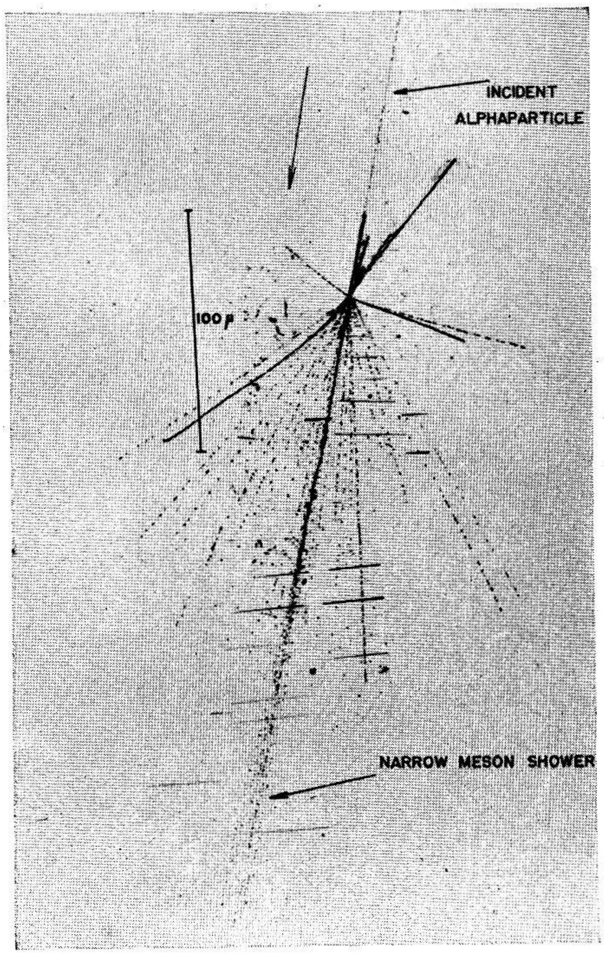

Fig. 12.
Photomicrograph (incomplete) of the R-shower. Only a small fraction of the 18 heavy prongs are shown. The narrow core of the shower contains 23 ± 2 relativistic particles, the diffuse shower contains 33 relativistic tracks.

to a very narrow penetrating shower of 23 relativistic singly charged particles together with a more diffuse shower of 33 relativistic particles. This star consists of 74 prongs (Fig. 12) representing a minimum of 81 units of charge of which about 56 correspond to minimum ionization for singly charged particles. These constitute the shower part of the star.

Incident α-particle.

The star is initiated by an alpha particle whose energy loss is 7.7 MeV/(gr/cm^2). As minimum ionization for an alpha particle corresponds to an energy loss of 6.7 MeV/(gr/cm^2) this particle has an energy loss 15% above its minimum. Since the event initiated by this particle represents a release of a great amount of energy, one is justified in ascribing this difference, if it is real*), to the increase of energy loss beyond the minimum on the energy loss curve.

Non-Relativistic Portion of Star.

There are 18 non-relativistic heavy particles (i. e. protons or heavier particles) emitted in the nuclear explosion, corresponding to at least 23 units of charge. One prong has a grain density corresponding to an energy loss 12% above minimum for a singly charged particle and another has one corresponding to an alpha particle with an energy loss of 8.6 MeV/(gr/cm^2) (constant loss of 8.6 MeV/(gr/cm^2) over a distance of 1.5 cm, in which distance any singly charged particle would appreciably increase its energy loss). We estimate for the energy in the non-shower part of the star a value of 2.8 Bev. (apart from the fast secondary alpha particle with an energy of some 2.5 Bev.). Since the collision leads to the emission of 79 units of charge (of either sign) we have, on the assumption that all are positive, an excess of 30 units of charge over the atomic number of Ag(^{47}), the heaviest abundant nucleus of the emulsion. We thus conclude that at least one-half of our 56 shower particles cannot possibly be protons of the target nucleus; they are most probably mesons.

Shower Portion of Star.

I. *Narrow core.*

The dense core (C) of the shower of relativistic particles, the majority of which are assumed to be mesons, has a total projected angular spread of 2.5^0. The axis of this core is an exact continuation of the direction of the incident α-particle. The core is so dense that not all individual tracks are resolvable in the immediate vicinity of the star. At larger distances where resolution is possible all these tracks prove to be minimum ionization tracks. By counting over a given distance the number of grains in the core and dividing by the number of grains of a minimum ionization track we obtain

*) It should be remarked, however, that no relativistic increase of grain density in emulsion has been observed for electrons up to energies of $E/mc^2 \sim 10^4$.

$N_{ch}^{(0)} = 23 \pm 2$ as estimate of the number of charged relativistic particles in the narrow shower.

The region where the narrow shower C, after passage through some 2 cm of glass penetrates the next plate of the stack (the dotted area C of 0.25 cm^2 in Fig. 13) was surveyed for tracks of such length and orientation that they can be assumed to be caused by the explosion; 44 tracks of relativistic particles were found, 38 inside and 6 near the edge of the dotted cone C, that is 21 more tracks than were originally contained in the narrow shower. In surveying an additional area $A = 0.29$ cm^2 for tracks satisfying identical criteria only 6 more tracks were found, all but one lying very close to the core and probably also belonging to the shower. We conclude that

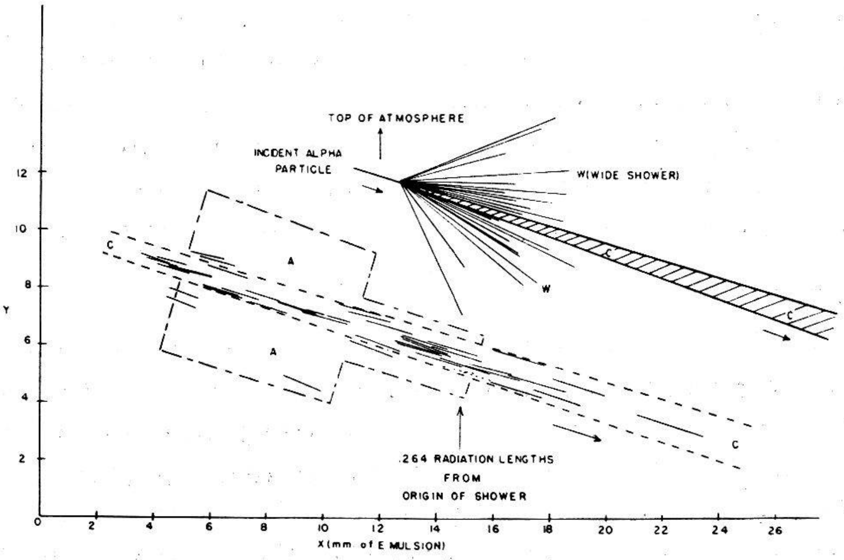

Fig. 13.

Schematic diagram of the appearance of the shower in the first and second emulsions. (The dotted lines above the vertical arrow refer to the event of Fig. 9). The area of the core C and the control area A where completely surveyed for tracks of such orientation that they could have been associated with the explosion.

considerable multiplication of singly charged relativistic particles (electrons) has occurred in the core. Two electron pairs, one with an energy of ~ 10 Bev (pair no. 2, event of Fig. 9) and one with an energy of ~ 50 Bev (pair no. 1) were created in the emulsions of the second and the first plate. One of the pairs within the core (pair no. 2) gives rise to another pair; this fact supports the assumption that the additional charged particles appearing in the narrow core are fast electrons produced by high energy γ-rays (or possibly other neutral particles).

As the core of the shower is not well-defined (due to scattering and the opening angles of the pairs created by the γ-rays) the correction to be applied from the control area A is somewhat indeterminate. Taking 3 as the number of random tracks to be expected in the area C we find that the most probable value for the number of pairs created is 8 ± 2. As these are created in the core over an average path length of 0.26 radiation units of glass and emulsion, the number of gamma rays in the core must therefore at least be equal to $N \sim 35$.

Pair Productions in the Core.

Pair no. 1.

In the first plate (the plate in which the star occurs) we observe a track on the outer fringe of the core C, that originates in the emulsion (approximately 500 μ from the star), has the direction of the core and corresponds to exactly twice minimum ionization over a length of approximately 2000 μ of emulsion. This track is resolvable after 2000 μ into two minimum ionization tracks (projected separation $d = 0.5$ μ after a track length of 2390 μ) and can therefore be regarded as the track of an electron pair. According to figure 11 this separation corresponds to an energy of $\sim$ 50 Bev.

Pair no. 2.

In the dotted cone another pair (Fig. 9) is observed to originate in the emulsion; the position of the pair is indicated by the dotted lines in fig. 13. After a track length of 700 μ of emulsion (at which point the projected separation is 0.5 μ) another pair (3.4) is created along the path of the original pair (1.2). The latter pair has a projected separation of 1 μ after a path of 345 μ in the emulsion and the mean projected separation between (12) and (34) (taking the origin at the point of creation of the second pair) is 5 μ in a path length of 345 μ. The original pair (12) leaves the emulsion 720 μ from the point at which (34) was created. The energy of the original pair (12) is estimated to be $\sim$ 10 Bev.

Although the increase in the number of tracks from the first to the second plate shows that at least six pairs have been created in the 2 cm of glass between the emulsions, the tracks of these pairs have already separated in the second emulsion sufficiently such that (except in one case where the separation is only 2.5 μ) no unique correlation between the associated tracks is apparent. The six smallest distances between possibly associated tracks are 2.5 μ, 9 μ, 18 μ, 19μ, 23 μ, and 39 μ. Hence the average separation between the tracks of the pairs created in the glass must be at least 15—20 μ. According to curve 2, Fig. 11 the average energy of these pairs can therefore not greatly exceed $E_\gamma \sim 10$ Bev.

Angular Distribution of the mesons.

In fig. 17a, b the angular distribution of the relativistic tracks in the narrow core C (excluding those correlated as pairs) is plotted in the histogram, the direction of the incoming alpha-particle being taken as the polar axis. This distribution was determined from the orientation of the tracks in C of Fig. 13 by assuming that each track originated directly from the star. Due to scattering the distribution curve obtained in this way is somewhat broader than the true angular distribution.

II. *Diffuse Shower.*

The grain density was measured for all the tracks in the diffuse shower. The maximum deviation of any of these 32 tracks from minimum ionization is $\pm$ 10% (minimum ionization for singly charged particles is 22 grains/100 μ). The maximum polar angle is found to be $\sim 60^0$. One third of all the tracks in the diffuse shower are contained between cones of opening $\Theta = 2^0$ and $\Theta = 6^0$.

The histogram of Fig. 16 represents the overall angular distribution of the shower (diffuse + narrow core).

V. Discussion of the R-Shower.

The main conclusions concerning the shower described in the previous section are:

(1) Production of both a well defined core and a diffuse shower of comparable intensity.

(2) a) Multiple production of gamma rays.
b) Multiplication of charged particles (electrons) in the core.

We will consider two possible modes of production of the narrow shower and the diffuse shower.

A. The narrow core C and the diffuse shower were produced simultaneously in the initial encounter between the nucleons of the incoming alpha particle and the nucleons of the target nucleus.

B. Only the narrow core C was produced in the initial encounter and the diffuse shower was produced in secondary and tertiary collisions of the nucleons of the incoming alpha-particle and the target nucleons.

We will assume, in view of the recent experiments at BERKELEY[4]), that the gamma rays arise from the decay of a neutral meson, π^0, whose rest mass is $\approx 300\, m_e$, the observed multiplication in the core then being due to the pair-production by these gamma-rays.

We consider first the information obtained from the observed multiplication of tracks in the narrow core C.

(i) *Estimate of Meson Energies from the Analysis of Electron Pairs.*

With the assumption concerning the origin of the gamma-rays stated above, we can use the information obtained from the electron pairs in order to estimate the average energy of the neutral meson, which we assume to be equal to the average energy $\langle\gamma_0\rangle$ of the charged mesons.

The result of the survey of area A (Fig. 13) shows that the γ-rays are essentially contained within a cone of opening $\sim 2.5^0$. This fact, according to eq. (21) provides a lower limit for the meson energy:

$$\langle\gamma_0\rangle \gtrsim 35$$

or

$$\langle E_\pi\rangle \gtrsim 5 \text{ Bev}$$

An upper limit for the energy of $\langle E_\pi\rangle \sim 20$ Bev results from the estimate of the γ-ray energy $E_\gamma \leqslant 10$ Bev arrived at in section IV. (The pair of tracks separated by 2.5 μ in the second emulsion may be ascribed to a pair produced in the glass with an energy possibly as high as ~ 50 Bev.)

Hence $\langle E_\pi\rangle = 15$—$20$ Bev, or $\langle\gamma_0\rangle \sim 120$ seems to be a reasonable estimate for the average energy of the mesons in the core C.

(ii) *Estimate of the Lifetime of the Neutral Meson.*

If the γ-rays result from the decay of a neutral meson of rest mass $\approx 300\ m_e$ we can, by assuming that the γ-rays are converted immediately into electron pairs, obtain an upper limit for the proper lifetime of the neutral meson. The appearance of pairs after a path length of ~ 2 cm shows that at least an appreciable fraction of the neutral mesons must have decayed within a time interval of $\sim 10^{-10}$ sec in the lab. system or an interval $\sim 10^{-12}$ sec in the rest system of the meson ($\langle\gamma_0\rangle \sim 120$). Hence $\tau_0 \lesssim 10^{-12}$ sec. A better estimate results from the following consideration:

Let $N_\pi(t)$ be the number of neutral mesons in the narrow core at a time t after the collision occurred, $N_\gamma(t)$ the number of γ-rays. Then

$$\frac{d N_\pi}{d t} = -\lambda_\pi N_\pi \tag{25}$$

$$\frac{d N_\gamma}{d t} = m \lambda_\pi N_\pi - \lambda_\gamma N_\gamma \tag{26}$$

where λ_π is the decay constant of the neutral meson, λ_γ the pair conversion probability per unit time of the γ-ray and m ($1 \leqq m \lesssim 2$)

the number of γ-rays arising from the decay of the neutral meson and contained with the core C.

Since we estimated $\langle\gamma_0\rangle \sim 120$, 90% of the γ-rays, assumed to be emitted in pairs isotropically in the rest system of the meson, are contained within a cone of opening angle $\Theta \sim \frac{3}{\langle\gamma_0\rangle} \sim 1\frac{1}{2}^0$ (see eq. (22)), that is well within the narrow core. Hence we have $m \approx 2$.

Solving (25) and (26) with the initial conditions $N_\gamma(0) = 0$, $N_\pi(0) = N_0$ we obtain for the number of electron pairs P:

$$P = m\,[N_0 - N_\pi] - N_\gamma = m\,N_0 \left[1 + \frac{\lambda_\pi e^{-\lambda_\gamma t} - \lambda_\gamma e^{-\lambda_\pi t}}{\lambda_\gamma - \lambda_\pi}\right]$$

$$\frac{P(z)}{m\,N_0} = 1 + \frac{e^{-z} - x\,e^{-z/x}}{x-1} \tag{27}$$

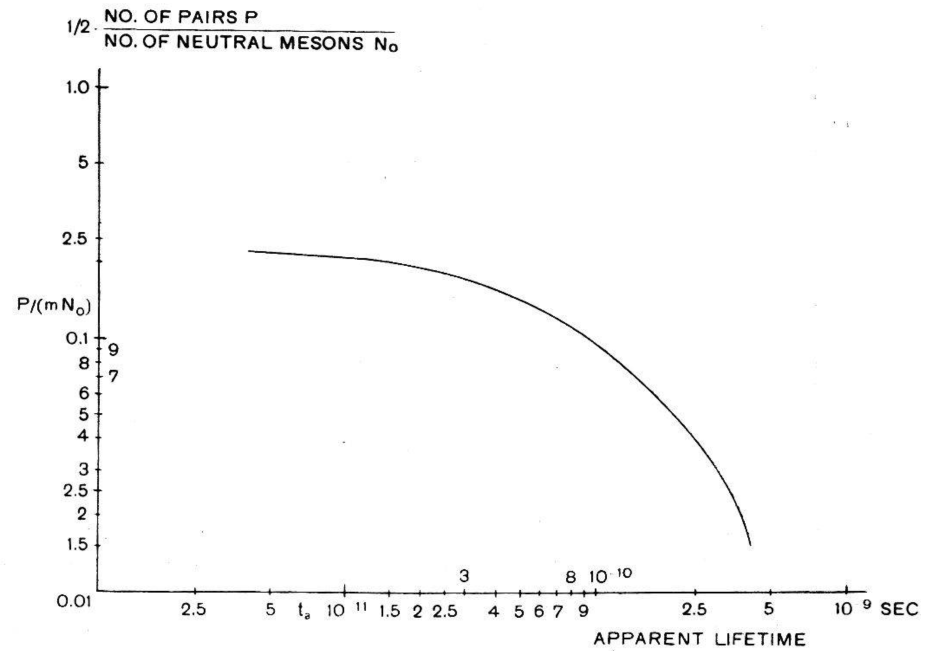

Fig. 14.

Rate of pair production versus life time of neutral mesons. Plotted is $^1/_2$ the number of pairs per neutral meson after a path length of 2 cm glass (0.26 radiation units) versus the apparent lifetime of the neutral meson in the L. system.

where z is the distance from the origin of the shower in units of the radiation length l_0 and

$$x = \frac{\lambda_\gamma}{\lambda_\pi} = \frac{c}{l_0\,\lambda_\pi}$$

With $m = 2$, $N_0 \lesssim N_c = 23 \pm 2$ (number of neutral mesons $\lesssim$ number of charged mesons) and $P = 8$ pairs after a distance

$$l = 2\text{ cm. glass}\left(\frac{l}{l_0} = z = 0.264\right)$$

we obtain from Fig. 14 with

$$\frac{P}{m N_0} \approx 0.2$$

$$\tau_\pi \lesssim 3 \times 10^{-11} \text{ sec.}$$

This time refers to the laboratory system. In the rest system of the meson the corresponding times are shorter by a factor $\sim \langle \gamma_0 \rangle$, hence we obtain

$$\tau_\pi^{(0)} \lesssim 3 \times 10^{-13} \text{ sec}$$

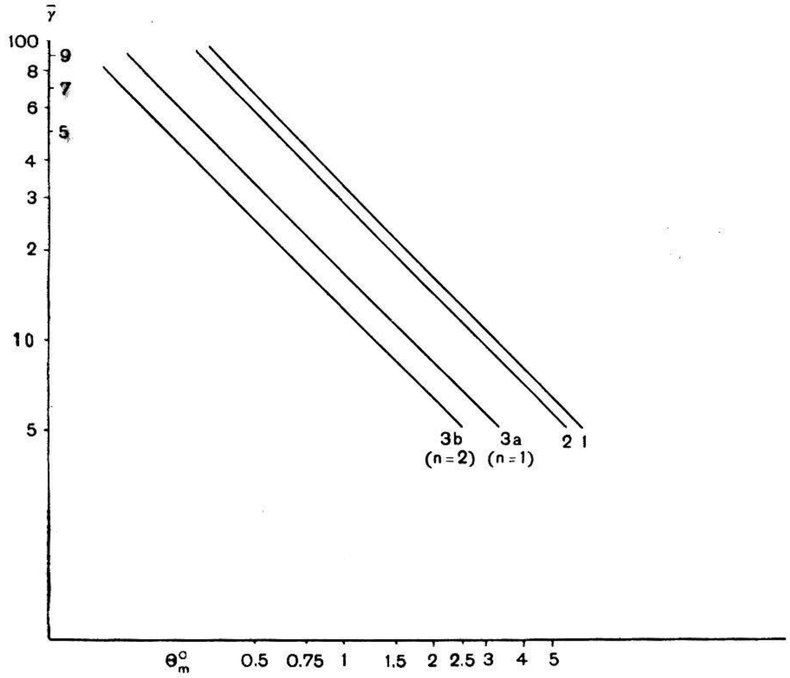

Fig. 15.

Energy of incident nucleon in the C. system as a function of Θ_m, the laboratory angle in degrees for which the angular distribution function assumes its maximum value.

Curve 1 refers to Case I (isotropy and equal energy for mesons in the C. system).

Curve 2 refers to Case II (isotropy and meson energy spectrum $\sim \frac{\overline{p}_0^2}{\overline{E}_0^4} \, d\overline{p}_0$ Curve).

3a, b refers to Case III (equal energy and an angular distribution of mesons proportional to $\cos^2 \vartheta$ and $\cos^4 \vartheta$ respectively).

as an estimate for the upper limit of the lifetime of the neutral meson.

(iii) *Angular Distribution of Mesons and Energy of the Shower.*

For any one of the three assumptions (Cas eI, II and III) of section II, the energy of the nucleons in the C. system $\overline{\gamma}$ is for sufficiently large meson energies in the C. system determined by Θ_m, the angle for which the angular distribution function assumes a maximum value (Fig. 15). Hence, in any case, $\bar{\gamma}$ can be deduced from the observed angular distribution of the mesons in the laboratory system. We know the multiplicity N and, as discussed above, we know also approximately the average meson energy.

Hence we can derive the average energy of the mesons in the C. system

$$\langle \bar{\gamma}_0 \rangle = \frac{\langle \gamma_0 \rangle}{\bar{\gamma}}$$

(eq. (4)) and the degree of inelasticity

$$K = \frac{N \langle \bar{\gamma}_0 \rangle}{13\,(\bar{\gamma}-1)} \quad \text{(see definition).} \tag{28}$$

A. *Simultaneous Production of Narrow plus Diffuse Shower.*

There are a total of $23 + 33 = 56$ charged particles in the complete shower. If we assume that the number of neutral mesons is $^1/_2$ the number of charged mesons, we have a total of 84 mesons or $N = 21$ mesons produced per incoming nucleon, assuming that all four nucleons of the α-particle contribute equally to the shower (see discussion below).

The observed angular distribution for the complete shower (Fig. 16) is summarized in table I:

Table I.

Angular Interval	Number of Particles	%
0—3	25	15
3—9	15	27
9—20	9	16
20—60	7	12
	56	100

Case I. The value of $\bar{\gamma}$ as determined from Fig. 15 necessary to fit the observed maximum at $\Theta_m \approx 1.5^0$ is $\bar{\gamma} = 30$. Hence $\langle \bar{\gamma}_0 \rangle = \frac{\langle \gamma_0 \rangle}{\bar{\gamma}} = 4$. With these values a cut-off angle $\Theta_c \sim 8^0$ is derived from eq. (5c). Therefore the assumption of equal energy and isotropy in the C. system is unable to explain the simultaneous production of the narrow core and the wide shower.

Case II. The value of $\bar{\gamma}$ as determined from Fig. 15 necessary to fit the observed maximum is $\bar{\gamma} = 20$. More than 92% of the mesons then must lie inside a cone of opening $\Theta = 9^0$. This is obviously not in agreement with Table I either.

Case III. The value of $\bar{\gamma}$ as determined from fig. 15 is $\bar{\gamma} = 12$ for $n = 1$. Hence we obtain for $\langle\bar{\gamma}_0\rangle$ a value $\langle\bar{\gamma}_0\rangle = 10$. From equation (28) we see that with $N = 21$, we obtain $K = 1.4 > 1$, but $K = 1$, the maximum value that K can assume, lies still within the statistical error. For any higher degree of anisotropy ($n \geqslant 2$) K would turn out to be much too large.

We conclude from the above discussion that in order to explain the simultaneous production of the narrow core and the diffuse shower, some considerable anisotropy is necessary. In Fig. 16 the angular distribution in the L. system derived for $n = 1$ ($\cos^2 \vartheta$-distribution in the C. system) is superimposed on the observed an

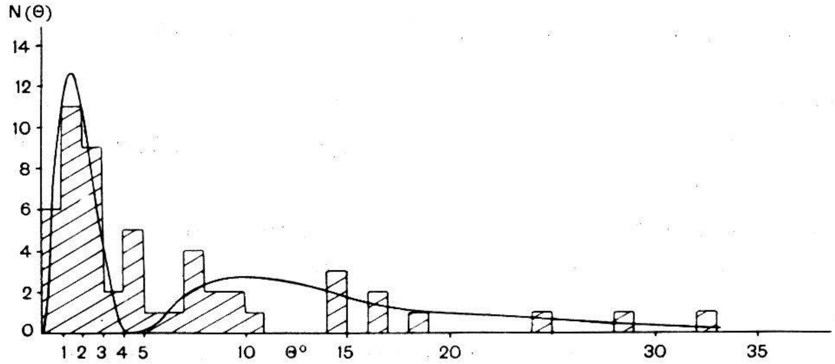

Fig. 16.

Angular distribution of all mesons in the R-shower. The histogram gives the observed distribution in both the narrow core and the wide shower (4 particles in the interval 40°—60° not shown). The curve is calculated for the assumptions of Case III, Section II:
$\bar{\gamma} = 12$ (Energy of incident α-particle $E_\alpha = 1.1 \times 10^{12}$ eV);
$n = 1$ (Angular distribution of mesons in the C. system proportional to $\cos^2 \vartheta$).

gular distribution. The graph shows that for this case the agreement is fair. It therefore appears that an anisotropic angular distribution in the C. system can explain the simultaneous production of the narrow and the wide shower.

B. *Production of the Narrow Core in the Primary Encounter.*

There are 23 particles in the narrow core. If we assume as before that $^1/_2$ as many neutral mesons as charged mesons are produced, and that all 4 of the nucleons of the incoming alpha particle contribute equally, we obtain $N = 9$ as the multiplicity per incident nucleon. For the three cases I, II and III the value of $\bar{\gamma}$ is determined from Fig. 15 to fit the peak of the angular distribution occurring at $\sim 1.5°$. Figs. 17a, 17b, 17c show the angular distribution function $N(\Theta)$ (eqs. 11, 15, 16) for all 3 cases. The observed distribution, a distribution which, but for the influence of the broadening effect discussed previously should reproduce the actual angular distribution of the mesons in the laboratory system is plotted in the same figure. The values of the incident energy which are required for an approximate fit with the observed angular distribution and the results of (ii) are listed in table II.

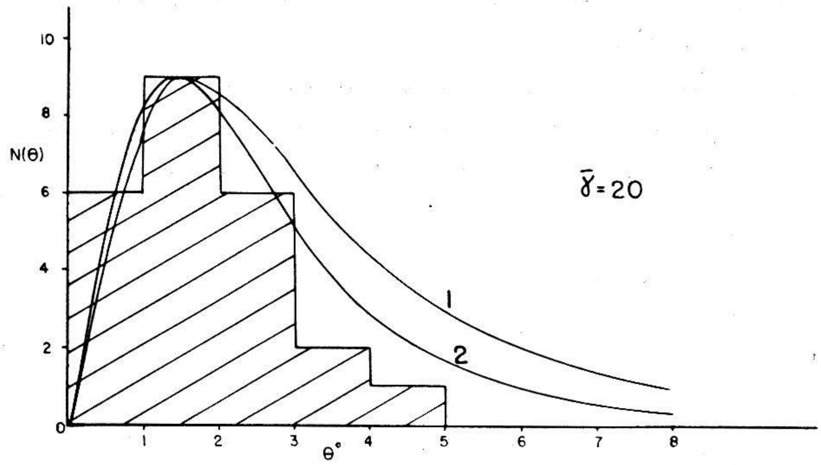

Fig. 17 a.

Angular distributions of mesons in the core of the R-shower. The histogram gives the observed angular distribution. The curves give the angular distribution calculated with the assumption of isotropic emission in the C. system.
Curve 1 refers to equal energies for all mesons in the C. system (Case I).
Curve 2 refers to a meson energy spectrum $\sim \frac{\bar{p}_0^2}{\bar{E}_0^4}\, d\,\bar{p}_0$ (Case II). Both curves have been adjusted to give the maximum intensity at $\Theta_m \approx 1.5^0$ corresponding to $\bar{\gamma} = 20$ or an energy of 3.2×10^{12} eV for the incident α-particle.

Table II.

Case I (isotropy and uniform meson energy)		Case II (isotropy and energy spectrum) $F(\bar{E}_0) \sim \frac{\bar{p}_0^2}{\bar{E}_0^4}\, d\,\bar{p}_0$	Case III (single meson energy and angular distribution $F(\vartheta)$)	
			$F(\vartheta) \sim \cos^2\vartheta$ $(n = 1)$	$\sim \cos^4\vartheta$ $(n = 2)$
$\langle\gamma_0\rangle$	120	82	120	120
$\bar{\gamma}$	30	20	12	10
K	0.1	0.15	0.6	1.0
$\gamma = 2\,\bar{\gamma}^2 - 1$	1800	800	290	200
$E_\alpha = 4\,\gamma\, Mc^2$ (in 10^{12} eV)	7.2	3.2	1.2	0.8
$\langle\bar{\gamma}_0\rangle = \frac{\langle\gamma_0\rangle}{\bar{\gamma}}$	4.0	4.1	10.0	12.0

In case I and III, we use the average observed meson energy ~ 20 Bev, corresponding to $\langle \gamma_0 \rangle = 120$. The observed Θ_m gives then a value for $\overline{\gamma}$. From $\overline{\gamma}$, $\langle \gamma_0 \rangle$ and the observed multiplicity N we obtain a value for the inelasticity K.

In case II Θ_m and the multiplicity N determine both $\overline{\gamma}$ and K. $\langle \gamma_0 \rangle$ is then already determined and the value obtained agrees within the experimental uncertainty with the estimates made from the pairs found in the core.

Row 2 gives for each case the value of $\overline{\gamma}$ for the best fit. Row 5 gives the energy of the incident alpha particle. Rows 6 and 1 give the average energy of the mesons in the C. system and the L. system,

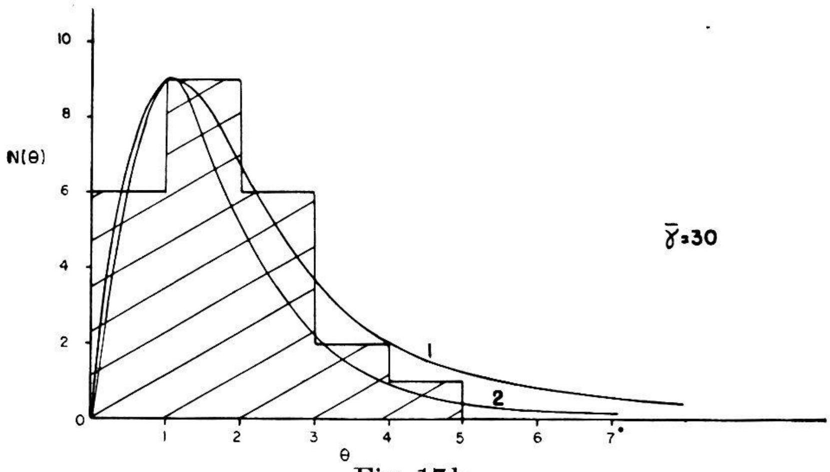

Fig. 17 b.
Angular distribution of mesons in the core of the R-shower. Same as Fig. 17a, except that the curves are calculated with $\overline{\gamma} = 30$, corresponding to an energy of 7.2×10^{12} eV for the incident α-particle.

respectively, determined to fit the results of section (ii), and row 3 the inelasticity determined by them.

Case I. For this case (isotropy and equal energy in the C. system) it is now possible to fit the observed data. The "degree of inelasticity" turns out to be small: $K = \frac{N \langle \overline{\gamma}_0 \rangle}{13\,(\overline{\gamma}-1)} \sim 0.1$. It is possible however that part of the energy not transferred to the mesons may appear in other processes (as for instance production of nucleon-anti-nucleon pairs).

Case II. For this case all factors are determined by the choice of $\overline{\gamma}$. The multiplicity is given as a function of $\gamma\, K^2$ in Fig. 6. With $\overline{\gamma} = 20$ determined to fit the observed maximum at $\Theta_m \sim 1.5^0$, $K = 0.15$ is then determined from the above figure and $\langle \overline{\gamma}_0 \rangle = 4.1$ from the relation (28). It is interesting to note that $\langle \gamma_0 \rangle = 82$ so determined is within the limits estimated in section (ii).

Case III. Both for $n = 1$ and $n = 2$ the angular distribution for this case can be fitted to the observed distribution. The contributions to the larger angles for this case may well be contained in the more diffuse shower, (see section IV). The value $K \sim 0.5 - 1.0$ deduced from the average meson energy in the L. system $\langle\gamma_0\rangle = 120$ is much closer to unity than for case I and II.

It is noteworthy that these rather widely different assumptions I, II and III give within one order of magnitude the same estimate for the energy of the primary α-particle: $E_\alpha = 10^{12} - 10^{13}$ eV.

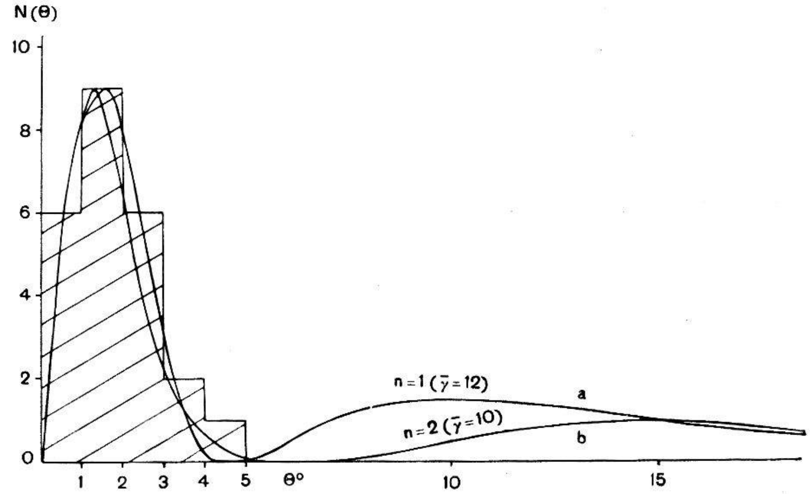

Fig. 17c.

Angular distribution in the core of the R-shower.
The histogram gives the observed angular distribution. The curves refer to Case III, Section 2.

Curve a is calculated for $\bar{\gamma} = 12$ ($E_\alpha = 1.1 \times 10^{12}$ eV) $n = 1$ ($\cos^2 \vartheta$ distribution).
Curve b is calculated for $\bar{\gamma} = 10$ ($E_\alpha = 0.8 \times 10^{12}$ eV) $n = 2$ ($\cos^4 \vartheta$ distribution).

(iv) *The Diffuse Meson Shower.*

The discussion of assumption A in the beginning of this section shows that the diffuse meson shower can be considered to result from the primary encounter only if the angular distribution of mesons in the C. system is anisotropic. There is no need to explain all of the mesons by this assumption, since it is quite probable that the nucleons which emerge from the primary encounter will collide again within the same nucleus and produce additional mesons.

The mean free path of fast nucleons in nuclear matter λ_N can be calculated from the known mean free path of singly charged cosmic ray primaries (protons) in air

$$\Lambda_{\mathrm{Air}} \sim 100 \text{ gr/cm}^2$$

$$\lambda_N = 5.4 \times 10^{-13} \text{ cm}$$

The large number of heavy prongs of the star associated with the meson shower shows that the collision occurred in a heavy nucleus (Ag or Br) of the emulsion. In order to estimate the maximum number of secondary, tertiary, etc., mesons we assume that the primary α-particle made a central collision with an Ag nucleus, whose diameter is $2R_{Ag} = 14.3 \times 10^{-13}$ cm. Hence the 4 nucleons of the α-particle have to penetrate an amount of nuclear matter equivalent to $\alpha = \frac{2R_{Ag}}{\lambda_N} = 2.66$ mean free paths. The probability that one of them passes through the center of the nucleus without colliding:

$$P_1 = 4\,e^{-a}\,(1 - e^{-a})^3 = 0.23$$

is three times smaller than the probability that all four nucleons collide

$$P_4 = (1 - e^{-a})^4 = 0.75$$

and we shall therefore assume the latter to be true. We have then 8 secondary nucleons resulting from primary collisions. A calculation performed by Dr. S. A. Wouthuysen*) shows that most probably 6 of them will collide again and only 2 will escape from the nucleus:

$$\bar{n}_2 = 5.92 \pm 1.38$$

is the calculated average number of secondary collisions. An average of 12 tertiary nucleons will result from these 6 secondary collisions, of which an average of 8 will collide again and produce $\bar{n}_3 = 8.0 \pm 2.8$ tertiary collisions.

In order to estimate the number of mesons to be expected in the wide shower, we have to make some assumption on the degree of inelasticity and the energy dependence of the multiplicity $N = N(E)$.

Table III shows the number of mesons to be expected as a result of primary, secondary and tertiary collisions for the assumption of complete inelasticity $K = 1$ and the multiplicity-energy relation given by Lewis, Oppenheimer and Wouthuysen:

$$N\,(E) \sim E^{(\frac{1}{4})}$$

Column (1) indicates the order of the collision, column (2) the energy γ_{i-1} of the nucleon (in units Mc^2) producing the collision i ($\gamma = 1800$ for $i = 1$ is taken from table II). Column 3 lists $\gamma_{i-1}^{(\frac{1}{4})}$, column 4 the multiplicity of mesons N_i ($N_1 = 10$ from table II), column 5 the number of ejected fast protons n_i resulting from the

*) To appear in "Physica".

collision of order i, column 6 the total number of charged particles N_i charged ($^1/_3$ of the mesons assumed to be neutral) resulting from the collision of order i and column 7 the tangent of the half width angle.

Table III.

Order of Collision	γ_{i-1}	$\gamma_{i-1}^{(1/3)}$	N_i	n_i	N_i^{charged}	$\text{tg}\,\Theta_{1/2} = \frac{1.34}{\gamma_i}$	
1	2	3	4	5	6	7	
Primary $i = 1$	1800	12.2	$4\times10=40$	0	27	0.45	2.5^0
Secondary $i = 2$	30	3.1	15	2	12	0.33	18^0
Tertiary $i = 3$	4	1.6	11	8	15	1	45^0

Table III shows that the expected number of charged relativistic particles from the secondary and tertiary collisions $N_2^{ch} + N_3^{ch} \sim 27$ is approximately equal to the number of charged relativistic particles from the primary collision and is only sligthly smaller than the number of relativistic tracks actually observed in the diffuse shower: $N_{ch} = 33$.

For complete inelasticity there will be no significant contribution to meson production from 4th and higher order collisions.

For partial inelasticity of the primary encounter secondary collisions will produce mesons only within a cone of a few degrees opening, but the 4th and 5th order collisions will contribute a number of mesons to the wide shower comparable to the number of mesons produced in the secondary collisions. Hence the general picture in the case of incomplete inelasticity of the primary encounter might not be too much different from the case of complete inelasticity, as far as the relative numbers of mesons from primary and from subsequent collisions are concerned. This discussion shows that in the case under consideration we may indeed expect a narrow core resulting from primary collisions of the nucleons of the α-particle to be accompanied by a diffuse shower resulting from subsequent collisions containing roughly the same number of charged particles. Hence we may consider the shower of Fig. 12 as an illustration of pluro-multiple meson production.

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