Aharonov-Bohm scattering : comment on a paradox

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Aharonov-Bohm scattering: comment on a paradox

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Abstract. Although the acceleration is zero in a very strong sense for the scattering set-up of Aharonov-Bohm effect, the Schrödinger equation predicts a diffraction pattern. The reason is simply that the velocity operator has no eigenstate for noninteger flux.

1. Introduction

The Aharonov-Bohm scattering [1] is an idealized experiment where a beam of charged particles scatter a whisker of magnetic flux. The problem is two-dimensional and classically no effect at all is expected.

In quantum theory, the vector potential appearing in the Schrödinger equation gives rise to a diffraction pattern corresponding to an infinite total cross section [2]. This fact, although strange, is not really a paradox. Quantum theory is more basic than classical mechanics and we have to accept the result.

An apparently deeper question arises if we look at the acceleration operator which is zero almost everywhere in the configuration space. The scattering states (with two exceptions) are eigenvectors of the acceleration with eigenvalue zero [3].

We consider this fact a real paradox. Why does the beam spread? To answer this question we shall look at the velocity components v_x and v_y in the plane perpendicular to the magnetic whisker. The magnetic field is supposed to be concentrated at the origin of the coordinate system and so strong that its flux ϕ remains finite.

2. Search for eigenstates of velocity operators

Following ref. [3] we introduce

$$v_+ = v_x + iv_y = e^{i\theta}\left[-i\frac{\partial}{\partial r}+\frac{1}{r}\frac{\partial}{\partial\theta}+i\frac{\alpha}{r}\right] \tag{1}$$

$$v_- = v_x - iv_y = e^{-i\theta}\left[-i\frac{\partial}{\partial r}-\frac{1}{r}\frac{\partial}{\partial\theta}-i\frac{\alpha}{r}\right] \tag{2}$$

where α is related to the flux by

$$\alpha = -\frac{e}{2\pi\hbar c}\phi \tag{3}$$

and the mass is chosen such $\hbar/m = 1$. We chose here periodical wave functions of θ with period 2π as in the conventional calculations where the mentioned paradox arises.

The operators $v_\pm$ transforms easily under velocity change

$$e^{-i\mathbf{k}\cdot\mathbf{x}}v_\pm e^{i\mathbf{k}\cdot\mathbf{x}} = v_\pm + (k_x \pm ik_y)\mathbb{1} \tag{4}$$

For this reason it is enough to study the eigenvalue zero. From the fact that ψ is periodic we have

$$\psi = \sum_m f_m(r)e^{im\theta} \tag{5}$$

and

$$0 = v_+\psi_+ = \sum_m \left[-if'_m + i\frac{m+\alpha}{r}f_m\right]e^{i(m+1)\theta} \tag{6}$$

and

$$f_m = c_m r^{m+\alpha}$$

Then

$$\psi_+ = \sum_m c_m(x+iy)^m r^\alpha \tag{7}$$

Each term diverges either at the origin or at infinity, unless α is an integer. In this case, for $m = -\alpha$, ψ_+ is simply

$$\psi_+ = c(x+iy)^{-\alpha}r^\alpha = ce^{-i\alpha\theta} \qquad \alpha \in Z \tag{8}$$

The same calculation for v_- gives

$$\psi_- = \sum c_m(x+iy)^m r^{-(2m+\alpha)} \tag{7'}$$

Again, each term diverges either at the origin or at infinity, unless α is an integer. In this latter case, for $m = -\alpha$

$$\psi_- = c(x+iy)^{-\alpha}r^\alpha = ce^{-i\alpha\theta} \tag{8'}$$

is identical with ψ_+.

As a result, we see by comparison of (7) and (7′) that v_+ and v_- have no common eigenvectors unless α is integer. This is a necessary condition to diagonalize simultaneously the two components of the velocity.

3. Commutation properties

Taking a regular vector potential we find

$$[v_x, v_y] = i\frac{e\hbar}{mc}B_z$$

In our case, and returning at $v_\pm$ we have

$$[v_+, v_-] = -4\pi\alpha\delta^{(2)}(\mathbf{x}) \tag{9}$$

It tells immediately that it is difficult to diagonalize simultaneously v_x and v_y, v_+ is not a normal operator.

This answer is enlightening but insufficient. We do not see the periodicity character in α of the problem from the commutation relations.

4. Conclusion

Although the acceleration plays no role in the Aharonov-Bohm set-up, interesting phenomena arise due to the impossibility of finding eigenvectors to the two components of the velocity operator. The velocity spread is not the effect of a force but of the boundary conditions. It is therefore not surprising that controversy concerning the Aharonov-Bohm effect is generally centered on the question what the appropriate boundary condition should be [3].

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