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Elastic Scattering of Deuterons by He^4 [1])

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Abstract. A model of the $d + He^4$ interaction is developed and compared to the data on the ground state of Li^6 and the $d + He^4$ elastic scattering data up to 4.5 MeV (deuteron laboratory energy). New phase shift analyses of the 8 and 10.3 MeV elastic scattering data are made, and quantities relevant to the production or analysis of beams of polarized deuteron calculated.

I. Introduction

A model of the $d + He^4$ interaction is developed. The model is in agreement with the ground state properties of Li^6 as deduced by FOLDY [1][4]) from the γd reaction in Li^6. Phase shifts calculated from this model are in agreement with phase shifts found in GALONSKY and MCELLISTREM's [2] analysis of GALONSKY *et al.*'s [3] experimental data on elastic $d + He^4$ scattering in the energy range up to 4.5 MeV (laboratory deuteron energy). Phase shifts calculated from the model make it possible to extend the phase shift analysis to the experimental data of ALLRED *et al.* at 10.3 MeV, and BURGE *et al.* at 8 MeV [4]. From the phase shifts for 8 and 10.3 MeV, we have calculated the quantities relevant to experiments designed to produce or analyze beams of polarized deuterons.

II. The Model

We reduce the problem to a three body problem by treating He^4 as a 'fundamental particle' or 'lump', and imagining the nucleons in the deuteron to interact with He^4 'lump' via the nucleon-He^4 optical model potential [5] and with each other via the nucleon-nucleon potential. We

[1]) This work done under the auspices of the U.S. Atomic Energy Commission and has been published in the Physical Review.

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[4]) Numbers in brackets refer to References, page 421.

solve this three body problem only approximately by applying a no distortion approximation; that is, we imagine the deuteron is not distorted in the course of the $d + \mathrm{He}^4$ interaction.

Thus we neglect the possibility of deuteron break up in the interaction. We hope to estimate the effects of the breakup in future work (in progress with Mr. JOHN H. CHRISTY). Our calculation of the deuteron spin polarization quantities is thus suspect at 8 and 10.3 MeV but may serve as a guide in the planning of experiments.

In order to formulate the model mathematically, we wrote the deuteron wave function in the form

$$\varphi(\boldsymbol{r}) = [u(r) + \frac{1}{\sqrt{8}}\, w(r)\, S_{12}]\, x_1^m$$
$$S_{12} = \frac{3\, \boldsymbol{\sigma}_n \cdot \boldsymbol{r}\, \boldsymbol{\sigma}_p \cdot \boldsymbol{r}}{r^2} - 1 \tag{1}$$

where $\boldsymbol{r}$ is $\boldsymbol{r}_p - \boldsymbol{r}_n$, x_1^m is a triplet spin function, and $\boldsymbol{\sigma}_n$ and $\boldsymbol{\sigma}_p$ are spin operators for neutron and proton, respectively. The total wave function is taken to be

$$\Psi = \varphi(\boldsymbol{r})\, F(\boldsymbol{q}) \tag{2}$$

where $\boldsymbol{q}$ is the vector from the He^4 lump to the center of gravity of the deuteron

$$\boldsymbol{q} = \frac{1}{2}\,(\boldsymbol{r}_p + \boldsymbol{r}_n) - \boldsymbol{r}_{He}. \tag{3}$$

Substituting Ψ into the Schrödinger equation, we obtain the no distortion approximation by multiplying by $u(r) + w(r)\, S_{12}$ and integrating over $\boldsymbol{r}$. In this way we obtain an equation for $F(\boldsymbol{q})\, x_1^m$, in which $V(\boldsymbol{q})$ appears as a potential, where

$$V(\boldsymbol{q}) = \int d\boldsymbol{r} \left(u + \frac{1}{\sqrt{8}}\, w\, S_{12}\right) \left[2\, V_c + V_{LS}\, (\boldsymbol{\sigma}_n + \boldsymbol{\sigma}_p) \cdot \left(\boldsymbol{q} + \frac{1}{2}\, \boldsymbol{r}\right)\right] \times$$
$$\times \left(\frac{1}{2}\, \nabla_q + \nabla_r\right)\Big] \left(u + \frac{1}{\sqrt{8}}\, w\, S_{12}\right). \tag{4}$$

In equation (4), V_c and V_{LS} (the nucleon He^4 optical model potentials) have $|\, \boldsymbol{q} + 1/2\, \boldsymbol{r}\,|$ as argument. Terms with $\boldsymbol{q} - 1/2\, \boldsymbol{r}$, which may also appear with $1/2\, \nabla_q - \nabla_r$, have been transformed by the substitution $\boldsymbol{r} \to -\boldsymbol{r}$ in deriving equation (4).

One may wonder about the question of conservation of total angular momentum. The $V(\boldsymbol{q})$ given by equation (4) is an operator; we have not yet made a partial wave expansion of $F(\boldsymbol{q})\, x_1^m$.

We find by performing the azimuthal integrations in equation (4) (referring $\boldsymbol{r}$ to $\boldsymbol{q}$ as polar axis)

$$V(\boldsymbol{q}) = V_c(q) + V_{LS}(q)\,(\boldsymbol{\sigma}_n + \boldsymbol{\sigma}_p)\cdot \boldsymbol{q}\times\nabla_q + V_T(q)\,\frac{\boldsymbol{\sigma}_n\cdot\boldsymbol{q}\;\boldsymbol{\sigma}_p\cdot\boldsymbol{q}}{q^2} \tag{5}$$

where [6]

$$\left.\begin{aligned}
V_C\,(q) &= 2\int d\boldsymbol{r}\,u'(r)\,V_C\left(\left|\boldsymbol{q}+\frac{1}{2}\,\boldsymbol{r}\right|\right)u'(r) + \\
&\quad + 2\int d\boldsymbol{r}\,u'(r)\,V_C\left(\left|\boldsymbol{q}+\frac{1}{2}\,\boldsymbol{r}\right|\right)w'(r)\,, \\
V_{LS}\,(q) &= \frac{1}{2}\int d\boldsymbol{r}\,u'(r)\,V_{LS}\left(\left|\boldsymbol{q}+\frac{1}{2}\,\boldsymbol{r}\right|\right)u'(r) + \\
&\quad + \frac{1}{4}\int d\boldsymbol{r}\,u'(r)\,V_{LS}\left(\left|\boldsymbol{q}+\frac{1}{2}\,\boldsymbol{r}\right|\right)u'(r)\cos(\boldsymbol{q},\boldsymbol{r})\,\frac{r}{q} + \\
&\quad + \frac{1}{2}\int d\boldsymbol{r}\,u'(r)\,V_{LS}\left(\left|\boldsymbol{q}+\frac{1}{2}\,\boldsymbol{r}\right|\right)w'(r)\sin^2(\boldsymbol{q},\boldsymbol{r})\,, \\
V_T\,(q) &= 4\int d\boldsymbol{r}\,u'(r)\,V_C\left(\left|\boldsymbol{q}+\frac{1}{2}\,\boldsymbol{r}\right|\right)w'(r)\,P_2(\boldsymbol{q},\boldsymbol{r})\,.
\end{aligned}\right\} \tag{6}$$

In equation (6), we have used the substitutions

$$u' = u - \frac{w}{\sqrt{8}}\,, \tag{7}$$

$$w' = 3\,\frac{w}{\sqrt{8}}\,.$$

Also $(\boldsymbol{q}, \boldsymbol{r})$ means the angle between $\boldsymbol{q}$ and $\boldsymbol{r}$. We have neglected integrals involving w'^2 (because the percent of D state in the deuteron is small).

In calculations, we have used for the deuteron wave function [7]

$$u(r) = N\cos\delta\,[1-\exp(-\beta\,x)]\exp(-x)\,.$$

$$w(r) = N\sin\delta\,[1-\exp(-\gamma\,x)]\exp(-x)$$

$$\left[1+\frac{3\,(1-\exp(-\gamma x))}{x}+\frac{3(1-\exp(-\gamma x))^2}{x^2}\right], \tag{8}$$

$$x = \alpha\,r$$

$$N = 0.875041 \qquad \gamma = 2.0170 \qquad \cos\delta = 0.99947$$

$$\beta = 4.7533 \qquad \sin\delta = 0.03356 \qquad \alpha = 0.23181750\,f^{-1}$$

and for the nucleon He^4 optical model potential [5]

$$\left.\begin{aligned} V_C(r) &= V_C \Big/ \left[1 + \left(\frac{r}{D} - 1\right) \exp\left(\frac{r-R}{D}\right)\right], \\ V_{LS}(r) &= V_{LS} \exp\left(\frac{r-R}{D}\right) \Big/ \left[\qquad\qquad\right]^2, \end{aligned}\right\} \tag{9}$$

where (these values are rough average values from reference [5])

$$\left.\begin{aligned} V_C &= 57.6 \text{ MeV} \\ V_{LS} &= 16.75 \text{ MeV} \\ R &= 1.775 \text{ f} \\ D &= 0.8875 \text{ f} \end{aligned}\right\} \tag{10}$$

Graphs of $V_C(q)$, $V_{LS}(q)$, and $V_T(q)$ so calculated are shown in figure 1.

III. Qualitative Features of the Model

The nucleon He^4 optical model predicts a bound state of Li^5(or He^5) if the exclusion principle is not taken into account. When the exclusion principle is taken into account, the bound state disappears because the amplitude of the properly anti-symmetrized wave function formed from the un-anti-symmetrized wave function corresponding to this bound state vanishes. Essentially, one is trying to put three protons (or neutrons) in the 1 S shell in anti-symmetrizing the wave function. These spurious bound states arise in other problems [8] and are well understood.

We must anticipate that spurious bound states will arise from the deep central potential shown in figure 1. In all, two S_1 bound states and one P_0, one P_1, and one P_2 bound states (the first and last with admixtures of D_1 and F_2 states, respectively) are found to exist with the potentials shown in figure 1.

The lowest S state is deep (more than 10 MeV of binding), as are the three P states (more than 5 MeV of binding). It is known from the mass defects of d, He^4, and Li^6 that the binding energy of Li^6 relative to separated d and He^4 is 1.52 MeV, and the binding energy of the second S state agrees closely with this value.

Without further study, we accept the lowest S level and the three P levels as spurious in the sense that if we knew how to anti-symmetrize the corresponding wave functions, we would find the anti-symmetrized wave function to vanish. We accept the 2 S level as the ground state of Li^6.

From a study of the γd reaction in Li^6, FOLDY [1] has concluded that $\langle r^2 \rangle$ in Li^6 is given approximately by the size of a deuteron, and that

$\langle q^2 \rangle$ is much larger than $\langle r^2 \rangle$. We get automatically that $\langle r^2 \rangle$ is given by the size of a deuteron in our model, of course. $\langle q^2 \rangle$ will turn out to be large for two reasons: first, it will be related to the binding energy of d and He^4 relative to Li^6 (1.52 MeV) which is less than the deuteron binding energy (2.22 MeV). The fact that the bound state is a $2\,S$ level means that there is a node in the wave function, which further increases $\langle q^2 \rangle$ over what one might expect. We find

$$\left.\begin{array}{ll} \langle r^2 \rangle = 11\ f^2 \text{ calculated} & \langle r^2 \rangle = 12\ f^2 \text{ Foldy} \\ \langle q^2 \rangle = 15\ f^2 & \langle q^2 \rangle = 22\ f^2 \end{array}\right\} \tag{11}$$

whereas, as FOLDY points out

$$\left.\begin{array}{l} \langle r^2 \rangle = 41.80\ f^2 \quad \text{shell modell} \\ \langle q^2 \rangle = 10.45\ f^2 \end{array}\right\}. \tag{12}$$

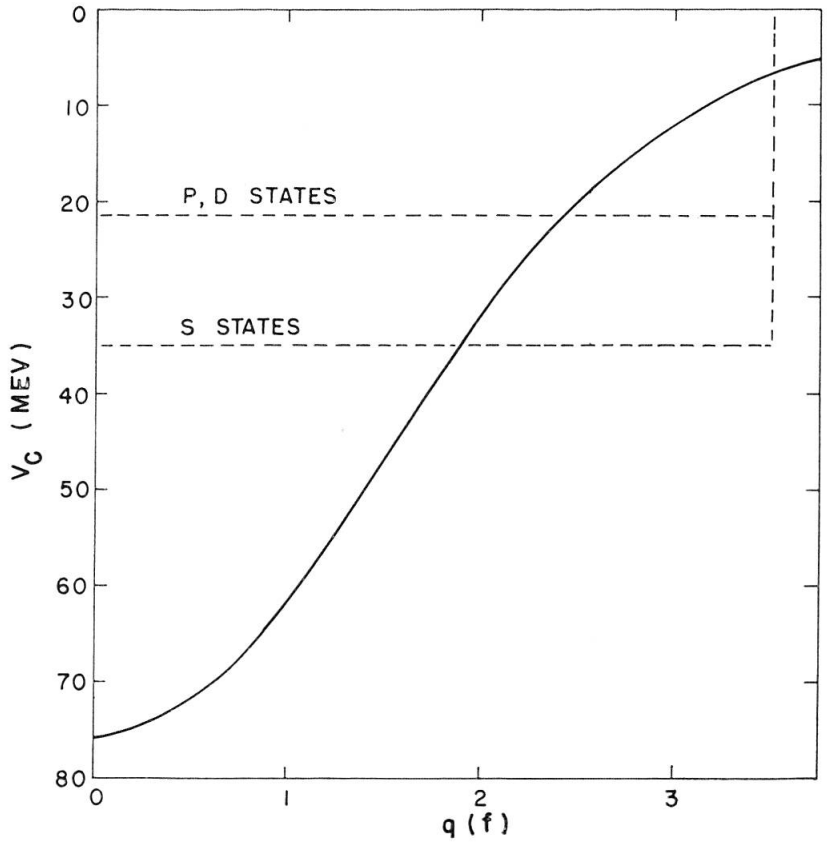

Figure 1a

The central term in the $d + He^4$ interaction. The solid curve is calculated from the model; the dashed curve is an equivalent square well potential adjusted to fit certain data.

It has to be concluded that the $d + He^4$ model of Li^6 is superior to the shell model in this respect. We now calculate the location of the virtual (unbound) D levels in order to show that the $d + He^4$ model also works for these levels. We treat the virtual D levels by calculating scattering phase shifts [9], of course.

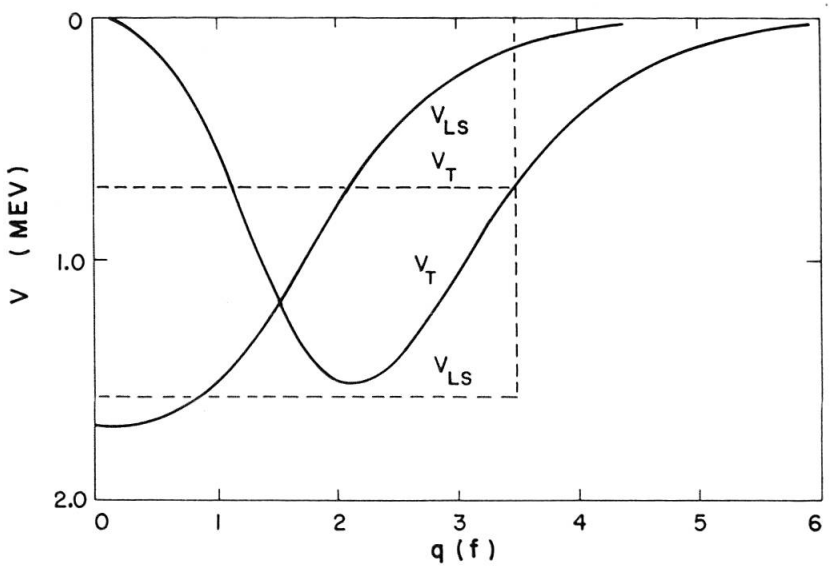

Figure 1 b

The tensor and spin orbit terms in the $d + He^4$ interaction. The solid curves are calculated from the model; the dashed curves are equivalent square well potentials adjusted to fit certain data.

We anticipate the following qualitative features. The S phase shift starts at 2π at zero energy and decreases with energy; that is (when the 2π is substracted from it) the S phase shift will look something like the S phase shift for hard sphere scattering for some radius hard sphere. This behavior agrees with the result of GALONSKY and MCELLISTREM [2]. The P phase shifts start at π at zero energy and decrease with energy. Because the tensor and spin orbit terms shown in figure 1 are weak, compared to the central term, there will not be much J splitting unless there is a resonance. Calculation shows there is no resonance and (as expected) little J splitting. This behaviour also agrees with the results of GALONSKY and MCELLISTREM, who found small, negative unsplit P phase shifts.

By inspection of figure 1, and from a knowledge of the interval rules [10] for the spin orbit and tensor terms in equation (5), it can be seen that the order of the D levels as given by GALONSKY and MCELLISTREM is the same as the order given by the $d + He^4$ model.

Only detailed computation can confirm whether or not the magnitude of the S phase shift is correct, whether or not the D phase shifts pass

through 90° at the right energy and with the right level width, and whether or not the tensor coupling of the $^3S_1 + {}^3D_1$ state is negligible as GALONSKY and MCELLISTREM assumed. We may summarize our calculations very shortly by saying simply that everything calculated from the $d + He^4$ model does, in fact, agree with the results of GALONSKY and MCELLISTREM, and that the $d + He^4$ model makes it possible to extrapolate their work to 8 and 10.3 MeV, at which energies the tensor coupling is important.

IV. Summary of Calculations Based on the Model

The $^3S_1 + {}^3D_1$ phase shifts, including the coupling constant, are shown as a function of energy in figure 2. The coupling parameter 2ε is negligible for $E_d < 4.5$ MeV, and the $^3S_1$ and $^3D_1$ phase shifts are in reasonable (if not excellent) agreement with the phase shifts of GALONSKY and MCELLISTREM.

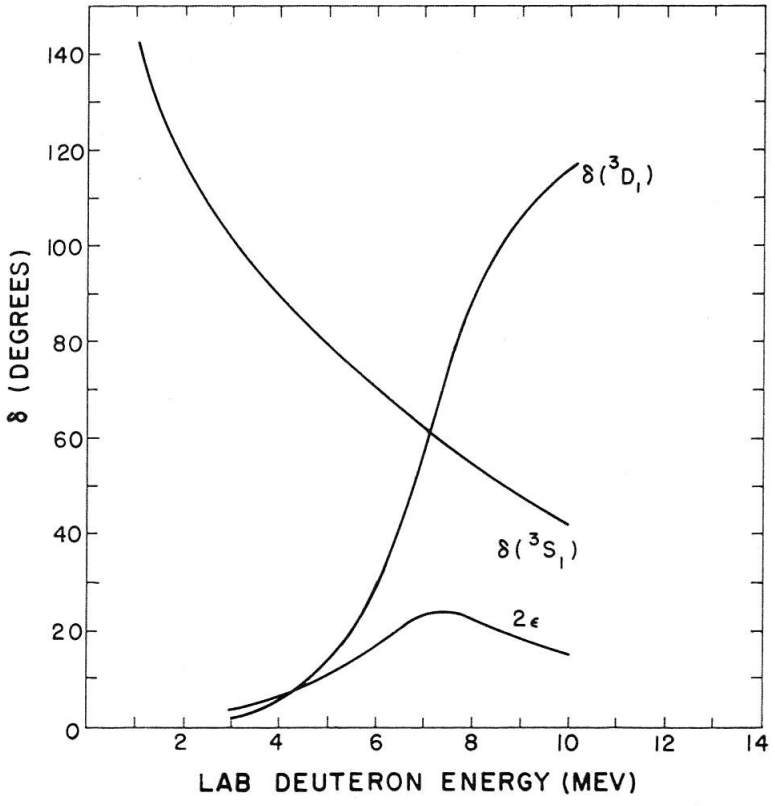

Figure 2

$^3S_1 + {}^3D_1$ phase shifts calculated from the model as a function of energy

Rather than calculating the $^3D_2$ and $^3D_3$ phase shifts directly from the model, we have adjusted a 'phenomenological' square well potential

central, tensor, and spin orbit terms to fit the following data: the $2\,S$ level must have the right binding energy (1.52 MeV), the $^3D_3$ and $^3D_2$ phase shifts must pass through 90° at the energies given by GALONSKY and MCELLISTREM, and the width of the $^3D_3$ resonance must agree with the width given by GALONSKY and MCELLISTREM. This last condition determines the range of the square well (we assume that all three terms have the same range) and the first three conditions determine the depth of each term[5]). This square well potential is compared with the model potential in figure 1. The agreement is reasonable. P and $^3D_2$ and $^3D_3$ phase shifts calculated from the square well potential are given in table 1 as a function of energy.

We have used phase shifts taken from table 1 and figure 2 as a starting point for a phase shift analysis of the 8 and 10.3 MeV data. The phase shifts which we find to give the best fit to the data are similar to the phase shifts used as a starting point as shown in table 2a; however, the magnitude of the P phase shift is much smaller than expected from table 1, and also smaller than expected from a graphical extrapolation of GALONSKY and MCELLISTREM's phase shift. The quality of the fit to the data is shown in tables 2b and 2c; the *rms* deviation is a little over twice the errors assigned by the experimentalists in the case of 10.3 MeV.

The quantities relevant to experiments designed to produce or analyze beams of polarized deuterons are the expectation values of four operators [11]. These four quantities are denoted differently by various authors.

WOLFENSTEIN's notation [12]		STAPP's notation
$\langle T_{20}\rangle$	=	$t/\sqrt{2}$
$\langle T_{11}\rangle$	=	$-u/\sqrt{3}$
$\langle T_{21}\rangle$	=	$-v/\sqrt{3}$
$\langle T_{22}\rangle$	=	$-w/2\sqrt{3}$

(13)

The quantities t, u, v, w are given in table 3 as a function of angle for the two energies and also 4.5 MeV. They have been calculated at lower energies (in the vicinity of the $^3D_3$ resonance, namely 1.1 MeV) by PONDROM [13] and also at 2.5 and 3.5 MeV by PHILLIPS [14]. We have checked these calculations.

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[5]) This is not really so since as indicated in fig. 1 we have used different central terms in the S and D states. Thus the square well potentiel in fig. 1 is 'a' potential (and not 'the' potential) which fits the D_2 and D_3 phase shifts of GALONSKY and MCELLISTREM (see also remarks in footnote to table I).

Table 1

Phase shifts versus energy calculated from the square well potentials shown in fig. 1*. The φ_l are Coulomb phase shifts $\varphi_0 = 0$, $\varphi_l = \varphi_{l-1} + 2 \arctan \eta/l$.

E (MeV)	$\delta(P_0) + \frac{\varphi_1}{2}$	$\delta(P_1) + \frac{\varphi_1}{2}$ (radians)	$\delta(P_2) + \frac{\varphi_1}{2}$	$\delta(D_2) + \frac{\varphi_2}{2}$	$(\delta D_3) + \frac{\varphi_2}{2}$
1	0.3692	0.3749	0.3828	0.6428	0.8114
2	0.1377	0.1530	0.1757	0.4969	0.3531
3	−0.0417	−0.0189	0.0165	0.5509	0.2237
4	−0.1947	−0.1663	−0.1206	1.1065	0.1168
5	−0.3290	−0.2964	−0.2424	−0.9075	0.0226
6	−0.4489	−0.4131	−0.3520	−0.5949	−0.0632
7	−0.5572	−0.5187	−0.4515	−0.5368	−0.1429
8	−0.6558	−0.6150	−0.5422	−0.5435	−0.2178
9	−0.7461	−0.7033	−0.6252	−0.5734	−0.2887
10	−0.8291	−0.7844	−0.7012	−0.6133	−0.3562
11	−0.9057	−0.8592	−0.7710	−0.6576	−0.4207
14	−1.1029	−1.0509	−0.9468	−0.7980	−0.6532

*) The square well potentials in fig. 1 would give the D phase shifts shown; however, the potential actually used in this calculation is $V_C = 22.100$ MeV, $V_{LS} = 1.3667$ MeV, $V_T = 0$, so that the P phase shifts should be slightly different. However, the main point is that the P phase shifts are negative and not much split; their precise magnitude is not important.

Table 2a

Phase shift analysis for 8 and 10.3 MeV

E	8 MeV		10.3 MeV	
	starting point	*end point*	*starting point*	*end point*
$\delta(^3S_1)$	0.9840	0.9489	0.7120	0.7339
$\delta(^3D_1) + \varphi_2/2$	1.5411	1.7764	2.0516	2.2595
2ε	0.3990	0.3990	+0.2500	+0.2500
$\delta(^3D_2) + \varphi_2/2$	−0.5435	−1.266	−0.6133	−0.6263
$\delta(^3D_3) + \varphi_2/2$	−0.2178	−0.2965	−0.3562	−0.1760
$\delta(^3P_0) + \varphi_1/2$	−0.6558	+0.1566	−0.8291	+0.4111
$\delta(^3P_1) + \varphi_1/2$	−0.6150	−0.1566	−0.7844	−0.1212
$\delta(^3P_2) + \varphi_1/2$	−0.5422	−0.1566	−0.7012	−0.1627

For 8 MeV, the Coulomb phase shifts are $\varphi_1/2 = 0.1566$ radians, $\varphi_2/2 = 0.2354$ radians, and for 10.5 MeV, $\varphi_1/2 = 0.1383$ radians, $\varphi_2/2 = 0.2077$ radians.

V. Summary

The model we have given for the d + He^4 interaction may be of interest for d + heavy nucleus interaction.

Table 2b

Quality of fit to differential cross section at 8 MeV

θ(cm) (degrees)	σ_{exp}(barns)	σ_{calc}(barns)
30	0.3300	0.3520
40	0.1760	0.1838
50	0.0920	0.0820
60	0.0395	0.0385
70	0.0415	0.0395
80	0.0660	0.0623
90	0.0830	0.0837
100	0.0950	0.0900
110	0.0860	0.0822
120	0.0730	0.0734
130	0.0750	0.0809
140	0.1070	0.1153
150	0.1860	0.1747
160	0.2400	0.2431

Table 2c

Quality of fit to differential cross section at 10.3 MeV

θ(cm) (degrees)	σ_{exp} (barns)	σ_{calc}(barns)	θ(cm) degrees	σ_{exp}(barns)	σ_{calc}(barns)
18 4	0.452	0.5445	100.6	0.0679	0.0713
22.0	0.356	0.3797	101.2	0.0709	0.0710
23.0	0.326	0.3479	103.6	0.0668	0.0694
29.4	0.186	0.2093	109.2	0.0616	0.0629
34.0	0.143	0.1460	110.6	0.0587	0.0608
36.6	0.111	0.1176	112.4	0.0576	0.0580
40.8	0.0761	0.0803	114.6	0.0519	0.0542
43.8	0.0571	0.0593	119.8	0.04447	0.0453
48.2	0.0369	0.0361	120.6	0.0418	0.0440
51.0	0.0223	0.0256	124.6	0.0378	0.0381
55.4	0.0105	0.0155	127.2	0.0362	0.0351
58.4	0.00975	0.0125	129.3	0.0370	0.0332
62.5	0.0118	0.0129	133.6	0.0354	0.0312
65.4	0.0171	0.0158	134.4	0.0371	0.0312
69.4	0.0217	0.0225	137.8	0.0400	0.0320
72.2	0.02535	0.0285	140.6	0.0436	0.0340
76.1	0.0343	0.0377	141.7	0.0473	0.0351
78.8	0.04115	0.0444	145.3	0.04655	0.0401
82.6	0.0481	0.0532	147.4	0.05365	0.0437
85.2	0.0548	0.0586	149.0	0.05845	0.0468
85.4	0.05785	0.0590	152.3	0.0706	0.0540
91.6	0.0629	0.0684	154.1	0.07275	0.0582
95.2	0.0694	0.0712	155.4	0.07425	0.0613
97.7	0 0691	0.0718	157 4	0.0824	0.0662

Table 3a

The function t versus θ

θ(cm) (degrees)	4.5 MeV	8 MeV	10.3 MeV
20	0.2155	—	− 0.005
30	0.3327	− 0.1044	− 0.045
40	0.1938	− 0.2256	− 0.115
50	− 0.0899	− 0.4663	− 0.285
60	− 0.3618	− 0.6060	− 0.240
70	− 0.2698	− 0.0285	0.150
80	0.1778	0.3170	0.220
90	0.4155	0.3712	0.235
100	0.2675	0.3192	0.250
110	− 0.0790	0.1777	0.255
120	− 0.2920	− 0.0621	0.230
130	− 0.1749	− 0.2526	0.080
140	0.1300	− 0.2323	− 0.195
150	0.4247	− 0.1285	− 0.305
160	0.6339	− 0.0465	− 0.310
170	0.7527	—	− 0.310

Table 3b

The function u versus θ

θ(cm) (degrees)	4.5 MeV	8 MeV	10.3 MeV
20	− 0.2449	—	0.23
30	0.4961	0.6615	0.395
40	0.7173	0.9326	0.595
50	0.9201	1.093	0.61
60	0.9944	0.382	− 0.40
70	0.7324	− 0.740	− 0.76
80	0.2773	− 0.514	− 0.31
90	0	− 0.024	0
100	− 0.1056	0.433	0.285
110	− 0.1726	0.810	0.57
120	− 0.2509	0.884	0.875
130	− 0.3019	0.423	0.98
140	− 0.2934	− 0.093	0.52
150	− 0.2403	− 0.273	0.095
160	− 0.1660	− 0.243	—
170	− 0.0841	—	—

It is our hope that the analysis of the 8 and 10.3 MeV data has led to correct estimates of the quantities required to plan experiments with polarized deuteron beams.

Table 3c

The function v versus θ

θ(cm) (degrees)	4.5 MeV	8 MeV	10.3 MeV
20	0.1139	—	0
30	0.3292	0.0424	0
40	0.5421	0.0910	0
50	0.7426	0.1650	0
60	0.8433	0.1913	−0.06
70	0.6664	0.0043	−0.11
80	0.2916	−0.1044	−0.115
90	0.	−0.1421	−0.12
100	−0.2155	−0.1762	−0.125
110	−0.4485	−0.2270	−0.115
120	−0.6624	−0.2794	−0.07
130	−0.7369	−0.2647	0.08
140	−0.6596	−0.1812	0.24
150	−0.5074	−0.1060	0.225
160	−0.3363	−0.0577	0.13
170	−0.1664	—	—

Table 3d

The function w versus θ

θ(cm) (degrees)	4.5 MeV	8 MeV	10.3 MeV
20	−0.0885	—	−0.045
30	−0.3802	−9.2434	−0.163
40	−0.8268	−0.5699	−0.44
50	−1.3568	−1.115	−0.98
60	−1.5953	−1.226	−0.8
70	−0.8091	0.3131	0.60
80	0.6021	1.029	0.73
90	1.2460	1.047	0.65
100	0.8842	0.8240	0.52
110	−0.0782	0.3703	0.315
120	−0.9736	−0.3351	−0.055
130	−1.2392	−0.8482	−0.62
140	−0.9820	−0.7421	−0.85
150	−0.5860	−0.4186	−0.52
160	−0.2615	−0.1743	−0.23
170	−0.0648	—	—

Appendix

The partial wave expansion of $F(\boldsymbol{q})\ x^1_m$ is exactly the same as it is for the triplet state in nucleon-nucleon scattering. The interval rule for

the term $(\boldsymbol{\sigma}_n + \boldsymbol{\sigma}_p) \cdot \boldsymbol{q} \times \nabla_q$ is the same as the interval rule for the spin orbit term; namely

$$\frac{1}{2}[J(J+1) - L(L+1) - S(S+1)]$$

and since $\boldsymbol{\sigma}_n \cdot \boldsymbol{q}\, \boldsymbol{\sigma}_p \cdot \boldsymbol{q}/q^2$ is $(S_{12}+1)/3$, its properties when operating on states of definite J and L may be read from a table of the properties of S_{12} (see, for example, ASHKIN and WU, Phys. Rev. *73*, 982 (1948) their equation (26)).

We have used the so called nuclear bar phase shifts in parametrizing the scattering matrix. The formulas for the elements of the scattering matrix in terms of these phase shifts are given in H. P. STAPP's thesis (UCRL 3098, unpublished), p. 107, equation A. 20. GALONSKY and MCELLISTREM introduce 5 scattering amplitudes A, B, C, D, E and give formulae for these in terms of the elements of the scattering matrix (see their Appendix I, final equation). STAPP, on the other hand, uses only 4 scattering amplitudes *a*, *b*, *c*, *d*. (See his equation (22), p. 75.) Therefore Galonsky and McEllistrem's amplitudes must be connected; we find

$$\frac{E \sin^2\theta}{\sqrt{2}} = A - B + \cos\theta\,(C - D)$$

and

$$b = -\frac{(C+D)\sin\theta\, i}{2}$$

$$d = \frac{-C+D}{2}$$

$$a = \frac{1}{3}(2A + B)$$

$$c = -2A + 2B - \frac{3}{2}C\cos\theta + \frac{3}{2}D\cos\theta$$

This last formula is, indirectly (via GALONSKY and MCELLISTREM's last equation in their Appendix I and STAPP's equation A. 20), an expression for STAPP's amplitudes in terms of the bar phase shifts.

For the calculation of quantities relating to deuteron spin polarization phenomena, we use STAPP's equations on his pp. 76 *et seq.*

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[9] The literature on the theory of $d + He^4$ scattering (definition and calculation of phase shifts and connection between phase shifts and scattering amplitudes) is reviewed in the Appendix.
[10] See the Appendix for the interval rules.
[11] See the Appendix for an outline of the theory.
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