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Localised Chaos in extended geometries*

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Although the chaotic behavior of physical systems with a few degrees of freedom is by now reasonably well understood [1], there is still a lot to be made concerning the same question but in extended systems. By "extended" I mean systems that are infinite in some spatial dimension-or at least very long- and are translationally invariant in that direction. They are obvious example of this in fluid mechanics, that are the so called parallel shear flows as plane or circular Poiseuille, Blasius boundary layer, etc...It is known from experiments that those flows share some common features in the way they become turbulent as one increases the constraint on them, as measured by the Reynolds number. When this number gets bigger than some more or less well defined value, localised patches of turbulence, with various names depending on which flow one is looking at, may be triggered by localised and sufficiently large external perturbations. Beside being convected downstream at constant velocity, those turbulent patches may have three different kind of behavior: they grow or shrink at a constant speed or they keep a constant size [2]. Probably the most remarkable example of this are the Emmons spots occuring in Blasius boundary layers.

As noticed in a previous work on this subject [3], the growth or decay of those turbulent patches may be explained by a sort of "thermodynamics" analogy: extending the classical Landau amplitude equation to finite amplitude perturbations, with a diffusion term added to model the large scale evolution of space dependent structures, one gets a dynamical van der Waals equation, relevant for first order phase transitions. There the order parameter (equivalent to the amplitude of the turbulent fluctuations in fluid mechanics) may relax toward either one of two linearly stable states, but the coexistence of two thermodynamic phases-each representing a minimum of the thermodynamic potentials- is not stationary in general, because the most stable phase tends to replace the metastable one (the metastability here means that this phase has the highest potential). The consequence of this is that the front separating two phases tends to move at a constant speed, as observed when a turbulent spot invades a laminar flow. However, the connection between first order phase transitions in thermodynamics and subcritical instabilities in flows cannot be made too tight.

There is nothing like a free energy in fluid mechanics, at least in the same sense it exists in thermodynamics (the deep reason for that is however quite subtle). The real thing, in a subcritical situation is the possibility of two different and linearly stable homogeneous states (one turbulent, the other laminar), and those two states-when filling a half space- are separated by a sharp front moving at a constant speed. In one dimensional situations- relevant for pipe flows for instance- the thermodynamic metaphor does not introduce any crucial-or observable- discrepancy between theory and experiment: it is a matter of semantics to say that, if one state invades the other it has the lowest "energy", as the sign of the velocity of expansion constitutes a well posed problem without reference to potential model. In anisotropic 2d flows ,as for instance plane Poiseuille, the matter is more complicated. If one had a true potential energy of the turbulent flow to compare to the one of the laminar one, that would imply that for a given value of the control parameter (i.e. the Reynolds number in hydrodynamical situations) either the turbulent region expands or shrinks independently on the orientation of the normal to the front, because that would be dictated by the difference of "energy" between the two possible homogeneous and linearly stable states. But, if no such energy exists [4] it may happen that one state wins over the other one when their boundary is oriented in some direction, with a reversed situation in other directions. This was already noticed, although in a slightly different context by Toom [5].

Other peculiarities, due to specific fluid mechanical effects may also change this general picture. In bounded flows, like between rotating Taylor-Couette cylinders, there is a feedback between the eventual growth of turbulent spots and the general flow conditions [6]. This feedback, as well as other things explains the remarkable occurence of turbulent spirals in Taylor-Couette flows with counterrotating cylinders. Finally it is also worth mentionning the possibility of localized finite amplitude perturbations, as seemingly occuring in recent numerical computations by Javier Jimenez linked to the so called excitability of some shear flows. A relevant numerical observation of those localised structures has been made by Thual and Fauve for a Landau-Ginzburg model [7].

As a conclusion, I would like to stress how real turbulence in fluid mechanics does continue to be a rich source of inspiration for scientists interested in Complex Systems.

References:

[1] P.Bergé, Y.Pomeau, C.Vidal "Order in chaos", Wiley, N.Y. (1987).

[2] Y.Pomeau, Physica D**23**, 1 (1986), P.Clavin, Y.Pomeau, Proceedings of the NATO workshop on "Geometry of nonequilibrium" held at Cargese, August 1988.

[3] Y.Pomeau, Proceedings of the conference "Fluctuations and Instabilities" held in Valparaiso (Chile), december 1987, edited by E.Tirapegui and L.Villaroel (Reidel) to appear.

[4] Those non potential terms appear in general when the turbulence is due to propagating waves. Then the amplitude equation has complex coefficients and has no simple Lyapunov function in general.

[5] A.L. Toom, in "Multicomponent random systems" edited by R.L. dobrushin in Advances in probability, vol. 6 Dekker, N.Y. (1980), p.549-575.

[6] J.Hegseth, D.C. Andereck, F.Hayot, Y.Pomeau "Spiral turbulence and phase dynamics" Ohio State University preprint, August 1988.

[7] O.Thual, S.Fauve, to appear in J.de Physique.