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Nuclear Systematics

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Introduction

The process of understanding natural phenomena is admittedly a very involved one. There is, however, one important feature in this process which repeats itself in all branches of natural sciences, namely the break-up of complicated phenomena into a superposition of simpler phenomena. In many cases a phenomenon is considered to be 'understood' if it can be described in terms of a number of other, previously established, phenomena, which in their turn may or may not be 'understood', again in the same sense.

It is therefore only natural that in exploring new fields of natural sciences the first task, after the nature of the phenomena has been established, is almost always that of looking for systematics in the phenomena observed. Nuclei, for instance, are rather complicated structures which may possess a great variety of properties: They have mechanical properties such as mass, moment of inertia, angular momentum, a certain flow pattern of the constituents of the nucleus, etc. They have electrical properties usually describable by a certain charge and current distribution over the nucleus, with a possible distribution of intrinsic magnetization as well. Finally nuclei have nuclear properties which are usually described in terms of nuclear forces whose existence is not known outside nuclear physics.

All these properties are not independent of each other. It is clear, for instance, that the flow pattern of the constituents of the nucleus is determined by the forces acting between them; the electric current distribution depends on the general flow pattern, etc. The purpose of the study of nuclear systematics is therefore to try and find some relations, both quantitative and qualitative, between the various parameters which are characteristic of each nucleus and nucleus. Once such relations are experimentally established we may have better chances of determining whether the notions and concepts of the physical world explored until now are valid also for the phenomena taking place inside the nucleus, or whether it is necessary to modify our concepts in order to have a unified description of the whole physical world, including the smallest nucleus on the one end and the biggest galaxis on the other.

We are still very far from having a comprehensive study of nuclear phenomena, let alone the study of the systematics of these phenomena. For the study of some type of data, such as the magnetic moments of excited nuclear states, there are not yet well developed experimental methods to allow systematic studies. Other data still require long and tedious experiments for their clarification. It is, however, comforting to realize that despite the relatively few systematic studies of nuclear phenomena which have been carried out to date, a number of important conclusions could be made about the structure of nuclei, and we have already reached a stage where some nuclear data can be predicted with a reasonable degree of certainty.

Nuclear Systematics

The nucleus is believed to consist of a certain number A of nucleons, Z of them being protons and $N = A - Z$ neutrons. It is not quite clear to what extent these nucleons retain their identity in the nucleus, but there is evidence of a general nature showing that the nucleons in a nucleus obey the Fermi-Dirac statistics. It is therefore most natural to choose the number of nucleons as the parameter determining the systematic changes of nuclear properties. In doing so, however, it was found already some time ago that striking similarities between nuclei exist only if we compare nuclei differing by two protons or by two neutrons. The addition of a single nucleon, or two different nucleons, to a given nucleus usually results in an entirely different 'type' of nucleus.

This observation results in the distinction between four classes of nuclei: even Z – even N, (even-even nuclei), even Z – odd N, odd Z – even N (both referred to as odd-even nuclei and sometimes included in one class), and odd Z – odd N (odd – odd nuclei). Thus it has been known for a long time that even-even nuclei are generally more tightly bound than odd – even nuclei, and that odd – odd nuclei are generally less tightly bound than odd – even nuclei. In addition, the characteristic spectra of even – even nuclei, odd – even nuclei and odd – odd nuclei, are quite different from each other.

There is probably a good reason why such general sort of behaviour could be expected in nuclei. The primary factors responsible for it are the attractive nuclear interaction and the exclusion principle which the nucleons have to obey. Thus it was pointed out by BARDEEN, COOPER and SCHRIEFFER [1][1]) and by BELAYEV [2] that one gets an exceptionally good wave function for the ground state of a system of fermions with

[1]) Numbers in brackets refer to References, page 156.

attractive interactions if one constructs this wave function by combining as many 'saturated pairs' as possible. Such saturated pairs have zero total momentum and zero intrinsic spin in the case of infinite systems, zero total angular momentum in spherically symmetric systems, or zero z-projection of the total angular momentum in cylindrically symmetric systems. Whatever the case may be, the saturated pair represents a pair of fermions whose matter distributions overlap each other as much as possible, taking into account the Pauli principle. With attractive interactions between the fermions they therefore represent an especially stable configuration. The pair of identical particles which is added to the ground state of a nucleus A to produce the nucleus $A + 2$, is most probably added as a 'saturated pair', and therefore, in some sense as an *inert pair*. We can thus understand, at least qualitatively, why regularities in nuclear properties may show up if we compare nuclei differing by two neutrons or two protons, rather than nuclei differing by a single proton or neutron.

The regularities which have been mostly studied are those referring to magnetic dipole and electric quadrupole moments of nuclear ground states, binding energies, separation energies and low excitations of even – even and odd – even nuclei (in particular in the vicinity of closed shells), and deformation parameters of highly deformed nuclei. Of these regularities the latter, and to some extent also the electric quadrupole moments, have been the subject of a number of first class surveys [3] and we shall therefore not treat them here.

The systematics of nuclear magnetic moments has also been known for some time, and very little additional insight into this special subject has been added in the last few years. It was pointed out by SCHMIDT [4] some twenty years ago that it is useful to divide the observed magnetic moments μ of odd – A nuclei into two groups: those of odd Z – even N nuclei and those of odd N – even Z nuclei. Plotting then μ against j, the nuclear spin, one obtains a very striking regularity: The magnetic moments in each of the above groups are very close to what one would expect to find by assuming that they are solely due to the last odd nucleon. Later, when more information was gained on properties of nuclear states, people concentrated mainly on understanding the *deviations* of the magnetic moments from the single-particle Schmidt lines; but it should not be forgotten that it was at all possible to take up the behaviour of these *deviations* only because the *moments themselves* showed such a remarkable regularity.

If we assume that there is a strong pair correlation in nuclei we can understand this regularity under quite general conditions. Thus if the nuclear wave function for an even number of nucleons is such that whenever a certain particle state is occupied also its time reversed state is occupied, then such pairs will not contribute to the total magnetic moment of the system, since magnetic moments change sign under time reversal. Therefore, in an odd – even nucleus the major contribution to the magnetic moment will come from the last odd, unpaired, nucleon, which may be in a relatively pure single particle state.

The deviations of magnetic moments from the Schmidt lines show some interesting regularities. The most remarkable one is that they all lie on one side of the relevant Schmidt line and are not scattered *around* the Schmidt lines as one is inclined to expect. Furthermore it was noticed [5] that if this behaviour is attributed to the quenching of the anomalous moment of the odd nucleon in nuclear matter, then, for nearly all nuclei, the effective intrinsic magnetic moment of the odd nucleon lies between the Dirac value and the observed anomalous value of the free nucleon. No theory has as yet been able to account for this quenching in a quantitative way, nor is it at all clear that this is the main mechanism responsible for the deviations of the magnetic moments from the Schmidt lines. An elaborate discussion of this point can be found in the excellent review article by BLIN-STOYLE [6].

There are other, less striking, regularities, which show up when the deviations of the magnetic moments are examined more closely. Thus if one adds two neutrons or two protons to a given odd – A nucleus its magnetic moment nearly always comes closer to the relevant Schmidt line provided its spin remains unchanged [6, 7]. No explanation of this regularity has as yet been offered.

Another interesting behaviour of the deviations shows up if we compare an odd Z – even N nucleus (Z, N) with an even Z – odd N nucleus (Z', N'), both having the same spin and satisfying in addition $Z = N'$. The deviation of the odd Z – even N nucleus is then seen to be about $-1{\cdot}2$ times that of the even Z – odd N nucleus. This observation can be explained with very few assumptions [8]. Thus if we assume that the even group of nucleons in an odd – A nucleus does not contribute to the magnetic moment, we can write for the g factor of a given nucleus

$$g\,\boldsymbol{J} = \sum g_l^{(i)}\,\boldsymbol{l}_i + g_s^{(i)}\,\boldsymbol{s}_i = g_l(\sum \boldsymbol{l}_i + \boldsymbol{s}_i) + (g_s - g_l)\sum \boldsymbol{s}_i = g_l\,\boldsymbol{J} + (g_s - g_s)\,\boldsymbol{S} \quad (1)$$

Here we have used the fact that all the nucleons contributing to the magnetic moment are identical so that $g_l^{(i)} = g_l$ and $g_s^{(i)} = g_s$, both independent of i and equal to the proper values for free protons or neutrons as the case may be. $\boldsymbol{J}$ is the total angular momentum operator und $\boldsymbol{S}$ is the total intrinsic spin of the group. The single particle Schmidt value for g is obtained by taking average values of both sides of (1) with the proper single particle (s. p.) wave function. On the other hand the actual value of g is obtained by taking the average value of (1) with the *real* wave-function. Since both wave-functions have the same total angular momentum we conclude that

$$\delta\mu = \langle gJ_z \rangle_{real} - \langle gJ_z \rangle_{s.p.} = (g_s - g_l)\{\langle S_z \rangle_{real} - \langle S_z \rangle_{s.p.}\} \quad (2)$$

Therefore, for the pair of nuclei considered above, with $Z = N'$, where we can perhaps assume that the averages of S_z in the odd group are the same, the deviations of the odd Z – even N and odd N – even Z nuclei should be in the ratio

$$\frac{\delta\mu^P}{\delta\mu^N} = \frac{(g_s - g_l)^P}{(g_s - g_l)^N} = -1 \cdot 20 \quad (3)$$

Systematics of Energies and Energy Levels

By far the most striking regularities associated with nuclear structure are those of binding energies, separation energies and spectra of excited states of related nuclei. A typical example of the latter is offered by the spectra of the odd – A isotopes of Au (Figure 1) studied by the Zurich group and others [9]. Particularly instructive is the comparison between ^{193}Au and ^{195}Au which are nearly identical with each other as far as their spectra are concerned (the few levels missing in ^{193}Au cannot be excited, energetically, by the methods studied so far). Thus the addition of two neutrons to ^{193}Au does not change appreciably the sequence and relative

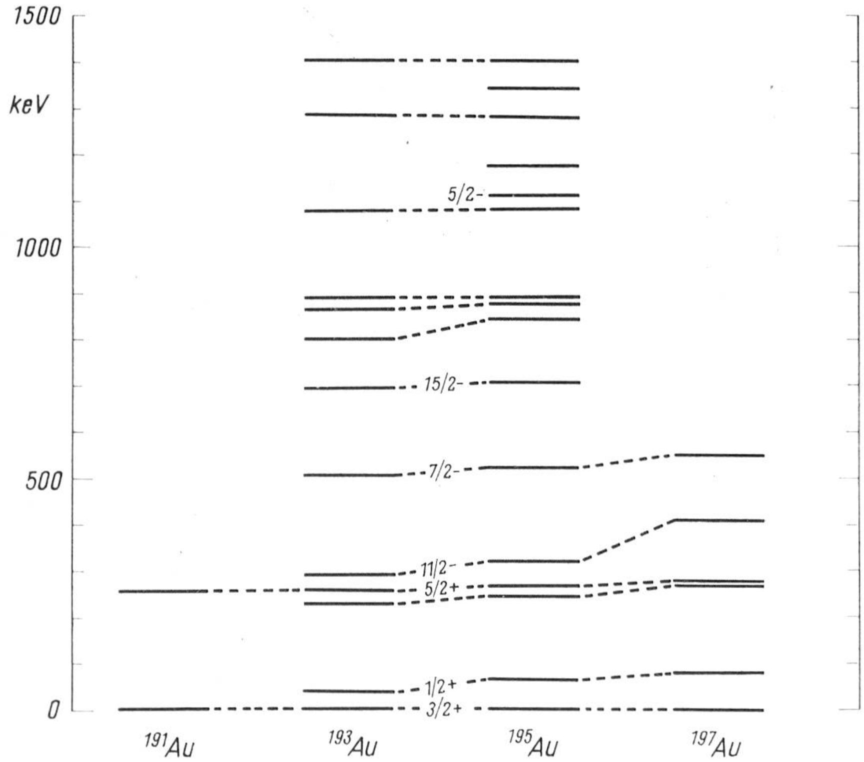

Figure 1
Energy levels of some Au isotopes.

positions of at least its first 16 levels! The available data on ^{197}Au indicates that the same holds true if we continue adding another pair of neutrons.

Such regularities are known in many parts of the periodic table, although they have not been studied to such high excitations in most of the cases. A slow, smooth variation of the separation between two levels j and j' in a series of nuclei $A, A+2, A+4, \cdots$ was pointed out by GOLDHABER and HILL [10] and by the Zurich group [11]. The latter, concentrated around P. SCHERRER, and including, at various periods, J. BRUNNER, A. DE-SHALIT, H. GUHL, J. HALTER, O. HUBER, F. HUMBEL, R. JOLY, D. MAEDER, CH. PERDRISAT, H. SCHNEIDER and W. ZUNTI, studied primarily the region of Pt, Au and Hg. Other regions were studied by other groups [12]. The results can be summarized by saying that in odd – even nuclei we find series of nuclei which differ from each other by two protons or two neutrons, and whose energy level spectra are very similar to each other; the relative separation between corresponding levels in such nuclei is a smooth function of the atomic number. A typical example is given in Figure 2.

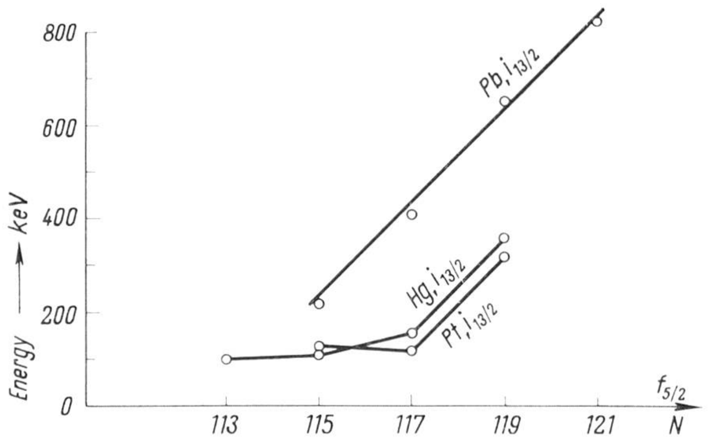

Figure 2
The spacings between $f^5/_2$ and $i^{13}/_2$ in some nuclei.

An attempt to explain these regularities semi-quantitatively on the basis of the shell model has been made by ZELDES [12]. To understand it we should remember that according to the jj-coupling shell model the energy of each nuclear level is composed of a main contribution coming from its total energy in the average central field, plus a small addition representing the effects of the residual two-body interactions. This small addition, according to the shell model, can be well approximated by the average value of the interaction in the state considered. Assume now, we have in a nucleus with A particles a certain state $\psi_1(A)$ with an energy $E_1(A)$. Let us add to this state $n/2$ pairs of particles, all in the same orbit, each pair coupled to $J = 0$. By doing so we have added to the nucleus the energy $E(n)$ of the additional n particles (including their mutual interaction) plus the interaction energy $E_1(n, A)$ of these n particles with the A particles in $\psi_1(A)$. Of these two quantities $E(n)$ is independent of the state $\psi_1(A)$ whereas $E_1(n, A)$, representing the interaction of the $n/2$ pairs with the initial nucleus, does, of course, depend on which state we are considering. Denoting the state derived from $\psi_1(A)$ by the above procedure by $\psi_1(A+n)$ and its energy by $E_1(A+n)$, we have

$$E_1\,(A+n) = E_1(A) + E(n) + E_1(A, n). \qquad (2)$$

If we consider now the energy difference between two states in the nucleus A and the two corresponding states in the nucleus $A+n$ we find from (2) that

$$E_1(A+n) - E_2(A+n) = [E_1(A) - E_2(A)] + [E_1(A, n) - E_2(A, n)]. \quad (3)$$

It can now be shown fairly easily [13] that if the n particles are added in the form of $n/2$ pairs with $J = 0$ then

$$E_1(A, n) = n\,A\,\varepsilon_1, \qquad (4)$$

where ε_1 is independent of n. The physical picture behind this relation is simply that a pair in the spherically symmetric $J = 0$ state can feel nothing but the monopole part of an interaction and therefore contributes to the energy bi-linearly in the number of particles. Substituting (4) into (3) we therefore find that

$$E_1(A+n) - E_2(A+n) = E_1(A) - E_2(A) + n\,A\,(\varepsilon_1 - \varepsilon_2). \qquad (5)$$

Thus we see that if the same equivalent $J = 0$ pairs are added to each one of the levels of a nucleus A we can expect a shift of the relative position of the levels which will be linear in the number of particles added. The notion of the 'inert pairs' proves therefore to be sufficient for the qualitative understanding of the regularities observed in many nuclear spectra. One can, of course, go into a much more detailed study of these regularities along the present lines, but we shall not do it here. For a detailed account of such studies the reader is referred to [12].

The regularities in the energies of related nuclei discussed above become even more impressive if we study the variations in the probabilities for transitions from corresponding levels as a function of the number of nucleons. Such studies have been particularly carried out by the Swedish group [14] in the lead region obtaining very interesting regularities which are as yet unexplained. It is worthwhile to note that while energies of states are usually not very sensitive to slight modifications in the model employed for their calculation, transition probabilities are usually much more sensitive to such modifications. This is so because the energies derived from a given model are eigenvalues of the Hamiltonian of that model and are therefore stationary with respect to small variations. Transition probabilities, on the other hand are off diagonal elements of a given operator, and as such are not necessarily stationary with respect to small variations. Thus the regularities found in transition probabilities may be very meaningful and may ultimately turn out to be of even greater importance than those found in energies of excited states.

Regularities similar to those mentioned above for odd A nuclei have also been observed in even – even nuclei by SCHARFF-GOLDHABER [15] and by PREISWERK and STÄHELIN [16]. The interpretation of these regularities is probably less straightforward. The nature of the excited states of some even – even nuclei is not at all clear, especially for non-deformed nuclei, and it is therefore hard to suggest one definite interpretation of their regular behaviour as a function of the nucleon number. It is, however, good to know that practically any model one adopts predicts a spin 2 + for the first excited state of an even – even nucleus, and a more or less regular variation of this first excited state with nucleon number.

Recently, through the works of TALMI and co-workers [17], it became possible to study much more complicated regularities, which, on the surface, may not appear as regularities at all. Without going into much details [18] the situation may be described in the following way: If one assumes the validity of the Shell-Model and a certain coupling scheme, say the jj-coupling, then it is possible to establish some general relations between the spectra of different nuclei. A simple example of such general relations is the theorem that the spectrum of energy levels of the configuration $j_1 j_2$ is identical with that of $j_1^{\,2j_1} j_2^{\,2j_2}$, i. e. the equivalence of the spectra of particle-particle and hole-hole configurations. More complicated examples are offered by the relations between particle-particle and particle-hole configurations [18], or by the expression of the energies in the configuration j^n in terms of those of j^2 [18, 19], both examples relying heavily on the use of the Racah algebra and RACAH's powerful methods in spectroscopy [20]. Thus for instance it is found that *irrespective* of what the central field is and what is the nature of the residual two body interaction, provided only the jj-coupling shell model is valid for nuclei, one must have the following relations in the $f^7/_2$ shell [18, 19, 21]:

$$E\,(j^3, {}^5/_2) - E(j^3, {}^7/_2)$$
$$= {}^1/_{132}\{187\,[E(j^2, 2) - E(j^2, o)] - 75\,[E(j^2, 4) - E(j^2, 0)] - 13\,[E(j^2, 6) - E(j^2, 0)]\},$$

$$E(j^3, {}^3/_2) - E(j^3, {}^7/_2)$$
$$= {}^1/_{84}\,\{19[E(j^2, 2) - E(j^2, 0)] + 135\,[E(j^2, 4) - E(j^2, 0)] - 91\,[E(j^2, 6\,) - E(j^2, 0)]\}.$$

Here $E(j^n J)$ stands for the energy of the state of total angular momentum J in the configuration $(^7/_2)^n$. Taking the experimental values of $E(j^2 J) - E(j^2\, o)$ from $^{50}_{22}Ti_{28}$ we obtain for the $(^7/_2)^3\, ^5/_2$ state, using the above formula, an excitation energy of 364 keV as compared with the experimental values of 373 keV in $^{43}_{20}Ca_{23}$, 320 keV in $^{51}_{23}V_{28}$, and 380 keV in $^{28}_{53}Mn_{28}$.

Many binding energies have been very succesfully analysed in a similar way, and an appreciable number of excited states of various nuclei have been associated with each other through such consideration. These studies are far from being complete, but they already indicate a great amount of order and systematic behaviour in nuclear energies.

Future Scope

The theoretical treatment of nuclear structure still suffers from the lack of systematic knowledge of many nuclear properties. Even in the relatively well investigated region of the deformed nuclei, most of the available information on the electromagnetic properties is related to the charge distribution in the nucleus and relatively little is known about the important question of the distribution of currents and magnetization in these nuclei. More systematic information is required on magnetic moments and magnetic transition probabilities of excited states of both deformed and non deformed nuclei. In particular it will be very instructive to have the magnetic moments of several states of the same configuration of an odd – odd nucleus, such as for instance the first four states of ^{40}K or ^{38}Cl. One might then be in a position to say something more definite about the mechanism causing the deviation of magnetic moments from the Schmidt lines.

Also in looking for level schemes of nuclei it is very advisable to look for regularities associated with a series of nuclei. In the first place this may give us an idea of what and where to look for, and this always improves the accuracy and reliability of the work. But even more important is the necessity of determining how far do the regularities go. We still do not know up to what energy are the spectra of, say, ^{193}Au and ^{195} Au similar to each other. It is obvious that new features must suddenly appear when we keep on adding $J = 0$ pairs to a given nucleus. Where do they appear? In what way? Is it just the addition of an extra few levels or is it a complete rearrangement of the whole spectrum? Are there related 'breaks' in the regular behaviour of other properties like electromagnetic moments or transition probabilities?

The answers to most of these questions are not known and any theory of nuclear structure may rise or fall depending on what they are. To be sure, these problems are not easily tackled experimentally, but their handling is not impossible. Thus, for instance, by going further away from the valley of stability the energies available for β-decay between two successive nuclei becomes bigger. This in its turn allows a study of the levels over a larger energy region in the daughter nucleus. Thus it may prove very fruitful to study decay schems of nuclei produced, say, by (p, $x\,n$) reactions where an increasing proton energy results in an increasing number of neutrons which come out, leading to nuclei further removed from the valley of stability. Products of (p, 8 n) reaction on Au could be relatively easily studied [22], and there are good reasons to believe that there are many other favourable regions in the periodic table which could be analysed along similar lines. The availability of high-energy heavy – ion beams should make such studies even more exciting.

Many other methods are nowadays available for accurate systematic study of nuclear properties. In choosing between such methods one should always keep in mind that we have passed the stage of just looking for levels; we are interested today in more information on the properties of various nuclear states and in more reliable information on the existence, *as well as* the non existence, of nuclear levels.

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