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Description of Unstable Particles by Nonunitary Irreducible Representations of the Poincaré Group[1])

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Abstract. A formalism is presented which allows a simultaneous treatment of independent unstable particles (in a beam) and unstable intermediate states in a scattering process. The unstable particles are described by non-unitary representations of the Poincaré group. The quantum mechanical formalism appropriate to cope with non-unitary group representations is presented. It is found that the spin of an unstable particle is not necessarily a well defined quantity. For an intermediate state formed by a particle which does have a well defined spin the usual resonnance formula is recovered.

1. Introduction

In S-matrix theory unstable particles are described as poles or resonances of the form $(s - m^2)^{-1}$ in the scattering amplitudes. The following properties, formulated in the angular momentum decomposition in the center of mass frame [1], are usually postulated [2]:

i) The pole appears only in the amplitudes for one fixed total spin j.

ii) The residue is factorizable.

Property (ii) expresses the assumption, characteristic for the particle interpretation, that decay and production are independent processes.

Property (i) is also supposed to be a consequence of the particle interpretation since a particle should have a well defined spin. However, while for a stable particle this is inferred from the representation theory of the Poincaré group [3], no such argument can be used for unstable particles. A simple physical argument illustrates this: The spin of a particle must be defined in its restframe. Thus to measure it (e.g. by studying the angular distribution of its decay) one has to know the velocity of the particle. However, to define the velocity exactly (e.g. by a time of flight measurement) one needs a measurement extending over an infinite distance, due to the uncertainty principle. This is impossible for an unstable particle. Thus from the point of view of what can be measured on the particle itself the spin of an unstable particle is not necessarily a well defined quantity [4].

[1]) Part of this work was supported by the US Atomic Energy Commission while the author stayed at Carnegie-Mellon University, Pittsburgh, Pennsylvania until Aug. 1969.

In the following we present a formalism which allows a simultaneous treatment of independent unstable particles (in a beam) and unstable particle intermediate states in a scattering process between stable particles (asymptotic states). The fundamental ingredients of the theory are:

a) Invariance under the restricted (connected) Poincaré group [5];

b) Description of unstable particles by nonunitary irreducible representations of the Poincaré group;

c) Causality.

As in the corresponding case of stable particles [3] b) is an assumption which is plausible but for which there exists no proof. It incorporates all intuitive notions about particles, in particular the one leading to (ii) above.

The formal equivalent of the above physical argument follows immediately: Only a very special class of 'useful' representations of the Poincaré group, the socalled degenerate representations, have the rotation group as little group and thus possess a well defined intrinsic spin.

Naturally quantum mechanics has to be adapted somewhat in order to accommodate non-unitary representations of the Poincaré group.

The main object of this paper is to present a consistent formalism for the description of unstable particles by non-unitary representations of the Poincaré group. This formalism is then applied to the special case of the degenerate representations yielding the familiar features generally attributed to unstable particles with a well defined spin.

The application to the general and particularly interesting case of the non-degenerate representations which correspond to particles which do not have a well defined intrinsic spin will be treated in a subsequent paper. Only some of their qualitative features are discussed here.

The relevant representations of the Poincaré group are briefly presented in sect. 2. In sect. 3 the quantum mechanical formalism is developped in a way which is adequate to handle states which transform according to nonunitary representations of the Poincaré group. This leads automatically to the sesquilinear systems introduced by Fell [6] to discuss general nonunitary group representations.

In sect. 4 we treat an unstable particle with well defined spin, i.e. transforming according to a degenerate representation, as intermediate state in a scattering process between stable particles, and recover the well-known resonnance formula. Qualitative features of the nondegenerate representations (which do not possess a definite spin) are discussed in sect. 5 and conclusions as well as some additional remarks are presented in the last section (sect. 6).

2. Nonunitary Irreducible Representations of the Poincaré Group

Nonunitary irreducible representations of the connected Poincaré group or its covering group i SL(2, C) are given, as a side result, by Roffman [7]. We discuss here only the relevant ones.

The translation group is represented by [8]

$$a \to e^{iap} \tag{2.1}$$

with, due to non-unitarity, complex p,

$$p = u - i\,v\,. \tag{2.2}$$

For the present application u and v are both real positive (for antiparticles negative) timelike vectors ensuring that the particle decays in the forward (backward) light cone as is required to insure causal behaviour.

Since u and v are transformed separately by a real Lorentz transformation one recognizes immediately the two fundamentally different cases characterized by whether u and v are proportional to each other or not.

A. *Degenerate representation*

Here v is proportional to u and with the restriction discussed after (2.2) we can represent p by

$$p = m\,q \tag{2.3}$$

where q is a real positive ($q^0 > 0$) timelike vector which we normalise to

$$q^2 = 1\,. \tag{2.4}$$

The complex invariant mass m has positive real and negative imaginary part.

As for real m the little group is the 3-dimensional rotation group (or SU(2)). Since it is compact one can always choose its representation unitary [10]. Thus the kinematics are essentially the same here as for unitary representations of i SL(2, C) which naturally are a special case of degenerate representations. The representations are, as usual, labelled by $[m, j]$, where j is the intrinsic spin.

B. *Nondegenerate representations*

Here u and v are linearly independent. The invariants are

$$u^2 = \mu^2\,, \qquad v^2 = \gamma^2\,, \qquad u\,v = \mu\,\gamma\,\xi\,, \tag{2.5}$$

and the causality restriction imposed on u and v as discussed after (2.2) implies

$$\xi > 1\,. \tag{2.6}$$

The little group in this case is the onedimensional rotation group U(1) as is easily seen if v is transformed to rest,

$$v = (\gamma, 0, 0, 0)\,, \tag{2.7}$$

and u brought to the form

$$u = (u^0, 0, 0, u^3) \tag{2.8}$$

by a rotation. The only transformation leaving u and v invariant is then a rotation around the third axis.

The irreducible nondegenerate representations of interest are thus completely labelled by $[\mu, \gamma, \xi, \sigma]$ where σ is a positive or negative integer or half integer [11] fixing the (unitary) representation of U(1):

$$\varphi \to e^{i\sigma\varphi}\,. \tag{2.9}$$

3. Quantum Mechanics with Nonunitary Representations

A. *Sesquilinear system and invariant scalar product*

The formalism presented here was originally introduced by Fell [6, 7] as a mathematical tool to construct nonunitary linear representations. The motivation to introduce it here is the requirement that we must be able to define states and measurements in a relativistically invariant way. In particular we have to require that the result of a measurement is invariant under simultaneous translations of the state (e.g. beam) and the measuring aparatus (e.g. counter).

A sesquilinear system is a pair of complex linear spaces E and $\tilde{E}$ together with a sesquilinear form (scalar product) $(\tilde{E}, E)$ obeying

$$(\pi, \lambda\,\psi + \mu\,\psi') = \lambda(\pi, \psi) + \mu(\pi, \psi')\,,$$
$$(\lambda\,\pi + \mu\,\pi', \psi) = \lambda^*(\pi, \psi) + \mu^*(\pi', \psi)\,, \tag{3.1}$$

and

$$(\pi, E) = 0 \Leftrightarrow \pi = 0\,,$$
$$(\tilde{E}, \psi) = 0 \Leftrightarrow \psi = 0\,, \tag{3.2}$$

with λ, μ complex numbers, $\pi, \pi' \in \tilde{E}$ and $\psi, \psi' \in E$.

If a representation $D(G)$ of a group G (here the Poincaré group) acts on E then we define the representation $\tilde{D}(G)$ acting on $\tilde{E}$ by requiring the scalar product to be invariant:

$$\big(\tilde{D}(g)\,\pi, D(g)\,\psi\big) = (\pi, \psi) \tag{3.3}$$

for all $g \in G$, $\pi \in \tilde{E}$ and $\psi \in E$. Together with the usual definition of the adjoint A^+ of an operator A by

$$(\pi, A\,\psi) = (A^+\,\pi, \psi) \tag{3.4}$$

this implies

$$\tilde{D}(g) = [D^+(g)]^{-1} = D^+(g^{-1})\,. \tag{3.5}$$

Clearly $\tilde{D} = D$ if D is unitary.

In this formalism the elements $\psi \in E$ are identified with the (pure) states of the particle in question while $\pi \in \tilde{E}$ represents, loosely speaking, a measurement or monitoring system applied to the state [12]. Invariance of the Theory is then guaranteed as usual in quantum mechanics by the invariance of the scalar product.

We give an explicit form of the above construction for a degenerate scalar ($j = 0$) representation with complex mass m. The generalisation, especially to degenerate representations with ($j \neq 0$) [13, 14], is straight forward.

E is a linear set of functions $\psi(q)$ with q on the hyperboloid (2.4). Its dual $\tilde{E}$ consists of the functionals $\pi(q)$ with

$$(\pi, \psi) = \int \pi^*(q)\,\psi(q)\,\frac{d^3\boldsymbol{q}}{2\sqrt{1+\boldsymbol{q}^2}}\,. \tag{3.6}$$

The action of an element $[a, \Lambda]$ of the Poincaré group is given by

$$\psi(q) \xrightarrow{[a,\Lambda]} D(a, \Lambda)\, \psi(q) = e^{i m q a}\, \psi(\Lambda^{-1} q)\,,$$
$$\pi(q) \xrightarrow{[a,\Lambda]} \tilde{D}(a, \Lambda)\, \pi(q) = e^{i m^* q a}\, \pi(\Lambda^{-1} q)\,. \tag{3.7}$$

One immediately verifies the invariance of (3.6).

As customary in quantum mechanics it is convenient to introduce momentum eigenstates $|q\rangle$ and $|\tilde{q}\rangle$ with the invariant sesquilinear form appropriate for continuum states

$$(|\tilde{q}\rangle, |q'\rangle) \equiv \langle\tilde{q}|q'\rangle = 2\sqrt{1 + \boldsymbol{q}^2}\, \delta\,(\boldsymbol{q} - \boldsymbol{q}')\,. \tag{3.8}$$

The transformation properties are then

$$|q\rangle \xrightarrow{[a,\Lambda]} D(a, \Lambda)\, |q\rangle = e^{i m a \Lambda q}\, |\Lambda q\rangle\,,$$
$$|\tilde{q}\rangle \xrightarrow{[a,\Lambda]} \tilde{D}(a, \Lambda)\, |\tilde{q}\rangle = e^{i m^* a \Lambda q}\, |\widetilde{\Lambda q}\rangle\,. \tag{3.9}$$

B. *Antiparticles and second quantisation*

In order to be able to construct a local theory we have to introduce also antiparticles. To do so it is convenient to use the language of second quantisation. No deeper meaning in the sense of a field theory is necessarily attributed to it, however. For simplicity the formula are given only for degenerate scalar ($j = 0$) representations. The generalization to degenerate representations with $j \neq 0$ is exactly the same as for stable particles [14], since the transformation properties under the homogeneous Lorentz group, in particular the rotation group, are exactly the same.

We introduce operators $a(q)$ and $\tilde{a}(q)$ and postulate

$$a^+(q) \xrightarrow{[a,\Lambda]} D(a, \Lambda)\, a^+(q)\, D^{-1}(a, \Lambda) = e^{i m a \Lambda q}\, a^+(\Lambda q)\,,$$
$$\tilde{a}(q) \xrightarrow{[a,\Lambda]} D(a, \Lambda)\, \tilde{a}(q)\, D^{-1}(a, \Lambda) = e^{-i m a \Lambda q}\, \tilde{a}(\Lambda q)\,, \tag{3.10}$$

$$a(q)\, |0\rangle = \tilde{a}(q)\, |0\rangle = 0\,, \tag{3.11}$$

$$[\tilde{a}(q), a^+(q)] = 2\, q^0\, \delta\,(\boldsymbol{q} - \boldsymbol{q}')\,, \tag{3.12}$$

all other commutators vanish, and $|0\rangle$ is the invariant vacuum.

The factor

$$q^0 = \sqrt{1 + \boldsymbol{q}^2} \tag{3.13}$$

ensures invariance of (3.12) under the transformations (3.10). Equation (3.10) implies that $\tilde{a}(q) \neq a(q)$ unless m is real i.e. $D(a, \Lambda)^+ = D(a, \Lambda)^{-1}$. In distinction to the case where $a = \tilde{a}$ (3.11) does not follow from the commutation relations but has to be postulated separately.

Clearly a^+ and $\tilde{a}$ are the creation and annihilation operators, respectively, of unstable particles. They are connected to the states in (3.9) by

$$a^+(q)\, |0\rangle = |q\rangle\,, \qquad \tilde{a}^+(q)\, |0\rangle = |\tilde{q}\rangle\,. \tag{3.14}$$

The invariance of (3.12) and thus the transformation properties (3.10) have to be postulated in order to ensure invariance of the theory (emission and reabsorption of a particle must be a translational invariant process).

Introducing antiparticle creation and annihilation operators $b^+(q)$ and $\tilde{b}(q)$ with the requirement that they obey the same postulates (3.10), (3.11) and (3.12) as $a^+(q)$ and $\tilde{a}(q)$ and that they commute with these, we construct the field operators

$$\varphi^+(x) = \left[\frac{|m|^3}{(2\pi)^3 m}\right]^{1/2} \int \frac{d^2\boldsymbol{q}}{2q^0} \left[a^+(q)\, e^{imqx} + \tilde{b}(q)\, e^{-imqx}\right]$$

$$\tilde{\varphi}(x) = \left[\frac{|m|^3}{(2\pi)^3 m}\right]^{1/2} \int \frac{d^3\boldsymbol{q}}{2q^0} \left[\tilde{a}(q)\, e^{-imqx} + \eta\, b^+(q)\, e^{imqx}\right]. \tag{3.15}$$

The role of η will be discussed below. With (3.10) one obtains the transformation properties

$$\varphi^+(x) \xrightarrow{[a,\Lambda]} D(a,\Lambda)\, \varphi^+(x)\, D(a,\Lambda)^{-1} = \varphi^+(\Lambda x + a)\,,$$

$$\tilde{\varphi}(x) \xrightarrow{[a,\Lambda]} D(a,\Lambda)\, \tilde{\varphi}(x)\, D(a,\Lambda)^{-1} = \tilde{\varphi}(\Lambda x + a)\,, \tag{3.16}$$

which of course are the motivation for the construction (3.15).

One easily finds the commutator

$$[\tilde{\varphi}(x), \varphi^+(y)] = i\,[\Delta^+(x-y) - \eta\,\Delta^+(y-x)]\,, \tag{3.17}$$

$$\Delta^+(x) = \frac{-i}{(2\pi)^3}\,\frac{|m|^3}{m} \int \frac{d^3\boldsymbol{q}}{2q^0}\, e^{-imqx}\,. \tag{3.18}$$

Since $Im\, m < 0$ this integral converges only for $(x)^2 \geqslant 0$ and $x^0 > 0$. For the time being we ignore this trouble. A regularization of the only relevant such integral in our theory, the causal propagator, will be given in the next section. Here we remark only the following: In order to have a local theory the right hand side of (3.17) must vanish for $(x-y)^2 < 0$. This is the case for

$$\eta = 1 \tag{3.19}$$

since

$$\Delta(x) = \frac{-i}{(2\pi)^3}\,\frac{|m|^3}{m} \int \frac{d^3\boldsymbol{q}}{2q^0}\, (e^{-imqx} - e^{imqx}) = \Delta^+(x) - \Delta^+(-x)\,, \tag{3.20}$$

(formally) disappears for spacelike x as can be seen by going to a Lorentz frame with $x^0 = 0$ and substituting $\boldsymbol{q}$ by $-\boldsymbol{q}$ in the second term which then cancels the first one. It will be seen in the next section that for the regularized integrals Lorentz invariance also requires (3.19).

If we had chosen the anticommutator in (3.12) the relative sign between the two terms in (3.17) would be opposite. However, we would be able to compensate for this by the choice of the sign of η. Thus, while locality (or Lorentz invariance) forces us to include antiparticles with the same weight as particles [14] it does not allow us to draw conclusions about the connection between spin and statistics in this way. The difference between our case and that of stable particles [14] arises from the fact that we are free to introduce independent factors in $\tilde{\varphi}$ and φ since, in contrast to the usual

formalism, these are not the same objects. We do not claim that anything is wrong with the usual connection between spin and statistics because of that. We only point out that one might have to be a little more careful in some of the proofs since even for stable particles the identification of $\tilde{\varphi}$ and φ is rather a convenient possibility than a necessity.

We finally remark that the fact that η was introduced in (3.15) in an asymmetric way has no meaning. In fact one notes the general feature of the formalism that one can multiply a^+ by any complex factor c, if at the same time one multiplies $\tilde{a}$ by c^{-1}, without affecting (3.12) and thus the whole theory. A similar statement holds clearly also for the formalism of sect. 3A. Naturally c has to be the same for all operators which can be transformed into each other by a Poincaré transformation (i.e. the transformation corresponding to it must be a multiple of unity on the space of each irreducible representation). This corresponds to the freely choosable phase in the usual formalism of quantum theory.

4. The Resonnance Formula

In this section we study unstable particles described by a degenerate representation as intermediate states in a scattering process between stable particles. This leads to the usual resonnance formula in the scattering amplitude. As before the calculations are performed only for $j = 0$. The generalization to arbitrary spin follows directly from Weinberg's work [14].

The object of interest is the Feynman propagator

$$\begin{aligned} i\,\Delta_F\,(x-y) &= \langle 0 \,|\, T\,\tilde{\varphi}(x)\,\varphi^+(y)\,|0\rangle \\ &= \vartheta\,(x-y)\,\langle 0\,|\,\tilde{\varphi}(x)\,\varphi^+(y)\,|0\rangle + \vartheta\,(y-x)\,\langle 0\,|\,\varphi^+(y)\,\tilde{\varphi}(x)\,|0\rangle\,. \end{aligned} \tag{4.1}$$

It has the formal representations

$$\Delta_F(x) = \vartheta(x)\,\Delta^+(x) + \vartheta(-x)\,\Delta^+(-x) = \frac{1}{2}\,[\varepsilon(x)\,\Delta(x) - i\,\Delta_1(x)]\,, \tag{4.2}$$

where $\Delta(x)$ is given by (3.18) and

$$\varepsilon(x) = \vartheta(x) - \vartheta(-x)\,, \qquad \Delta_1(x) = i\,[\Delta^+(x) + \Delta^+(-x)]\,.$$

The propagator Δ_F is a relativistic scalar in spite of the stepfunction due to the fact that $\Delta(x)$ vanishes for spacelike x, and thus due to (3.19).

We now proceed to regularize $\Delta_F(x)$ i.e. give it a meaning for all x even where the integrals in the original definition diverge. First of all we remark that both terms of the first form of (4.2) are well defined for time like or light like x. Assuming therefore $x^2 \geqslant 0$ for all manipulations we bring

$$P(x) \equiv \vartheta(x)\,\Delta^+(x) = \frac{-i}{(2\pi)^3}\,\vartheta(x)\,\frac{|m|^3}{m}\int \frac{d^3\boldsymbol{q}}{2\,q^0}\,e^{-im\,q\,x} \tag{4.3}$$

into a more suitable form.

Using $p = m\,q$ as the integration variable we write (4.3) as a complex integral

$$P(x) = \frac{-i}{(2\pi)^3}\,\vartheta(x)\int\limits_C \frac{d^3\boldsymbol{p}}{2\,\omega(\boldsymbol{p})}\,e^{-iw(\boldsymbol{p})x^0+i\boldsymbol{p}\cdot\boldsymbol{x}} \tag{4.4}$$

$$\omega(\boldsymbol{p}) = m\sqrt{1+\boldsymbol{q}^2} = \sqrt{m^2+\boldsymbol{p}^2}\,, \tag{4.5}$$

where the integration path C is the same for all p^k, $k = 1, 2, 3$ and given by the oriented straight line through $-m$ and m. The branch of the square root in (4.5) is defined by taking the positive sign of $\sqrt{1+\boldsymbol{q}^2}$. The path of integration can be brought back to the real axis by a counterclockwise rotation. In fact one notices that the contribution of the two arcs at infinity is zero in the second and fourth quadrant and that the singularity of $\omega(\boldsymbol{p})$ is only met in the first and third quadrant.

Inserting the representation

$$\vartheta(x) = \frac{-1}{2\pi i}\int\limits_{-\infty}^{\infty}\frac{e^{-i\eta x^0}}{\eta+i\,\varepsilon}\,d\eta\,,$$

for the stepfunction in (4.4) one obtains [15]

$$\begin{aligned} P(x) &= (2\pi)^{-4}\int\frac{d^3\boldsymbol{p}}{2\,\omega(\boldsymbol{p})}\int d\eta\,\frac{e^{-i[\eta+w(\boldsymbol{p})]x^0+i\boldsymbol{p}\cdot\boldsymbol{x}}}{\eta+i\,\varepsilon} \\ &= (2\pi)^{-4}\int\frac{d^3\boldsymbol{p}}{2\,\omega(\boldsymbol{p})}\int\limits_{C'} dp^0\,\frac{e^{-ip^0x^0+i\boldsymbol{p}\cdot\boldsymbol{x}}}{p^0-[\omega(p)-i\,\varepsilon]} \end{aligned} \tag{4.6}$$

where $p^0 = \eta + \omega(\boldsymbol{p})$ was substituted for the last form. The path of integration C' is parallel to the real axis in the lower half plane right above the pole at $\omega(\boldsymbol{p}) - i\,\varepsilon$. We can therefore shift it up to the real axis. Remarking that the two contributions at infinity vanish and that the path is now away from the pole such that we can put $\varepsilon = 0$ we get the final representation

$$\begin{aligned} P(x) &= (2\pi)^{-4}\int\frac{d^4\boldsymbol{p}\,e^{-ipx}}{2\,\omega(\boldsymbol{p})\,[p^0-\omega(\boldsymbol{p})]} \\ &= -\,(2\pi)^{-4}\int\frac{d^4\boldsymbol{p}\,e^{ipx}}{2\,\omega(\boldsymbol{p})\,[p^0+\omega(\boldsymbol{p})]} \end{aligned} \tag{4.7}$$

where the second form is obtained by substituting $-\,p$ for p.

With (4.7) we have brought (4.3) into a form which is well defined for arbitrary x. We use it as the definition of $P(x)$ for $x^2 < 0$.

Clearly (4.7) is not covariant for all x. However, this was not to be expected since $\vartheta(x)$ is not invariant on the side cone. Inserting (4.7) into (4.2) one sees, however, that the non-covariant terms cancel (due to (3.19)!). An elementary calculation yields the familiar result

$$\Delta_F(x) = P(x) + P(-\,x) = (2\pi)^{-4}\int\frac{d^4p\,e^{-ipx}}{p^2-m^2}\,. \tag{4.8}$$

Standard procedure now yields the scattering amplitude for a process going over an intermediate unstable particle. The propagator (4.1) is coupled at y to the incoming and at x to the outgoing states for which one can take plane waves with real momentum since they are supposed to be stable. Integration over x and y then yields the scattering amplitude in the momentum representation with the Fourier transform of $\Delta_F(x)$,

$$\Delta_F(p) = \frac{1}{p^2 - m^2} \equiv \frac{1}{s - m^2}, \tag{4.9}$$

as the pole term. The residue is $(-i)$ times the product of production and decay amplitude and thus factored as expected.

This procedure is here naturally not obtained from a formal argument as in the usual field theoretic perturbation expansion (Wick's theorem) but rather directly from the model that the particle is produced at y and decays at x. The corresponding anti-particle term, which is not quite as 'anschaulich', has to be included as usual in order to restore Lorentz invariance.

5. Some Qualitative Features of Nondegenerate Representations

If one characterizes unstable particles as stable particles which due to some additional interaction can decay then one is led in a natural way to degenerate representations for their description. However, this philosophy is hardly justified. In fact one could even take the opposite stand that all particles are unstable unless they happen not to have anything to decay into without violating some conservation law [16]. In this case there is a priori no reason to exclude nondegerate representations.

Of course the question is whether these representations correspond to some physical reality. A final answer to it has to await the results of a more complete treatment which allows one to compare predictions with experimental findings. However, some qualitative features can be discussed already without a deeper mathematical analysis.

First of all we remark that an appreciable effect of the nondegeneracy is expected in the first place for short lived particles. For almost stable (weakly decaying) particles v in (2.2) is almost zero and it should not make much difference whether it is almost zero times u or not. This is naturally in agreement with the fact that the effect of the uncertainty in the determination of the velocity discussed in the introduction is extremely small for these representations [17]. This does not exclude that the problem might be interesting from the principal point of view.

Short lived particles, on the other hand, are, at least in practice, not subject to direct measurements. Their properties have therefore to be infered from scattering measurements in which they appear as intermediate states.

Since the little group is not the Rotation group it is clear that the nondegenerate representations do not have a fixed spin. It is instructive to develop the representation for $v = (\gamma, 0, 0, 0)$ into irreducible representations of the rotation group. Naturally this destroys the simple features (2.1) of the representations of translations. Due to rotational invariance around one axis one sees that in the development each spin $j = |\sigma| + n$, n a nonnegative integer, appears exactly once [18]. If such an object is exchanged we therefore expect it to contribute to all these angular momentum

channels giving a connection between the amplitudes for all $j = |\sigma| + n$ which depends only on the three parameters μ, γ, ξ introduced in (2.5) to label an irreducible representation.

Naturally it is of doubtful value to make guesses concerning the interpretation of this behaviour as long as its exact features are not known. Nevertheless the conjecture suggests itself that such a nondegenerate representation corresponds to a whole Regge trajectory. This would mean that a whole trajectory is generated by the exchange of one single object. However, even without this especially exciting possibility one has eventually to face the fact that the 'particle interpretation' does not a priori imply that its contribution is confined to a single angular momentum channel.

6. Discussion

We have developed in this paper a quantum theoretical formalism which yields an unified description of processes involving unstable particles. In particular we were able to derive the usual resonnance formula as well as give a description of independent unstable particles (e.g. in a beam).

However, it turned out that the formalism is not restricted to the 'usual' unstable particles with well defined spin but contains also the possibility of particles with much more general transformation properties which have no correspondence among the stable particles. We have not yet given a sufficient mathematical analysis of these to be able to decide conclusivly whether they correspond to anything known experimentally. Nevertheless we found a structure which suggests that they might be connected to Regge trajectories.

It cannot be emphasized too much that the fact that an unstable particle might not have a well defined intrinsic spin has nothing what so ever to do with nonconservation of angular momentum. In fact we have constructed the formalism invariant under the Poincaré group. This automatically garantees angular momentum as well as energy-momentum conservation. The only new feature in this respect is that the intrinsic spin of an unstable particle is not directly (exactly) measurable. That the same is true for energy and linear momentum has been known for a long time.

It should also be pointed out that in spite of the appearance of nonunitary representations unitarity of the S-matrix does not have to be violated. Instead one uses the unitarity requirement to obtain restrictions on the production and decay amplitudes, as is usually done in S-matrix theory for the residue of a particle pole.

A somewhat weak point of the theory is naturally the regularization of the propagator in the side cone (sect. 4). We have presented a method which yields a unique result. This seems to be the 'most regular' regularization. However, some deeper mathematical analysis of this point would be desirable.

Some words about the 'philosophy' of the approach: We have on purpose not stated any sort of Feynman rules. Our approach should be understood rather in the sense of S-matrix theory. No 'iteration' of the result is necessary if the scattering matrix is unitary where, eventually, background terms are to be included which are not necessarily obtainable from our procedure. Thus in the construction of the resonnant amplitude in sect. 4 the vertex functions have to be understood as complete production and decay amplitudes and not as representing some elementary inter-

action. Granted we used microcausality in our derivation of the propagator which is not quite conform to this point of view. However, the reasons for this are mainly of technical nature: Macrocausality (or asymptotic causality) is very hard to handle [19] and moreover it seems that, at least for stable particle exchange, it yields the same pole terms [20]. This result can be understood intuitively: it is the long range part which gives rise to the pole while a change in the short range behaviour only leads to a redefinition of background and resonance term away from the pole.

We conclude this paper with some remark which we think is instructive: one way to think about unstable particles is to represent them as a superposition of stable (multiparticle) states. In the present theory this amounts to developing a nonunitary representation of the Poincaré group into unitary ones. This is not possible for complex p, otherwise the representation would be equivalent to a unitary one. The case is different, however, in the case of the 'causal description' of the particle (as a resonnance) by a propagator. Here the backward cone (forward cone for the antiparticle) is cut off. A decomposition into unitary representations, achieved by taking the Fourier transform, is now possible. The onedimensional analogon of this is that $e^{i\alpha t}$ has no Fourier transform for complex α while $\vartheta(t)\, e^{i\alpha t}$ does if $Im\,\alpha < 0$, yielding the 'resonance formula'

$$\vartheta(t)\, e^{i\alpha t} = \frac{1}{2\pi i} \int \frac{e^{i\eta t}}{\eta - \alpha}\, d\eta\,.$$

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Note added in proof: After submitting this paper it has been brought to the author's attention that E. G. Beltrametti, Nuovo Cim. *25*, 1393 (1962) and E. G. Beltrametti and G. Luzzatto, Nuovo Cim. *36*, 1217 (1965) have pointed out before that unstable particles do not necessarily have a well defined spin and might be described by nondegenerate representations of the Poincaré group. No quantum mechanical formalism apropriate to do this was proposed, however.

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[4] In a recent discussion of measurements in connection with unstable particles D. A. Jacobson, after developing a presumably complete theory of nonasymptotic measurements, assumes without discussion that the velocity of unstable particles can be measured inspite of the fact that his own formalism does not yield such measurements. D. A. Jacobson, Unstable Particles and Non-Asymptotic Measurements, Preprint. See also D. A. Jacobson, Nuovo Cim. *58 B*, 124 (1968) and *63 B*, 515 (1969).
[5] Inclusion of discrete symmetries does not lead to essentially new problems.

[6] J. M. G. FELL, Acta Mathematica *114*, 267 (1965).
[7] E. H. ROFFMAN, Commun. Math. Phys. *4*, 237 (1967).
[8] $p\,a$ is the Lorentz scalar product with the metric $p\,a = p^0\,a^0 - \boldsymbol{p}\cdot\boldsymbol{a}$.
[9] E. P. WIGNER, Ann. Math. *40*, 149 (1939).
[10] L. S. PONTRJAGIN, *Topologische Gruppen*, Teil 1 (Teubner, Leipzig 1957), Satz 28, p. 241.
[11] Inclusion of space inversion would connect $[\mu, \gamma, \xi, \sigma]$ to $[\mu, \gamma, \xi, -\sigma]$.
[12] To make this an exact statement one has to go to the corresponding Liouville space L which is the real linear space generated by the density matrices (see e.g. U. Fano, Liouville Representation of Quantum Mechanics, in Lectures on the Many-Body Problem Vol. 2, Academic Press, New York 1964. Naturally one has to adapt the formalism to our situation where $\tilde{E} \neq E$). All measurements are then expressed as elements of its dual $\tilde{L}$ ($\neq L$).
[13] R. F. STREATER and A. S. WIGHTMAN, *PCT, Spin and Statistics, and all that* (Benjamin, New York 1964).
[14] S. WEINBERG, Phys. Rev. *133*, B1318 (1964).
[15] The integrals with no path specified are of course understood as extending from $-\infty$ to ∞ on the real axis.
[16] Isn't this real democracy [2] among all particles? As a matter of fact, this is the philosophy behind the introduction of conservation laws for baryon number etc.
[17] While a preliminary estimate for the neutral K system suggests that the effects of non-degeneracy would be of the order of 10^{-5} times the CP violating effects (in amplitude) it might still be interesting to make a more complete analysis of this most sensitive system in particular if it should turn out against all expectations that the 'clean' tests of CP violation (P. K. KABIR, Phys. Rev. Letters *22*, 1018 (1969) and references cited therein) give negative results. However, due to the above estimate there is little hope that anything exciting can come out.
[18] The proof is the same as the one of the corresponding Satz 8 on p. 145 of M. A. NEUMARK, Lineare Darstellungen der Lorentz Gruppe, Deutscher Verlag der Wissenschaften, Berlin, 1963.
[19] C. CHANDLER, Phys. Rev. *174*, 1749 (1968); C. CHANDLER and H. P. STAPP, J. Math. Phys. *10*, 826 (1969); D. IAGOLNITZER and H. P. STAPP, Comm. Math. Phys. *14*, 15 (1969) and references cited therein.
[20] A. PERES, Ann. Phys. (N.Y.) *37*, 179 (1966).