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The supercurrent and β-functions vanishing to all orders in strictly massless supersymmetric Yang-Mills theories

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ABSTRACT

It is shown that the one-loop criteria for having β functions vanishing to all orders in SYM-theories survive the limit in which all masses vanish. This is non-trivial due to off-shell infrared divergences. The supercurrent, chiral insertions of dimension three and the supersymmetric descent equations have to be constructed as operators which are BRS invariant, gauge parameter independent and covariant under supersymmetry which is explicitly broken by an infrared regulator.

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1. Introduction. Statement of the problem

In a previous paper[1)] [1] we have devised a way how to produce with *one*-loop calculations supersymmetric Yang-Mills (SYM) theories whose β-functions vanish to *all* orders of perturbation theory. The main input were the well-known non-renormalization theorems for chiral vertices in supersymmetric models [2] and the concept of reduction of couplings [3]. In order to avoid all off-shell infrared problems associated with the fact that the $N = 1$ vectorsuperfields have canonical dimension zero, hence develop a $1/(k^2)^2$ singularity in the propagator, we have given supersymmetric masses to *all* fields, even to the gauge fields. If in the corresponding theory the generating functional is denoted by $\Gamma(m)$ its properties can be summarized in the following Ward-identities (WI):

$$W_\alpha \Gamma(m) = W_\alpha^h \Gamma(m) = 0 \tag{1.1}$$

— expressing exact supersymmetry (h=homogeneous)

$$s(\Gamma(m)) \sim 0 \tag{1.2}$$

— broken Becchi-Rouet-Stora (BRS) invariance

$$W_R \Gamma(m) \sim 0 \tag{1.3}$$

— broken R-invariance

$$W_\omega \Gamma(m) \sim 0 \tag{1.4}$$

— (potentially) broken invariance under rigid gauge transformations.

Exact supersymmetry means: all vertices. all propagators etc. are naively supersymmetric, subtractions too are performed in a supersymmetric manner; the breaking of BRS, R- and rigid invariance is due to terms with power counting three.

Since β-functions are believed to reveal the ultraviolet properties of a model and can very often be constructed as mass-independent quantities even in massive theories nothing is wrong with (1.1)-(1.4) as a starting point. But, of course, eventually

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1)Cf. this reference also for notations not explained below.

one wants to have a strict Slavnov identity (r.h.s. of (1.2) vanishing) for securing unitarity, i.e. one would like to distinguish gauge invariant masses from non-symmetric ones and also discuss other "soft" aspects of the theory as spontaneous symmetry breaking, decoupling of heavy modes and the like. Due to the ennoying fact mentioned above that massless vectorsuperfields produce off-shell infrared divergences there exists at present no other possibility than introducing an infrared regulator μ^2 [4] which violates explicitly supersymmetry, but maintains BRS-invariance[2)] I.e. one constructs a vertex functional $\Gamma(\mu^2, \mu_0^2)$ which satisfies

$$W_\alpha \Gamma \equiv (W_\alpha^h + \mu^2 \int \frac{\delta}{\delta \mu_\lambda^\alpha}) \Gamma(\mu^2, \mu_0^2) = 0 \tag{1.5}$$

$$s(\Gamma(\mu^2, \mu_0^2)) = 0 \tag{1.6}$$

$$W_R \Gamma(\mu^2, \mu_0^2) = 0 \tag{1.7}$$

$$W_\omega \Gamma(\mu^2, \mu_0^2) = 0 \tag{1.8}$$

(The parameter μ_0^2 will be explained below. $u = (\ldots, \mu_\lambda^\alpha, u_D)$ is an external superfield coupled to the supersymmetry breaking induced by the infrared regulator.) The question is now, how it can be shown in this theory, whether β-functions vanish to all orders, where the explicit breaking of supersymmetry (the inhomogeneous term in (1.5)) prevents the immediate use of the non-renormalization theorem for chiral vertices. The answer is provided by a suitably constructed functional $\Gamma(m, \mu^2, \mu_0^2)$ which "interpolates" between the two limits $\Gamma(m, \mu_0^2) = \Gamma(m, \mu^2, \mu_0^2)|_{\mu^2=0}$ and $\Gamma(\mu^2, \mu_0^2) = \Gamma(m, \mu^2, \mu_0^2)|_{m=0}$, the transition $\Gamma(m, \mu_0^2) \to \Gamma(m)$ then being achieved by coupling constant redefinition. Its matching will be performed on the normalization conditions of the model in different "regions" defined by m and μ^2.

[2)]The alternative – working with non-linear field tranformations in the Wess-Zumino gauge – has not yet been fully worked out [5].

2. Preparation

In order to make the present paper reasonably self-contained we recall from [1,4], a certain minimum of notation. We think of a SYM-theory with simple gauge group G and a classical action given by

$$\begin{aligned}\Gamma_{inv} = & -\frac{1}{128g^2}Tr\int d^4xd^2\theta F^\alpha F_\alpha + \frac{1}{16}\int d^4xd^4\theta \bar{A}_R\, e^{\phi_i T^i_R} A^R \\ & + \frac{1}{6}\lambda_{(rst)}\int d^4xd^2\theta A^r A^s A^t + \frac{1}{6}\bar{\lambda}^{(rst)}\int d^4xd^2\bar{\theta}\bar{A}_r\bar{A}_s\bar{A}_t\end{aligned} \tag{2.1}$$

$F^\alpha \equiv \overline{DD}(\bar{e}^\phi D^\alpha e^\phi)$, $\phi \equiv \phi_i \tau^i$

$A_r \equiv$ chiral superfields

$r \equiv (R,\rho)=$ (irrep. R, label within R)

$T_R \equiv$ generator of irrep. R

For quantization we add gauge fixing and Faddeev-Popov terms:

$$\Gamma_{\phi\pi} = Tr\int d^4xd^4\theta\Big((BDD\phi + \bar{B}\overline{DD}\phi + \alpha B\bar{B}) - (DDc_- + \overline{DD}\bar{c}_-)s\phi\Big) \tag{2.2}$$

where $s\phi = c_+ - \bar{c}_+ + \ldots$ is the BRS-variation of ϕ and c_+, c_- the chiral ghosts and anti-ghosts. In order to regularize the infrared we introduce on the one hand auxiliary supersymmetric mass terms for all fields

$$\begin{aligned}\Gamma_m = & (m^2 + M^2(s-1)^2)Tr\int d^4xd^4\theta\phi^2 \\ & + \Big(\int d^4xd^2\theta\Big\{(m + M(s-1))A^R A^R + (m^2 + M^2(s-1)^2)Tr(c_-c_+)\Big\} \\ & + conj.\Big)\end{aligned} \tag{2.3}$$

s is a parameter varying between 0 and 1 and participates in the subtractions [6]. Its physical value is $s = 1$, at which the theory is independent of the auxiliary mass M. m is the mass governing the interpolation alluded to in the introduction.

On the other hand an infrared regulator μ^2 is introduced by a kind of field renormalization

$$\phi \rightarrow \phi + \frac{\mu^2}{2}\theta^2\bar{\theta}^2\phi \tag{2.4}$$

This field redefinition is compatible with BRS but not with supersymmetry. Hence the μ^2-dependent terms break supersymmetry but not BRS and regulate the $1/(k^2)^2$

terms appearing in the supersymmetric propagator of the supervectorfield ϕ. (An explicit expression is given in [4], first paper, equ. (A.6)). The entire μ^2-dependence of the theory is controlled algebraically by use of a Grassmann variable ν^2 which forms together with μ^2 a BRS doublet and contributes to the Slavnov-identity (1.6).

From the results obtained in [4] and [1] it is clear that to all orders of perturbation theory a functional $\Gamma(m, \mu^2, \mu_0^2)$ can be constructed with the following properties

(i) it satisfies the Ward and Slavnov identities

$$\left(W_\alpha^h + \mu^2 \int \frac{\delta}{\delta \mu_\lambda^\alpha}\right)\Gamma(m, \mu^2, \mu_0^2) = 0 \tag{2.5}$$

$$s(\Gamma(m, \mu^2, \mu_0^2)) \sim 0 \tag{2.6}$$

$$W_R \Gamma(m, \mu^2, \mu_0^2) \sim 0 \tag{2.7}$$

$$W_\omega \Gamma(m, \mu^2, \mu_0^2) \sim 0 \tag{2.8}$$

(ii) it fulfils the following (off-shell) normalization conditions:

$$\Gamma_{D^i D^j}\Big|_{\substack{p^2=-\kappa^2 \\ s\,=\,1 \\ \mu^2=\,\mu_0^2}} = \frac{\delta_{ij}}{4g^2} \quad (2.9a) \qquad \Gamma_{F^i A^j A^k}\Big|_{\substack{p=0 \\ s=0}} = \lambda_{ijk} \quad (2.9b)$$

$$\Gamma_{\eta^i A_+^j}\Big|_{\substack{p^2=-\kappa^2 \\ s\,=\,1}} = \delta_{ij} \quad (2.9c) \qquad \Gamma_{\sigma^i A_+^j A_+^k}\Big|_{\substack{p^2=-\kappa^2 \\ s\,=\,1}} = f_{ijk} \quad (2.9d)$$

and quite a few others (s. [4] second ref. equ. (2.12)) which are not very relevant in the present context.

(iii) it satisfies a Callan-Symanzik (CS) equ.

$$(\mu_0^2 \partial_{\mu_0^2} + \mu^2 \partial_{\mu^2} + \kappa^2 \partial_{\kappa^2} + m\partial_m + \beta_g \partial_g + \beta_\lambda \partial_\lambda - \gamma \mathcal{N} + \gamma_u \mathcal{N}_u)\Gamma = 0. \tag{2.10}$$

Some comments are in order:

To (ii) The appearance of the special value μ_0^2 in (2.9a),(2.9b) is important. Since normalization conditions like (2.9c), (2.9d) are intended to fix gauge dependent, i.e. μ^2-dependent counterterms, they must be imposed for any value of μ^2. On the other hand (2.9a) fixing a gauge invariant counterterm – the Yang-Mills action – requires the choice of a particular value $\mu^2 = \mu_0^2$. (See [7] for a general discussion on gauge parameter dependence.) In this respect μ_0^2 resembles very much the normalization point κ^2 for the momenta: infrared divergences prevent the use of $\kappa^2 = 0$ in a massless theory, hence some auxiliary variable has to be introduced for the purpose of normalization. The case of (2.9b) - taken at the non-physical value zero of the infrared subtraction parameter s - is special and will be explained in sect. 4.

To (iii): The β-functions can depend on mass ratios $\frac{\mu_0^2}{\kappa^2}, \frac{m}{\kappa}$ beginning with two loops; by construction (cf. [4, 7]) they do not depend on gauge parameters like μ^2.

Since the supersymmetric m-mass terms regularize already alone the infrared, the limit $\mu^2 \to 0$ exists (at $m \neq 0$) and defines vertex functions depending on μ_0^2 with

(i) the Ward and Slavnov identities

$$W_\alpha^h \Gamma(m, 0, \mu_0^2) = 0 \tag{2.11}$$

$$s(\Gamma(m, 0, \mu_0^2)) \sim 0 \tag{2.12}$$

$$W_R \Gamma(m, 0, \mu_0^2) \sim 0 \tag{2.13}$$

$$W_\omega \Gamma(m, 0, \mu_0^2) \sim 0 \tag{2.14}$$

(ii) normalization conditions inherited from $\Gamma(m, \mu^2, \mu_0^2)$

$$\Gamma_{D^iD^j}(m, 0, \mu_0^2)\Big|_{\substack{p^2=-\kappa^2 \\ s=1}} = \lim_{\mu^2 \to 0} \Gamma_{D^iD^j}(m, \mu^2, \mu_0^2)\Big|_{\substack{p^2=-\kappa^2 \\ s=1 \\ \mu_0^2}} = \frac{\delta_{ij}}{4g^2} f(\mu_0^2) \tag{2.15a}$$

$$\Gamma_{F^iA^jA^k}(m, 0, \mu_0^2)\Big|_{\substack{p=0 \\ s=0}} = \lim_{\mu^2 \to 0} \Gamma_{F^iA^jA^k}(m, \mu^2, \mu_0^2)\Big|_{\substack{p=0 \\ s=0 \\ \mu_0^2}} = \lambda_{ijk} \tag{2.15b}$$

where $f(\mu_0^2) = 1 + \mathcal{O}(\hbar))$
etc.

(iii) fulfilling a CS-equ.

$$(\mu_0^2 \partial_{\mu_0^2} + \kappa^2 \partial_{\kappa^2} + m\partial_m + \beta_g \partial_g + \beta_\lambda \partial_\lambda - \gamma \mathcal{N} + \gamma_u \mathcal{N}_u) \Gamma(m, 0, \mu_0^2) \sim 0. \tag{2.16}$$

As indicated by (2.11) supersymmetry holds strictly but due to the non-supersymmetric subtractions performed for the functional $\Gamma(m, \mu^2, \mu_0^2)$ it is non-naively realized: although propagators and tree vertices are supersymmetric (at $\mu^2 = 0$) the subtraction rules violate supersymmetry because of the normalization conditions at $\mu^2 = \mu_0^2$), hence Γ_{eff} (in the sense of Zimmermann [8]) contains non-supersymmetric counter-terms restoring the supersymmetry. The main observation is now that given the above normalization conditions (2.15) with μ_0^2-dependent right-hand-sides one can construct a theory $\tilde{\Gamma}(m, \mu_0^2)$ with the *same* normalization conditions, but being naively suprsymmetric (i.e. with supersymmetric Γ_{eff} and subtraction rules [9]. $\tilde{\Gamma}(m, \mu_0^2)$ will be identical to $\Gamma(m, 0, \mu_0^2)$ because any renormalized theory is uniquely defined once its symmetries and normalization conditions are prescribed. In particular the β-functions of this theory $\Gamma(m)$ and those of $\Gamma(m, 0, \mu_0^2)$ coincide. They coincide moreover with those of the theory $\Gamma(m, \mu^2, \mu_0^2)$ since by construction the latter are independent of μ^2.

3. Non-renormalization theorem and reduction

For the proof that certain coefficients of chiral anomalies are precisely of one-loop order (non-renormalization theorem) one needs a basis for local (i.e. non-integrated) chiral insertions of dimension three and R-weight minus two. Similarly one needs the supercurrent and its anomalies which describe the superconformal symmetries of the theory and their breaking. A third ingredient is the set of supersymmetric descent equations [1]. All of these objects are not only needed in the theory described by $\Gamma(m, \mu^2, \mu_0^2)$, but in particular in the limit theory $\Gamma(0, \mu^2, \mu_0^2)$ where they are a priori in danger not to exist because of the off-shell infrared divergences. The construction and existence proof of all needed operators is a technical problem whose solution is presented in the appendices A and B. So, we take over that result and write only down, what is needed explicitly: the basis for the chiral insertions.

$$S \sim \overline{DD}[rK^0 + J^{inv}]K^0 \tag{3.1a}$$
$$L_g \sim \overline{DD}\Big[\Big(\frac{1}{128g^3} + r_g\Big)K^0 + J_g^{inv}\Big] + L_g^{chiral}K^0 \tag{3.1b}$$
$$L_{rst} \sim \overline{DD}\Big[r_{rst}K^0 + J_{rst}^{inv}\Big] + L_{rst}^{chiral}K^0 \tag{3.1c}$$
$$L_k \sim \overline{DD}\Big[r_kK^0 + J_k^{inv}\Big] + L_k^{chiral}K^0 \tag{3.1d}$$
$$L_S^R \sim \overline{DD}\Big[r_S^RK^0 + J^{inv\ R}{}_S\Big] + L^{chiral\ R}{}_SK^0 \tag{3.1e}$$
$$L_{oa} \sim \overline{DD}\Big[r_{oa}K^0 + J_{oa}^{inv}\Big]K^0 \tag{3.1f}$$
$$L_{1K} \sim \overline{DD}\Big[r_{1k}K^0 + J_{1K}^{inv}\Big]K^0 \tag{3.1g}$$

$$L_\phi \sim \overline{DD}\big[J_\phi^{inv}\big]K^0 \tag{3.1h}$$
$$L_U \sim \overline{DD}\big[r_U K^0 + J_U^{inv}\big]K^0 \tag{3.1i}$$

S is the supercurrent anomaly. L_g etc. are defined by

$$\delta_x\Gamma \sim \int d^4x d^2\theta L_x + conj. \tag{3.2}$$

where δ_x spans all independent symmetric variations (with respect to fields or parameters). Exactly as in [1] the superconformal structure combined with the descent equations (cf. App. B) yields the following relation

$$r = \beta_g\Big(\frac{1}{128g^3} + r_g\Big) + \beta_\gamma r_\gamma + \gamma_{oa} r_{oa} + \gamma_k r_k \tag{3.3}$$

already in the theory $\Gamma(m,\mu^2,\mu_0^2)$. It is crucial that the coefficients r, r_{oa} in (3.1a,f) are the coefficients of very specific chiral anomalies. But although they are gauge parameters independent (like the β-functions) they need not have non-renormalization properties i.e. they could have in principle contributions to all orders in $\hbar$. But in the manifestly supersymmetric theory $\Gamma(m,0,\mu_0^2)$ the non-renormalization theorem holds [1]:

$$r = \hat{r}\hbar \quad , \quad r_{oa} = \hat{r}_{oa}\hbar \tag{3.4}$$

Since r and r_{oa} are independent of μ^2 they are pure one-loop quantities also in the theory $\Gamma(m,\mu^2,\mu_0^2)$. The relation between $\Gamma(m,0,\mu_0^2)$ and a theory [1] with vertex functional $\Gamma(m)$ given by normalization conditions

$$\Gamma_{D^iD^j}\Big|_{p^2=\kappa^2} = \frac{1}{4g^2} \qquad \Gamma_{A^iA^jF^k}\Big|_{\substack{p=0\\ \mu=0}} = \lambda_{ijk} \tag{3.5}$$

instead of (2.15) (without parameter s and auxiliary mass) is provided by a reparametrization of the coupling g

$$\frac{1}{4g^2} \longrightarrow \frac{1}{4g^2} f(\mu_0^2)$$

(cf. (2.15), f starts with 1). Under this reparametrization the β-functions change as usual and accordingly r_g, r_λ, r_k. But r and r_{oa} do not change, since they remain one-loop quantities. The equ. (3.2) then tells us how the changes of the β-functions and coefficients $r_x (x = g, \lambda, k)$ are interrelated. With these considerations the questions of principle are answered: one has the same criterion for $\beta = 0$ in the theory $\Gamma(m,\mu^2,\mu_0^2)$ as before in the theory $\Gamma(m)$ [1], since it is based on the same equation (3.2) with the same non-renormalized coefficients r and r_{oa}.

4. Relation $\beta_\lambda, \gamma_{matter}$

There remains an important problem of practice. Since the one-loop anomalous dimensions γ of matter fields are given by sesquilinear forms in $\lambda, \bar{\lambda}$ there will always be free phases for λ when reduced to g, hence according to the criterion [1] the necessity of fixing these phases. In the $SU(6)$ example this was done by using a very specific and simple normalization condition

$$\Gamma_{A^r A^s F^t}\Big|_{p=0} = \lambda_{rst} \tag{4.1}$$

i.e. *no* counter term in Γ_{eff} for Γ_{AAF}. Since the relevant diagrams are finite, this is even a natural condition to be imposed.

In order to have the same condition and in particular its consequence available in the theory $\Gamma(o, \mu^2, \mu_0^2)$ one has to control the purely chiral contributions L^c in the basis (3.1).

In S (3.1a) they are absent since R-invariance is exact at $m = 0$. In L_{oa} (3.1f), L_{1K} (3.1g), L_ϕ (3.1h), L_U (3.1i) they are absent due to the specific choice of basis. It is shown in Appendix A that a chiral insertion L^{rst} can be constructed whose F-component starts precisely with $N_4^4[A^{(r}A^s F^{t)}]$. Thus, taking the F-component of $\lambda_{rst}L^{rst}$ as the whole matter self-interaction term in $\mathcal{L}_{eff}$ is equivalent to impose the normalization condition (2.9b) (by virtue of the N-product normalization conditions [6]). (2.9b) is the analogue of (4.1) in the present massless case. It *defines* the coupling constants λ_{rst}. We have to make sure that in L_g^{chiral} and L_k^{chiral} this contribution does not occur. We first check L_g^{chiral}. In $\partial_g\Gamma$ there are in general contributions of the type $\partial_g\lambda' \int \psi\psi A$ (λ': coefficient occurring in $\mathcal{L}_{eff}$), but since $\partial_g\lambda = 0$ the term $\int AAF$ is missing, hence the complete insertion L^{rst} cannot appear in L_g^{chiral}: all corrections belong to $\overline{DD}[\cdots]$. i.e. $L_g^{chiral} = 0$. The same argument applies to L_k^{chiral}. We finally check the contributions in $L^{chiral}\, R_S$.

$$N^x{}_y\Gamma = \left[\int d^4x d^2\theta L^x{}_y + conj.\right] \cdot \Gamma \tag{4.2}$$

by the action principle and the form of $\mathcal{L}_{eff}$. In components:

$$N^x{}_y\Gamma = N_4^4\left[\int \lambda_{yst} A^x A^s F^t + cycl. + \; conj.\right] + \; corr. \tag{4.3}$$

which can be redefined by adding corrections to read

$$N^{x}{}_{y}\Gamma = N_4^4\Big[\int \lambda_{yst} L^{xst} + \ cycl. \ + \ conj.\Big] + \ corr. \tag{4.4}$$

Hence $L^{x}{}_{y}$ can be taken to be

$$L^{r}{}_{s} \sim \overline{DD}[\ldots] + \lambda_{sxy} L^{rxy} \ + \ cycl. \tag{4.5}$$

As a consequence of R-invariance (for hard terms) it follows thus that

$$\beta_{rst} = \lambda_{rsx}\gamma^{x}{}_{t} \ + \ cycl. \tag{4.6}$$

i.e. by the well-chosen definition of λ (2.9b) one can always arrange that the β-functions of matter self-couplings are given by the product of anomalous dimensions and the coupling! It is remarkable that this relation, (4.6), can be established on the basis of R-invariance alone, no finiteness properties of diagrams are needed, although the latte render our definition of λ more natural.

With this result we have reconstructed in the theory $\Gamma(\mu^2, \mu_0^2)$ all elements needed in principle and practice for establishing strictly massless models with β-functions vanishing to all orders. The off-shell IR-divergences are regulated by μ^2, physical quantities (like β-functions) are independent of μ^2.

5. Conclusion

Let us represent graphically our main result:

$$\begin{tikzcd}
& \Gamma(m,\mu^2,\mu_0^2) \arrow[dl, "m \to 0"'] \arrow[dr, "\mu^2 \to 0"] & \\
\Gamma(0,\mu^2,\mu_0^2) & & \Gamma(m,0,\mu_0^2) \arrow[d, "f \to 1"] \\
& & \Gamma(m)
\end{tikzcd}$$

Fig.

The functionals $\Gamma(m,\mu^2,\mu_0^2)$ and $\Gamma(0,\mu^2,\mu_0^2)$ have a priori no non-renormalization property for their chiral vertices, hence the relevant anomaly coefficients r, r_{oa} in (3.2) could in principle be renormalized arbitrarily.

Relation (3.2) and r, r_{oa} being non-renormalized, (3.4), is true for the functionals $\Gamma(m,0,\mu_0^2)$ and $\Gamma(m)$ (the latter being defined by normalization conditions (3.5)). But once within $\Gamma(m,\mu^2,\mu_0^2)$ the chiral insertions, supercurrent and descent equations are constructed such that $\Gamma(m,0,\mu_0^2)$ and $\Gamma(m)$ result with their desired properties in the limit $\mu^2 \to 0$, the β-functions are determined, the anomaly coefficients are of one loop only and one thus has for $\Gamma(0,\mu^2,\mu_0^2)$ the same criterion as for $\Gamma(m)$. I.e. β-functions vanish in $\Gamma(0,\mu^2,\mu_0^2)$ when they do so in $\Gamma(m)$. The proportionality of β- and γ-functions (4.6) which is important for practical applications of the criterion in [1] can also be established.

Appendix A. Supercurrent, chiral insertions

The aim is to construct a basis for the supercurrent V_μ and the basis (3.1) of chiral insertions S with dimension 3 and R-weight -2 in the theory with $\Gamma(\mu^2,\mu_0^2) = \Gamma(0,\mu^2,\mu_0^2)$ satisfying the WI's (1.5)-(1.8)

$$\Big[(W_\alpha^h + \mu^2 \int \frac{\delta}{\delta u_\lambda^\alpha}\Big]\Gamma(\mu^2,\mu_0^2) = 0 \tag{A.1}$$

$$s(\Gamma(\mu^2,\mu_0^2) = 0 \tag{A.2}$$

$$W_R\Gamma(\mu^2,\mu_0^2) = 0 \tag{A.3}$$

$$W_\omega\Gamma(\mu^2,\mu_0^2) = 0 \tag{A.4}$$

(For an explicit expression of the WI-operators s. [4].) These insertions V_μ, S should all be "superfields", i.e. supermultiplets with respect to the explicitly broken supersymmetry WI (A.1), they should be BRS-invariant and gauge independent, they should transform under R-variations as densities with prescribed weights and be rigidly invariant. Rigid invariance and R-invariance (resp. -covariance) are known to be renormalizable [9]. Supersymmetry and BRS invariance

$$W_\alpha X \cdot \Gamma = 0 \tag{A.6}$$

$$X = V_\mu, S$$

$$\mathcal{B}_{\bar\Gamma} X \cdot \Gamma = 0 \tag{A.7}$$

will be non-trivial to prove. ($\mathcal{B}_{\bar{\Gamma}}$ is the linearized form of the Slavnov operator s, see [9].) The standard procedure consists in introducing suitable external fields E_μ, E and to couple them to the desired operators:

$$\Gamma_{eff} \to \Gamma_{eff}{}^{(E)} = \Gamma_{eff} + \int dV E_\mu V^\mu + \int dsES + \int d\bar{s}\bar{E}\bar{S} \qquad (A.8)$$

(this holds in the tree approximation)
and to prove the WI's

$$W_\alpha \Gamma(\ldots, E_\mu, E) = 0 \qquad (A.9)$$

$$s(\Gamma(\ldots, E_\mu, E)) = 0 \qquad (A.10)$$

for the vertex functional depending on these external fields E_μ, E.

W_α also changed from (A.6) to (A.9) since E_μ, E are real, resp. chiral superfields. The operator $\mathcal{B}_{\bar{\Gamma}}$ however changed only due to the change of $\bar{\Gamma}$ and not in form since classically

$$sE_\mu = sE = 0 \qquad (A.11)$$

(V_μ and S are desired to be BRS-invariant.) The gauge independence of the insertions V_μ, S too is a consequence of (A.10) [7].

The combined cohomology problem which is posed by (A.9), (A.10) for insertions invariant or covariant under rigid gauge and R-transformations can be solved like in the case of vanishing external fields [1]. First we introduce a kind of local supersymmetry WI operator w_α by

$$W_\alpha = \int dx w_\alpha \qquad (A.12)$$

with the property

$$[w_\alpha, s] = 0. \qquad (A.13)$$

Then we proceed by induction and assume

$$w_\alpha \Gamma = \partial_\mu N_{7/2}^{7/2}[Q^\mu{}_\alpha] \cdot \Gamma + \mathcal{O}(\hbar^n) \qquad (A.14)$$

$$s(\Gamma) = \mathcal{O}(\hbar^n) \qquad (A.15)$$

$$\mathcal{B}_{\bar{\Gamma}}[Q_\mu \cdot \Gamma] = \mathcal{O}(\hbar^n) + \text{ improvement terms.} \qquad (A.16)$$

It follows that

$$W_\alpha \Gamma = \mathcal{O}(\hbar^n) \tag{A.17}$$

The terms of order n will now be specified as prescribed by the power counting of $w_\alpha \Gamma$ (namely $\rho = 5/2, \delta = 9/2$).

$$w_\alpha \Gamma = \delta_\mu [Q^\mu_\alpha]^{7/2}_{7/2} \cdot \Gamma + \hbar^n (R_\alpha + \partial_\mu S^\mu_\alpha) + \mathcal{O}(\hbar^{n+1}) \tag{A.18}$$

Here R_α has $\rho = 5/2, \delta = 9/2$ and can thus be integrated. Its integral gives rise to supersymmetry breaking which can be absorbed with the help of powers of the shifted external field u (see [4]).

Thus the supersymmetry WI holds at the next order:

$$W_\alpha \Gamma = \mathcal{O}(\hbar^{n+1}) \tag{A.19}$$

The extension of the Slavnov identity (A.15) to the next order in u requires listing of all candidates for infrared anomalies – i.e. terms which are not absorbable in $\mathcal{L}_{eff}$ in an infrared-regular way – and checking their coefficients. Like in [4] the individual infrared behaviour of the terms involved is better than mere power counting indicates and in fact all dangerous coefficients can be shown to vanish, hence

$$s(\Gamma) = \mathcal{O}(\hbar^{n+1}) \tag{A.20}$$

Having already absorbed R_α in (A.18) we are now left with the derivative term: S^μ_α has $\rho = 3/2$, $\delta = 7/2$, hence the integral of the insertion $\partial_\mu S^\mu_\alpha \cdot \Gamma$ does not vanish by power counting and thus could be the origin of infrared divergences. However the validity of the Slavnov identity (A.20) implies

$$\begin{aligned} \mathcal{O}(\hbar^{n+1}) &= \mathcal{B}_{\bar\Gamma} w_\alpha \Gamma \\ &= \partial_\mu \mathcal{B}_{\bar\Gamma}[Q^\mu_\alpha] \cdot \Gamma + \hbar^n \partial_\mu (b S^\mu_\alpha) + \mathcal{O}(\hbar^{n+1}) \end{aligned} \tag{A.21}$$

$(b \equiv \mathcal{B}_{\bar\Gamma_{classical}})$

From hypothesis (A.16) follows

$$\mathcal{B}_{\bar\Gamma}[Q^\mu_\alpha \cdot \Gamma] = \hbar U^\mu_\alpha + \text{improv. terms } + \mathcal{O}(\hbar^{n+1}) \tag{A.22}$$

and then from (A.21):

$$U^\mu_\alpha + b S^\mu_\alpha = \text{improv. terms} \tag{A.23}$$

A check of all possible contributions shows that S^μ_α has infrared power counting $\rho = 7/2$. Thus (A.14) and (A.16) hold at the next order $\hbar^{n+1}$ for Q^μ_α replaced by $Q^\mu_\alpha + \hbar^n S^\mu_\alpha$. (Once checks also that the improvement terms have correct power counting.) This finishes the inductive proof of (A.9) and (A.10). With these (very condensed) considerations the existence of a basis for the suprcurrent V_μ and a chiral basis S is established. Apart from additional terms associated with the external fields μ, E_μ and E the explicit form is that of [1] equ. (4.7).

Appendix B. Descent equations

The supersymmetric descent equations ([1] App. A) read in the classical approximation

$$\begin{aligned} sk^0 &= \bar{D}_\alpha k'^{\dot\alpha} \\ sk^{1\dot\alpha} &= (\bar{D}^{\dot\alpha} D^\alpha + 2D^\alpha \bar{D}^{\dot\alpha}) k^2_\alpha \\ sk^2_\alpha &= D_\alpha k^3 \\ sk^3 &= 0 \end{aligned} \tag{B.1}$$

with a special solution $\{k^q\}$ given by

$$\begin{aligned} k^0 &= Tr(\phi^\alpha \overline{DD} \phi_\alpha) \\ k^{1\dot\alpha} &= -Tr(D^\alpha c_+ \bar{D}^{\dot\alpha} \phi_\alpha + \bar{D}^{\dot\alpha} D^\alpha c_+ \phi_\alpha) \\ k^2_\alpha &= T_r(c_+ D_\alpha c_+) \\ k^3 &= \frac{1}{3} Tr c^3_+ \end{aligned} \tag{B.2}$$

$(\phi^\alpha \equiv e^{-\phi} D^\alpha e^\phi)$

The task is now to include external fields $u, v, u_{(p)}$ coupled to the objects above in the construction of the higher orders K^q out of k^q i.e. – again to check the potential effect of infrared anomalies.

The discussion is similar to the one of App. A and will not be reproduced here.

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