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# Some Vacua of the Heterotic String

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Symmetric orbifolds, as examples of consistent vacua of the $E_8 \times E_8$ heterotic string, are described. After a summary of their construction and of the conditions for the quantum consistency of the string theory, some phenomenologically relevant vacua are briefly discussed. Finally, the low-energy effective theory and its quantum symmetries are presented.

## 1 Introduction

Since the discovery [1] of the consistent supersymmetric solutions to the heterotic string [2] equations of motion with gauge groups $E_8 \times E_8$ or $SO(32)$ and ten-dimensional space-time, the exploration of the vacuum structure of heterotic strings has vastly progressed. It has been realized that, at the perturbative level, this vacuum is infinitely degenerate, with solutions corresponding to many different space-time dimensions, gauge groups and particle contents. The number of vacua increases fast when the space-time dimension decreases, and the class of consistent four-dimensional superstrings [3–10] is infinite.

In perturbation theory, solutions to the classical equations of motion remain solutions to arbitrary order if the vacuum symmetry includes $N = 1, 2$ or 4 space-time supersymmetry. Supersymmetric vacua are then expected to be stable in perturbation theory. On the other hand, consistent non-supersymmetric vacua are usually unstable: loop corrections develop non-zero tadpole (dilaton) expectation values spoiling perturbative stability (as well as a large cosmological constant). Even though a theorem has not been proven, one infers then that consistent, realistic string models should have space-time supersymmetry, and possess a mechanism for supersymmetry breaking at low energies.

The degeneracy of the vacuum offers the prospect of describing many different low-energy physics, some of them being realistic or close to realistic, most others having very little to do with expected particle phenomenology. On the other hand, perturbation theory cannot tell what is the true, exact vacuum of the string theory, whether the degenacy is lifted. It is actually unclear if this true vacuum has properties closely related to those of perturbative solutions, or if it turns out to be completely different. If the second possibility is realized, phenomenological studies of perturbative string vacua will only be crude approximations of the correct ground state of the string theory. This difficulty is particularly apparent for the problem of supersymmetry breaking in string theory. Perturbative stability certainly prefers supersymmetric vacua. On the other hand, supersymmetry must be broken in any realistic model. This requires a non-perturbative mechanism for spontaneous supersymmetry breaking. The most plausible candidate seems to be the formation of condensates of gauginos [11] in the hidden sector of the gauge group, which contains in general asymptotically-free forces which become strong at intermediate energies, $M_{weak} \ll \Lambda_{hidden} \ll M_{strings} \sim M_{Planck}$. Non-perturbative studies of globally supersymmetric field theories have shown that this mechanism is able to break supersymmetry under certain conditions [12]. In the case of strings however, analogous non-perturbative studies are beyond present technical abilities and gaugino condensation is presently more a model for supersymmetry breaking than a rigorously supported mechanism.

In view of these limitations, works on four-dimensional superstrings have mainly concentrated on two aspects. Firstly the construction of large classes of perturbative string vacua, using for instance lattice [3, 4] or fermionic [5, 6] constructions, Calabi-Yau compactifications [7], orbifolds [8] and generalizations [9, 10]. This category of work addresses the classification problem, and also the derivation of efficient methods for explicit construction of realistic vacua. Then, using the different formalisms for four-dimensional superstrings, various realistic heterotic string vacua have been constructed and their phenomenology has been studied in details. A class of such models, which will be briefly discussed later on uses the flipped $SU(5) \times U(1)$ unified gauge group [13]. Such models have been considered in Calabi-Yau compactifications [14, 15], in a fermionic construction [16] equivalent to a $Z_2 \times Z_2$ orbifold, and in $Z_6$ and $Z_{12}$ orbifolds [17].

The purpose of this paper is to review some aspects of orbifold heterotic vacua (section 2), and discuss some phenomenologically acceptable models constructed using simple symmetric orbifolds (section 3). The discussion is kept at a general level. In the last section, the effective low-energy field theory describing string massless states is briefly considered, including some recent results on explicit computations of string one-loop corrections to the effective action for the simplest $(2,2)$ orbifolds.

## 2 Some Orbifold Vacua

A very large class of string vacua, with very different gauge groups and particle contents, can be described using orbifold compactifications of the heterotic string [8]. In addition, their description is remarkably simple, and they can be studied exhaustively at the string level.

In general, strings on orbifolds can be completely and simply characterized by the set of boundary conditions satisfied by all closed string coordinates in the model. For the heterotic string, besides the space-time coordinates $X^\mu(\tau,\sigma)$, string coordinates include eleven complex left-moving bosonic fields $Z^I(\sigma+\tau)$, $I = 1,\ldots,11$, three complex right-moving bosonic fields $Z^k(\sigma-\tau)$, $k = 1,2,3$ and four complex, right-moving (Green-Schwarz) Weyl fermions $S^a(\sigma-\tau)$, $a = 1,\ldots,4$. Orbifolds are characterized by the appearance of twisted boundary conditions for bosons, of the form

$$Z(\sigma \pm \tau + \pi) = e^{2i\pi\theta} Z(\sigma \pm \tau) + \Delta, \qquad (\theta \neq 0). \tag{1}$$

This means that the right-moving ($-$ sign) or left-moving ($+$ sign) string $Z$ is closed on a circle (torus) with circumference $\Delta$, up to a phase defined by the twist angle $\theta$. The spectrum of physical states is the collection of an untwisted, $\theta = 0$, sector and several twisted sectors. The rules for constructing the spectrum are dictated by the consistency of the string theory. Finiteness of loop amplitudes imposes conditions on the choice of all boundary conditions applied to all string coordinates in all sectors. It also determines the particle content in each sector. For instance, the mode expansion corresponding to equation (1) is

$$Z(\sigma \pm \tau) = Z_0 + \frac{i}{2} \sum_{n \in Z} \frac{1}{n - \theta} \alpha_{n-\theta} e^{-2i(n-\theta)(\sigma \pm \tau)}, \tag{2}$$

with

$$Z_0(1 - e^{2i\pi\theta}) = \Delta. \tag{3}$$

This expansion does not contain any term linear in $\sigma \pm \tau$: a twisted string has zero center of mass momentum, it can only oscillate around each possible fixed point $Z_0$. The oscillator content of the state is as usual obtained by the action of the mode operators with non integer frequencies $\alpha_{n-\theta}$ on a vacuum $|Z_0>$ related to each fixed point.

In the case of heterotic strings on orbifolds, there are two general classes. Symmetric orbifolds [8] have twisted boundary conditions applied symmetrically on left- and right-movers. They have a direct interpretation in terms of Kaluza-Klein compactified ten-dimensional heterotic strings. Our discussion will concentrate on them. Asymmetric orbifolds [18] contain different twisted boundary conditions for left- and right-movers and their interpretation as a compactification as well as their construction are more obscure.

We will also concentrate on $N = 1$ (space-time) supersymmetric orbifolds which are known to be stable, consistent perturbative vacua of the heterotic strings, and also offer some interesting phenomenological applications. When supplemented by a mechanism for supersymmetry breaking, some classes of symmetric orbifolds lead to almost realistic models, and reproduce correctly the standard model of strong and electroweak interactions.

The untwisted sector of a symmetric orbifold corresponds to a compactification on a torus, with an additional GSO projection restricting physical untwisted states. Torus compactification of bosonic string fields corresponds to the boundary conditions

$$X^k(\sigma + \pi, \tau) = X^k(\sigma, \tau) + \pi n_i e_i^k, \tag{4}$$

where $e_i^k$ ($k = 1, \ldots, d$) is the $k$-th component of the basis vector $\vec{e}_i$ of a $d$-dimensional lattice $\Lambda$. The winding numbers $n_i$ are integers and the point $n_i\vec{e}_i$ belongs to $\Lambda$. This boundary condition leads to the mode expansion

$$X^k(\sigma, \tau) = x^k + p^k\tau + L^k\sigma + \frac{i}{2} \sum_{n \neq 0} \frac{1}{n} \left( \alpha_n^k e^{-2in(\sigma+\tau)} + \tilde{\alpha}_n^k e^{-2in(\sigma-\tau)} \right), \tag{5}$$

with

$$L^k = n_i e_i^k. \tag{6}$$

Quantization implies that admissible momenta $p^k$ belong to the dual lattice, $\tilde{\Lambda}$: $p^k = m_i \tilde{e}_i^k$ with $e_i^k \tilde{e}_j^k = \delta_{ij}$ and $m_i \in \mathbf{Z}$. With torus boundary conditions only, $N = 4$ space-time supersymmetry would always remain unbroken. In the orbifold untwisted sector, the spectrum of the torus compactification is truncated. This in particular reduces space-time supersymmetry. In the context of string theory, this truncation of the space of physical states is a Gliozzi-Scherk-Olive (GSO) projection.

The introduction of a GSO projection, when applied to the untwisted sector only, destroys however the consistency (finiteness) of the torus model, which is expressed in terms of modular invariance of string amplitudes. In order to restore modular invariance, twisted sectors specifically adapted to the orbifold GSO projection must be introduced.

From the point of view of Kaluza-Klein compactifications, six-dimensional orbifolds $O_6$ play the role of the internal, compact space on which ten-dimensional heterotic strings are compactified, leaving a four-dimensional Minkowski space-time, $M_4$: $M_{10} \to M_4 \times O_6$. Geometrically, the orbifold $O_6$ is the space obtained by taking the quotient of a six-dimensional torus by a discrete point group $P$ acting with fixed points on the torus. Since the torus is obtained by dividing the six-dimensional euclidean space $\mathbf{R}^6$ by the lattice $\Lambda$ with basis vectors $\vec{e}_i$, the orbifold can equivalently be viewed as the quotient of $\mathbf{R}^6$ by the space group containing lattice translations and point group transformations. The action of the point group on the lattice must be well defined, it must be a symmetry of $\Lambda$. If the point group $P$ does not possess fixed points of the torus, the resulting quotient is a manifold. On the other hand, if $P$ has fixed points, the quotient is an orbifold, with isolated singularities at the fixed points.

A very simple example of a non-compact orbifold would be a cone with opening angle $2\pi/N$. It can be obtained by dividing the complex plane by the discrete group $Z_N : z \in \mathbf{C} \to e^{2i\pi/N}z$. The origin is the unique fixed point of $Z_N$. A closed string propagating on the cone, which does not surround the singularity at the origin, corresponds to a curve with $z(\sigma + \pi) = z(\sigma)$. It is untwisted and can have an arbitrary center of mass momentum. A closed string around the singularity is such that $z(\sigma + \pi) = e^{2i\pi k/N}z(\sigma)$, $k = 1, \ldots, N-1$. It is called 'twisted' and can only oscillate around the singularity. Since the cone is non compact, it cannot be used as the internal space of a Kaluza-Klein theory: $\mathbf{C}$ should be replaced by a torus, for instance.

In the simple case of a symmetric six-dimensional orbifold, with an abelian point group $P = Z_N$, one can choose a complex basis on which $P$ acts as a diagonal matrix:

$$P = \{g^n\}, \qquad n = 0, 1, \ldots N-1, \tag{7}$$

the generator being

$$g = e^{2i\pi M}, \tag{8}$$

and

$$M = diag(\eta_1, \eta_2, \eta_3). \tag{9}$$

The three angles $\eta_i$, $i = 1, 2, 3$ define the action of $P$ on the six (real) internal, compactified bosonic string coordinates, denoted by $\vec{z}$, in the complex basis. Consistency of the theory requires that all possible sectors [$n = 0$ (untwisted), $1, \ldots, N-1$ (twisted sectors)] are present in the spectrum of the orbifold. According to eq. (3), fixed points in twisted sector $n$ satisfy then

$$\vec{z_f} = g^n \vec{z_f} + \pi \vec{l}, \tag{10}$$

where $\vec{l}$ is a lattice vector. The angles $\eta_i$ cannot be chosen arbitrarily since the point group must be a symmetry of the lattice. This is the so-called cristallographic condition. This turns out to be the case if the number of fixed points is integer: assuming $\eta_i \neq 0$, $i = 1, 2, 3$,

$$\prod_{i=1}^{3} |1 - e^{2i\pi\eta_i}|^2 = integer. \tag{11}$$

(in a three-dimensional space, with only one angle, this is the statement that the point symmetry of a crystal has order 2, 3, 4 or 6). This condition allows to classify the possible point groups and then to construct the appropriate space groups.

The spectrum of the untwisted sector corresponds to the torus spectrum, as obtained from eqs. (5) and (6), truncated by the orbifold GSO projection. Torus states have oscillator modes, winding numbers $n_i$ and discrete momenta $m_i$. The GSO projection simply specifies that only states left invariant by the action of the point group generator survive the truncation. This eliminates states with non-zero winding and momenta from the orbifold spectrum, and reduces space-time supersymmetry.

Altogether, the consistency of the orbifold is guaranteed in perturbation theory when three kinds of conditions are met. Firstly, the boundary conditions must be chosen in such a way that local world-sheet reparametrization invariance is accompanied by a fermionic local symmetry which is either world-sheet supersymmetry when the fermionic degrees of freedom are described using Neveu-Schwarz-Ramond fermions, or $\kappa$-symmetry when Green-Schwarz fermions are used [19]. These two possibilities can be shown to lead to equivalent classes of theories for arbitrary point groups [20]. This condition relates the twisted boundary conditions applied to the fermions to the point group element acting on the bosonic string coordinates. It is equivalent to the statement that the point group $P$ should be a discrete subgroup of $SO(6)$, with Green-Schwarz fermions and bosons respectively in representations **4** and **6** of $SO(6)$ [8, 10]. For $N = 1$ orbifolds, one has $P \subset SU(3)$, $P \not\subset SU(2)$. The second class of conditions is modular invariance of the one-loop, world-sheet torus, vacuum amplitude. Modular invariance is equivalent to a set of 'level-matching' conditions [8], applied on the set of boundary conditions for all heterotic string coordinates. They will for instance specify the gauge group of the orbifold model as a function of the point group. Thirdly, higher order modular invariance requires the existence of an orbifold GSO projection which completely defines the spectrum of physical states of the theory. This GSO generalizes the truncation applied in the untwisted sector to all twisted sectors, and completely defines (up to possible choices of free GSO phases) the spectrum of twisted, physical states. All these consistency conditions are directly related to the two-dimensional, world-sheet character of the dynamics of the string theory. They only depend on the space-time properties in the fact that a dimension of space-time is chosen: this tells us which string fields can have non-trivial, compactified boundary conditions.

Since symmetric orbifolds can be equivalently constructed using either two-dimensional consistency arguments only, as outlined above, or by following a Kaluza-Klein approach (find perturbative solutions to the equations of motion of ten-dimensional heterotic strings which reduce the space-time dimension to four), it is instructive to compare the origin of the various consistency conditions arising in both approaches.

In the compactification picture, consistency is expressed by two conditions. Firstly, the point group defining the twisted boundary conditions of the orbifold string states should be a symmetry of the lattice: the point group $P$ should act cristallographically on the lattice. This condition is simply necessary to define the orbifold:

$$Orbifold = \frac{Torus\ T}{Point\ group\ P}, \qquad T = \frac{\mathbf{R}^d}{\Lambda}, \tag{12}$$

$\Lambda$ being a $d$-dimensional lattice. The second condition for consistency of the orbifold compactification is the breakdown of the gauge group (from $E_8 \times E_8$ or $SO(32)$, in ten dimensions) in specific ways, depending on the point group. The simplest category is $(2,2)$ symmetric orbifolds for which $E_8 \times E_8$ breaks into

$$E_6 \times H \times E_8,$$

$H$ being either $SU(3)$ or $SU(2) \times U(1)$ or $U(1)^2$ depending on $P$.

These two conditions can be directly related to the consistency conditions found by solving the conditions for finiteness of the two-dimensional field theory. As already mentioned, finiteness corresponds to all order modular invariance of the orbifold partition function (the vacuum amplitude on world-sheet Riemann surfaces of arbitrary genus). This is well known to generate only three independent conditions (provided higher genus amplitudes factorize as expected: this should still be proved). The conditions arising from one-loop modular invariance can be reexpressed in the form of level-matching conditions, which in turn determine the breaking of the gauge group, in complete agreement with the compactification picture. The condition for two-loop and higher order modular invariance, leads to the existence of a specific Gliozzi-Scherk-Olive (GSO) projection. It should then correspond to the condition of cristallographic action of the orbifold point group. This equivalence is more subtle since the GSO projection in untwisted and twisted sectors provides different informations. The existence of a correctly normalized GSO projection implies first that the number of fixed points must be an integer, as in eq. (11). The partition function in each twisted sector is proportional to the quantity $\prod_{i=1}^{3} |1 - e^{2i\pi\eta_i}|^2$. Because of one-loop modular invariance, this quantity also appears in the partition function of the untwisted sector. If it would not be an integer, the spectrum would not be defined at all (non integer number of states). This tells us that $P$ is a point group but does not indicate for which lattice. This last information is contained in both the twisted and untwisted partition functions. In both cases, the GSO projection, generated by the point group must be able to remove the untwisted states with non-zero windings and lattice momenta, and also act on the various fixed point vacua $|Z_0>$ in twisted sectors. This is only feasible if the structure of windings, momenta and fixed points is tuned to the fixed point under consideration: this is the cristallographic condition. To convincingly demonstrate this last statement, the explicit use of a complete partition function, constructed in the two-dimensional picture by requiring all order modular invariance (as for instance in ref. [10]), is necessary: this is beyond the scope of the present discussion.

# 3 Some realistic orbifold vacua

To be considered as potentially realistic, a vacuum of the heterotic string must fulfil various conditions. Some of them are simple to impose: they use well-controlled aspects of string model building. Others are more involved since they involve detailed knowledge of massless state interactions, computed at the string level. The minimal conditions have to do with the gauge group, the matter content and supersymmetry of the string vacuum. The gauge group should be of the form:

$$G = G_{visible} \times U(1)^n \times G_{hidden}, \tag{13}$$

where $G_{visible}$ contains and maybe unifies the standard-model gauge group $G_{sm} = SU(3)_c \times SU(2)_L \times U(1)_Y$. By itself, this condition is straightforward to satisfy ($E_8 \times E_8$ is already of this form, with $n = 0$ and $G_{visible} = E_8$). The matter content should include three generations of quarks and leptons and the scalar multiplets for the spontaneous breaking of $G_{visible}$. This visible matter transforms in non trivial representations of $G_{visible} \supset G_{sm}$, is invariant under $G_{hidden}$ and allowed in general to have non zero $U(1)^n$ charges. In most cases, matter also includes fermions with non trivial $G_{hidden}$ and $U(1)^n$ quantum numbers, but singlets with respect to the non-abelian part of $G_{visible}$ (hidden matter). This definition of the visible and hidden parts of the matter content actually defines the hidden part of the gauge group as the part of the non-abelian subgroup of the gauge group which does not communicate with $G_{visible}$ through matter particles having both $G_{visible}$ and $G_{hidden}$ quantum numbers.

Since $G_{hidden}$ is non-abelian and contains in general asymptotically-free components, it gives rise to confining forces which have a characteristic energy scale $\Lambda_{hidden}$, the scale at which $G_{hidden}$ becomes strongly coupled (there can be several different scales if $G_{hidden}$ contains several asymptotically-free subgroups). Bound states with masses $\sim \Lambda_{hidden}$ will then be present in the spectrum at lower energies. This confinement mechanism in the hidden sector has two important consequences. Firstly gaugino condensates will form, providing a possible source for dynamical supersymmetry breaking [11, 21]. Secondly, a generic feature of realistic string vacua is the presence of fractionally charged massless and massive string states [22, 23]. Hidden forces can provide a mechanism for confining these unwanted particles into bound states with integer electric charges (this is for instance the case [24] for the flipped $SU(5)$ model of ref. [16]). Notice that the hidden sector would only be really hidden if all its matter would have zero $U(1)^n$ charges. In this case only, and for all energies below the Planck scale, would the visible and hidden sectors only communicate through interactions of gravitational strengths. Using however this criterion as a condition on realistic string vacua is much too strong, at least in the context of orbifold or fermionic models [23, 25].

The visible part of the gauge group is either $G_{sm} = SU(3)_c \times SU(2)_L \times U(1)_Y$ itself (for orbifold examples, see [26, 27]), or a larger, non-unified group like $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, or a flipped, unified group [14, 15], the most attractive possibility being 'flipped $SU(5)$' [13, 16], $G_{visible} = SU(5) \times U(1)$ (where $SU(5)$ contains $SU(3)_c \times SU(2)_L$ but not $U(1)_Y$). Standard unification for which $G_{sm} \subset G \subset G_{visible}$, with $G$ simple is however excluded with four-dimensional strings. This observation, which was originally made for the case of Calabi-Yau compactifications [14, 15], is related to the absence of massless scalar multiplets in the adjoint representation of $G$. It is then impossible to spontaneously break $G$ into $G_{sm}$, using the Higgs mechanism. The absence of massless adjoint scalar is a general fact in potentially realistic heterotic vacua, even though this is not a theorem valid for all vacua [28]. Flipped $G_{visible}$ are unified groups for which scalar multiplets in the same representations as quark-lepton generations are sufficient for the breaking into $G_{sm}$. The simplest of them is $SU(5) \times U(1)$.

The basic structure of flipped $SU(5) \times U(1)$ models is as follows. Each quark-lepton generation includes a right-handed neutrino and is as usual in the representation

$$\mathbf{10}_1 + \overline{\mathbf{5}}_{-3} + \mathbf{1}_5 \tag{14}$$

(the notation is $\mathbf{R}_q$, $\mathbf{R}$ being the $SU(5)$ representation and $q$ the $U(1)$ charge). $SU(5)\times U(1)$ is broken into $G_{sm}$ by a scalar multiplet in representation $\mathbf{10}_1 + \overline{\mathbf{10}}_{-1}$. The weak hypercharge is then a linear combination of $U(1)$ with charges $q$ and another $U(1)'$ group appearing in the subgroup $SU(3)_c \times SU(2)_L \times U(1)'$ of $SU(5)$. In comparison with the Georgi-Glashow $SU(5)$ unified theory, the assignement of states in a generation exchanges the right-handed, charge 2/3 and 1/3 quarks, and also the right-handed charged lepton and neutrino. An important property of (supersymmetric) string-based flipped $SU(5)$ models is a natural doublet-triplet mechanism [15], which follows from the absence of quadratic superpotential (mass terms). An example of a flipped $SU(5)$ heterotic vacuum has been constructed using the fermionic formalism in ref. [16]. By boson-fermion equivalence, this model could also be viewed as an orbifold with point group $Z_2 \times Z_2$. Its full gauge group corresponds to eq. (13), with $G_{visible} = SU(5) \times U(1)$, $G_{hidden} = SO(10) \times SO(6)$ and $n = 4$. Visible matter has four generations and one antigeneration, including the Higgses for $SU(5) \times U(1)$ breaking, plus $4(\mathbf{5}_{-2} + \overline{\mathbf{5}}_2)$ giving rise to the Higgs doublets for the electroweak breaking, after automatic doublet-triplet splitting. This model, which turns out to be remarkably attractive, has been analysed in great details [16, 29]: it is probably the realistic heterotic vacuum which is at present best understood at the string level.

This flipped model, constructed using $Z_2 \times Z_2$ orbifolds, is by far not unique. Using $Z_6$ or $Z_{12}$ orbifold (this choice was dictated by fixed point structures which favour obtaining three generations of quarks and leptons), further models have recently been constructed and studied [17]. The aim of this work is to compare various flipped models with the same visible gauge group and chiral matter, to look for possible common features in the superpotential or Yukawa couplings (the Kobayashi-Maskawa matrix). These common features would be new string predictions, beyond the constraints only due to gauge invariance and general properties of string flipped models (like the doublet-triplet splitting mechanism). The main problem when trying to construct orbifold flipped $SU(5)$ vacua is actually related to the Higgs multiplets necessary for the breaking of $SU(5) \times U(1)$. One representation $\mathbf{10}_1 + \overline{\mathbf{10}}_{-1}$ is needed, but the spectrum in general contains several complete ($generation + antigeneration$) multiplets (one only for the model of ref. [16]). The main difficulty is then to avoid unwanted massless or nearly massless charged, vector-like fermions, remnants of this Higgs structure. Giving a mass to these dangerous multiplets is a delicate problem, which requires the use of non-renormalizable contributions to the superpotential (terms up to order 6 were considered in [17]). From this small class of flipped models, it seems that the superpotential does not possess features which would be string predictions: different vacua lead to very different superpotentials and Yukawa couplings. The number of examples is however too small to draw strong conclusions and further work could be considered useful.

Analogous studies could certainly be performed for other classes of gauge groups, or for the vacua with $G_{visible} = G_{sm}$, which have up to now essentially been studied in the limited context of $Z_3$ orbifolds [26].

# 4 Low-energy effective field theory and string loop corrections

To discuss low-energy, $E \ll M_{Planck} \sim 10^{19} GeV$, physics, only the massless part of the string spectrum of physical states is relevant. In general, massless states of $N = 1$ supersymmetric heterotic strings belong to three categories. Firstly, the gravitational sector of the theory, which is universal in all four-dimensional models, contains the supergravity multiplet (graviton and gravitino) and one chiral supermultiplet, gauge singlet and containing a Majorana spinor and a complex scalar $S$. The real part of $S$ is related to the dilaton degree of freedom and $Im\, S$ is obtained by a duality transformation applied on an antisymmetric tensor $b_{\mu\nu} = -b_{\nu\mu}$. The second class of massless states has a geometrical origin. The equations of motion of ten-dimensional heterotic strings do not determine the sizes of the compactification space (i.e. the radii of the orbifold), but only its geometry [it should be a Ricci-flat space with holonomy group contained in $SU(3)$]. Any deformation of the compact space which does not modify the topology generates a new solution. These deformations will then also correspond to massless scalar states $T$ (and their supersymmetric spin 1/2 partners), called moduli. All expectation values $< Re\, T >$ give vacua of the theory with degenerate energies (the scalar potential is then flat in these directions), corresponding to the various radii and dimensions of the compact space left undetermined by the perturbative equations of motion. Fields in the third and last class are either charged under the gauge group (chiral supermultiplets generically denoted by $C$) or belong to the Yang-Mills supermultiplet (gauge fields and gauginos).

These three classes of states play specific roles in the effective field theory obtained from the string by integrating out the massive string modes, as an expansion in powers of $E/M_{Planck}$. This effective theory is specified by a local supergravity Lagrangian $\mathcal{L}(S,T,C)$ which, at least in the case of symmetric orbifolds, is well established at string tree-level (to all orders in the moduli $T$), and only very partially known at the level of string loop corrections.

In the case of symmetric, $N = 1$ orbifolds, the number of untwisted moduli is easily determined from the point group $P$. In the complex basis (9), $P$ acts with phases $e^{2i\pi\eta_i}$ on the three complex planes. It is clear that for each complex plane, the overall radius of the torus will be left undetermined, giving rise to one $(1,1)$ modulus $T$. If however, two phases $\eta_i$ are equal, there will be four $(1,1)$ moduli corresponding to the freedom of a unitary rotation of the orbifold planes. The unique case with three identical phases is the $Z_3$ orbifold with $\eta_i = 1/3$, $i = 1,\ldots,3$: the number of $(1,1)$ moduli is then nine. One can also have one $(1,2)$ modulus when some $\eta$ is one-half (this is only possible on one plane for $N = 1$ orbifolds), corresponding to the fact that antiperiodic boundary conditions do not mix the real and imaginary parts of a complex boson. One can then rotate the real part independently from the imaginary part, hence the existence of a further modulus.

For simplicity of the discussion, we will only consider here the case where all three twist phases $\eta_i$ defining the point group are different (and $\eta_i \neq 1/2$). This is only the case for the $Z_7$, $Z_8'$ and $Z_{12}'$ orbifolds [8]. The gauge group is then $E_6 \times U(1)^2 \times E_8$, and the massless spectrum contains three $(1,1)$ moduli $T_i$, three untwisted **27** generations $C_i$, zero $(1,2)$ modulus and untwisted antigeneration. In addition, the twisted sectors (indexed by $k$) contain generations **27**'s and antigenerations $\overline{\mathbf{27}}$'s (except for $Z_7$ which only possesses **27**'s) denoted by $C_T^k$ for each twisted sector. (Possible twisted moduli which can at most have $U(1)^2$ quantum numbers and singlets will be omitted).

Since the effective theory has local supersymmetry, it has the standard supergravity form [30], with $\mathcal{L}$ expressed in terms of two functions and their derivatives: the real Kähler function $\mathcal{G}$, and the gauge kinetic function $f_{ab}$, analytic in the chiral scalars. At tree-level, the Kähler function for the effective low-energy Lagrangian has the following form [31, 32]:

$$\begin{aligned}\mathcal{G}(S, T_i, C_i, C_T^k) \;=\; & -ln(S+\overline{S}) \\ & -ln\left[\prod_{i=1}^3 (T_i+\overline{T}_i - 2C_i\overline{C}_i) - \sum_k C_T^k \overline{C}_T^k \prod_{i=1}^3 (T_i+\overline{T}_i)^{\eta_i^k}\right] \\ & +ln|W|^2,\end{aligned} \tag{15}$$

where $W$ is the superpotential. The gauge kinetic function is simply [21, 31]

$$f_{ab} = S\delta_{ab}, \tag{16}$$

indices $a, b$ denoting the adjoint representation of the gauge group. This theory possesses target space modular invariance (or target space duality) [33]: it is invariant under the transformations

$$\begin{aligned} S &\rightarrow S, \\ T &\rightarrow \tfrac{aT-ib}{icT+d}, \\ C_i &\rightarrow e^{i\alpha(a,b,c,d)}|icT+d|^{-1}C_i, \\ C_T^k &\rightarrow e^{i\alpha'(a,b,c,d)}|icT+d|^{-2}C_T^k, \\ W &\rightarrow |icT+d|^{-3}W, \end{aligned} \tag{17}$$

where $a, b, c, d \in \mathbf{Z}$ and $ad - bc = 1$. The tree-level action is actually invariant under the continuous modular transformations, $a, b, c, d \in \mathbf{R}$, but only the discrete symmetry is preserved by quantum, perturbative corrections. More precisely, the Kähler function (15) has been checked to give the correct effective theory up to quadratic order in the charged fields $C_i$ and $C_T^k$, but to all orders in moduli $T_i$ [32]. The complete form (15) is deduced using target space duality. The superpotential $W$ is a cubic polynomial in the charged fields, at tree-level.

The scalar potential in this effective theory exhibits the correct flat directions, leaving $< Re\, S >$ and $< Re\, T_i >$ undetermined. Supersymmetry is unbroken for all its minima. When gaugino condensates are introduced, supersymmetry is broken provided the superpotential can acquire a non-zero vacuum expectation value, $< W >$. But at tree-level at least, there is no source for $< W > \neq 0$. This problem with supersymmetry breaking is deeply related to the particular dependence on $S$ and $T_i$ exhibited by the tree-level $\mathcal{G}$ and $f_{ab}$. It is plausible to expect that string loop corrections, which preserve target space duality, could modify these functions in such a way that the minimum of the scalar potential, once the effect of gaugino condensates has been included, would break supersymmetry with determined and finite $< Re\, S >$ and $< Re\, T_i >$.

Recently, a string one-loop calculation of the moduli dependence of gauge kinetic terms for $(2,2)$ orbifolds [34] has provided new informations on this problem. An interpretation compatible with supersymmetry of these results [35] shows that the one-loop effect at the origin of the computed correction is a mixed gauge – sigma-model anomaly, which can then be computed in the effective low-energy theory, as defined by eqs. (15) and (16). By supersymmetry, a general gauge kinetic term of the form

$$-\frac{1}{4}h(T_i,\overline{T}_i)F^a_{\mu\nu}F^{\mu\nu\, a}, \qquad h=\overline{h}, \tag{18}$$

is related to an anomaly term for two gauge fields $A_\mu$ and one composite vector field $V_\nu$:

$$-\frac{1}{4}\epsilon^{\mu\nu\rho\sigma}V_\mu\Omega_{\nu\rho\sigma}, \tag{19}$$

where $\Omega_{\nu\rho\sigma}$ is the Chern-Simons form for the gauge group of the theory,

$$\Omega_{\nu\rho\sigma}=Tr\left(A_{[\nu}F_{\rho\sigma]}-\frac{1}{3}A_{[\nu}A_\rho A_{\sigma]}\right), \tag{20}$$

and

$$V_\mu=i\frac{\partial h}{\partial T_i}\partial_\mu T_i+c.c. \tag{21}$$

is a gauge (holonomy) connection for the sigma-model defined by the Lagrangian of the moduli fields $T_i$. This connection can easily be computed from the tree-level effective $\mathcal{G}$. The function $h$ in eq. (18) is then obtained from the triangle anomaly diagram with two external gauge fields and one external holonomy connection $V_\mu$, computed for all massless fermions of the string theory. This approach has the important advantage that it only uses the effective theory for the massless modes. The effect of massive string states is to promote the function $h$ in eq. (18) to a modular function, invariant under transformations (17).

In addition, since the one-loop, moduli-dependent correction to gauge kinetic terms is due to a chiral anomaly, one can conjecture the existence of an Adler-Bardeen theorem which would imply that perturbative corrections can be computed to all orders, and give a new non-renormalization theorem for the low-energy effective field theory. Also, one can expect to generalize the results obtained in ref. [34] to larger and more interesting classes of string vacua. (2,2) orbifolds are phenomenologically irrelevant, but string loop computations are relatively simple. Realistic string vacua would in general require much more complicated one-loop string calculations to obtain similar results. The anomaly approach should be more powerful since it only uses a triangle diagram in the effective theory and the knowledge of the target space duality algebra corresponding to the string model, analogous to eq. (17). Such generalizations will be important for future phenomenological studies, and for the problem of obtaining a satisfactory scheme for supersymmetry breaking induced by gaugino condensation in the hidden sector [11], compatible [36] with the quantum duality symmetries of the string theory.

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