Critical Fields of Superconductive Hollow Cylinders in Transverse Magnetic Fields

by **B. Serin** and **E. A. Lynton**
Department of Physics, Rutgers University, New Brunswick, N. J.
and
J. Gittleman, R.C.A. Laboratories, Princeton, N. J.

(9. III. 1959)

Recently, JAGGI, MÜLLER, and SOMMERHALDER[1]) have criticized our interpretation of some experiments we performed several years ago[2]) on the magnetic properties of hollow superconductive cylinders. In these experiments a long hollow cylinder with plane ends was cooled below the transition temperature, T_c, in zero field. A magnetic field transverse to the cylinder axis was then applied, and we observed that the field value at which such cylinders pass into the intermediate state depends on the wall thickness. In particular when the wall is sufficiently thin, the transition occurs at fields less than $H_c/2$, where H_c is the critical field value. JAGGI, MÜLLER, and SOMMERHALDER have essentially verified these observations in their measurements on a cylinder having plane ends.

We suggested that some insight into these observations might be obtained by calculating the free energy of a hollow superconductive cylinder under the conditions which prevailed in the experiment (i.e. one cooled below T_c before the field is applied), and comparing this free energy with that of the normal phase. On this basis, we derived an expression for an upper limit, H_{oe}, of the applied field value at which a hollow cylinder could remain superconductive. This limiting value depended on wall thickness. Our measurements on cylinders of various wall thickness were in very good agreement with this result; the measured values at which the field first penetrated into the cylinder being quite close to but always smaller than the value H_{oe}.

JAGGI, MÜLLER, and SOMMERHALDER state that the thermodynamic variables we used in our derivation (the applied field, H_o, and the temperature, T) are not correct, since they do not uniquely specify all the states of the hollow cylinder. To illustrate this point, they calculate the free energy of a hollow cylinder in a longitudinal, rather than transverse, field when a uniform field equal to H_c is frozen inside the hollow. Under these circumstances, the free energies of the normal and superconductive phases become equal at H_c, independently of wall thickness. Furthermore, JAGGI, MÜLLER, and SOMMERHALDER measured the penetration fields in cylinders with caps, which were therefore essentially hollow ellipsoids, and also in a cylinder with one end conically shaped. In the former, penetration occurred at $H_c/2$, whereas in the latter it occurred at a value appreciably less than H_{0e}. They therefore conclude that our observations arose solely from the special form of the ends of our cylinders, and since they believe our thermodynamic argument to be wrong, we assume they suggest that the agreement between our measurements and our thermodynamic argument is fortuitous.

After reconsidering this matter, we find that we cannot agree with these criticisms. In deriving our expression for the free energy, our starting point is the fact that the change in free energy at constant temperature equals the reversible work done on the body[3]). In the case of superconductive cylinders, the work is the reversible magnetic work done by the batteries in establishing the current producing the field. For calculating this work we used the expression

$$-\int_0^{H_0} I dH_0 ,$$

where I is the total magnetic moment of the body and H_0, the external field. A derivation of this expression*) under the conditions identical to those which prevail in the experiment may also be found on pp. 23–26 of reference 3. In calculating the work it does not seem necessary to specify all possible states of the system, but only to evaluate it correctly for the experimental conditions. For our case, namely an infinite hollow cylinder cooled below T_c in zero field, there is no field inside the cylinder, and its total magnetic moment is specified uniquely by the external field value. Moreover, as long as the cylinder remains in the superconductive phase, the magnetic work can be done reversibly, since one can return the specimen and the battery to their initial states by reducing the field to zero. Thus, the essence of our argument is that by restricting attention to an admittedly *special case*, one can calculate rigourously and correctly the magnetic work done on the specimen along a reversible path which is experimentally realizable. Of course, as soon as the field penetrates into the cylinder, further increases of the field lead to irreversible processes, but we made no attempt to treat these. Rather, what we tried to show was that by considering only that part of the magnetization

*) We had essentially duplicated this derivation before publishing our paper[2]); see footnote 3 in that paper.

curve which can be treated by reversible thermodynamics, one can see that the free energy of a hollow superconductive cylinder can increase with increasing magnetic field at such a rate as to make it certain that this free energy equals that of the normal phase at fields less than $H_c/2$. For these reasons we believe our original calculation of the work and the free energy change is correct. It is to be noted that the example given by JAGGI, MÜLLER, and SOMMERHALDER of a cylinder in a longitudinal field is also not unique. Clearly if such a hollow cylinder is cooled below the transition temperature before the magnetic field is turned on, the field inside does not equal the critical value H_c, and their description does not apply to this case.

It does not seem necessary in this note to discuss observations on cylinders with caps. We feel that this matter was covered adequately in our original paper in our consideration of the earlier work of BABISKIN [4]) on the properties of a hollow sphere. As for the cylinder with a conical end, we are not surprised that the field first penetrates into it at a value less than H_{oe}. Since we compared only the superconducting and normal states of the cylinder we expect the transition to an intermediate state to occur at a value less than H_{oe}, although we cannot estimate the difference. On the contrary, what seems surprising to us is that the observed values for cylinders with plane ends fall so close to the values of H_{oe}. Moreover, on the basis of our observations it seems likely that if JAGGI, MÜLLER, and SOMMERHALDER had made measurements on specimens with the same conical end, but of varying wall thickness, they would have observed that the penetration field varied smoothly with thickness with values always somewhat less than our limiting field H_{oe}.

We do not think therefore, that it is appropriate to discard our thermodynamic argument; first because we believe it to be correct, and second because, despite its limited scope, we believe it gives insight into one aspect of the behavior of hollow superconductors.

One of us (B. S.) gratefully acknowledges illuminating discussions with Professor J. BARDEEN and Dr. M. GARFINKEL.

References

[1]) R. JAGGI, J. MÜLLER, and R. SOMMERHALDER, Helv. Phys. Acta *31*, 637 (1958).
[2]) B. SERIN, J. GITTLEMAN, and E. A. LYNTON, Phys. Rev. *92*, 566 (1953).
[3]) A. B. PIPPARD, *Elements of Classical Thermodynamics*, Cambridge University Press (1957), p. 56.
[4]) J. BABISKIN, Phys. Rev. *85*, 104 (1952).