Inflation-driven string instabilities ... and the other way around

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INFLATION-DRIVEN STRING INSTABILITIES ...
... AND THE OTHER WAY AROUND

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We review recent work showing that inflation can drive Jeans-like string instabilities, while it is not so easy for such strings to drive inflation.

1. Introduction

I ought to start this talk by a short explanation of the title:

The "instabilities" that we shall be dealing with have nothing to do with the usual break-up and decay of excited strings, a typical **quantum**, string-loop effect. Rather, we shall be dealing with a **classical** instability (of the Jeans type) caused by a period of fast, accelerated expansion of the Universe.

The "other way around" part of the talk will address the following question: is it possible that the equation of state of a fluid made of unstable strings (in the above sense) will sustain inflation, thus dispensing us from the need of a cosmological constant? Can we have, in short, a "self-sustained inflation" of the kind proposed [1,2] a few years ago?

For the sake of establishing notations and of being self-contained I will now recall a few known facts about homogeneous, isotropic cosmologies. The Friedmann-Robertson-Walker (FRW)-like element ($k = 0$, for simplicity)

$$ds^2 = dt^2 - R^2(t)d\vec{x}^2 \quad (c = 1)$$

satisfies Einstein's equations provided (in $D = 4$)

$$\begin{aligned}(\dot{R}/R)^2 &= \frac{8\pi G}{3}\,\rho \\ (\ddot{R}/R) &= -\frac{4\pi G}{3}\,(\rho + 3p)\end{aligned} \tag{1.1}$$

where dot denotes (in this section) derivative w.r.t. cosmic time t and the energy density and pressure of the (supposedly) perfect fluid are defined in terms of its energy momentum tensor $T_{\mu\nu}$ by:

$$T^0_0 = \rho; \quad T^j_i = -\delta_{ij} p \tag{1.2}$$

Ordinary matter satisfies

$$0 \le p \le \rho/3 \tag{1.3}$$

where the two extreme cases are reached by a non-relativistic and ultra-relativistic fluid, respectively.

By contrast, a cosmological constant term corresponds to $T^\nu_\mu \propto \delta^\nu_\mu$, i.e., to $p = -\rho$.

If ordinary matter dominates we have a standard cosmology with $\ddot{R} < 0$. This yields notorious problems - which we shall not review here - and which are (at least partly) solved by assuming that, in the past, the Universe has undergone a long period of inflation, i.e., a period during which $\ddot{R} > 0$. It follows from (1.1) that $p < -\rho/3 < 0$ is needed for inflation. A cosmological constant ($p = -\rho$) is perfectly suited, but by no means a necessity.

The kind of inflation brought by a cosmological constant is exponential since $p = -\rho$ implies through (1.1):

$$(\dot{R}/R)^2 = (\ddot{R}/R) \Rightarrow R = \exp(Ht), \quad H^2 = \frac{8\pi G\rho}{3} \equiv \Lambda/3 \tag{1.4}$$

Another kind of inflation which has been considered in the literature is the so-called power inflation:

$$R(t) \sim t^\gamma \quad (\gamma > 1) \tag{1.5}$$

It is characterized by $\ddot{R} > 0, \dot{H} \equiv \frac{d}{dt}\ (\dot{R}/R) < 0$. Finally one talks about super (or pole) inflation when $\dot{H} > 0$. An example is given by

$$R(t) \sim (-t)^{-\gamma} \quad (\gamma > 0) \tag{1.6}$$

The scale factor reaches an infinite value at a finite cosmic time (here taken to be $t = 0$).

As a final reminder we recall that it is useful sometime to work with conformal time η defined by

$$dt = R d\eta \Rightarrow ds^2 = R^2(\eta)(d\eta^2 - d\vec{x}^2) \tag{1.7}$$

One finds immediately that in terms of η the three kinds of inflation described above all correspond to scale factors of the type

$$R(\eta) = (-\eta)^{-\alpha}, \quad \alpha > 0 \tag{1.8}$$

where $0 < \alpha < 1$ gives superinflation, $\alpha = 1$ gives exponential (or de Sitter) inflation and $\alpha > 1$ gives power inflation. Notice that, in all cases, $R \to \infty$ for $\eta \to 0^-$.

2. Points and strings in FRW cosmologies

Let us compare the classical equations of motion of a relativistic (massless) point with those of a string. For a generic metric $g_{\mu\nu}$ the trajectory $X^\mu(\tau)$ described by the point is given by the solution of the geodesic-like equations:

$$\ddot{X}^\mu + \Gamma^\mu_{\nu\rho}\dot{X}^\nu\dot{X}^\rho = 0 \tag{2.1a}$$

$$\dot{X}^\mu\dot{X}^\nu g_{\mu\nu} = 0 \tag{2.1b}$$

where

$$\Gamma^\mu_{\nu\rho} \equiv \frac{1}{2}g^{\mu\lambda}\left[\partial_\nu g_{\rho\lambda} + \partial_\rho g_{\nu\lambda} - \partial_\lambda g_{\nu\rho}\right] \tag{2.2}$$

Similarly, the surface swept by the string $X^\mu(\sigma,\tau)$ is given by the geodesic-surface equations

$$\ddot{X}^\mu - X''^\mu + \Gamma^\mu_{\nu\rho}(\dot{X}^\nu + X'^\nu)\,(\dot{X}^\rho - X'^\rho) = 0 \tag{2.3a}$$

$$g_{\mu\nu}(\dot{X}^\mu\dot{X}^\nu + X'^\mu X'^\nu) = 0 \tag{2.3b}$$

$$g_{\mu\nu}\dot{X}^\mu X'^\nu = 0 \tag{2.3c}$$

In the above equations, dots and primes represent, as usual, derivatives w.r.t. τ and σ, respectively. Actually, Eqs. (2.1a) and (2.3a) represent the equations of motion while (2.1b) and (2.3b,c) represent the constraints following from τ reparametrization and $\sigma - \tau$ reparametrization invariance respectively.

In a FRW metric, Eq. (2.3a) splits into a space and a time equation

$$\ddot{X}^i - X''^i = 2H(X'^0X'^i - \dot{X}^0\dot{X}^i) \tag{2.4a}$$

$$\ddot{X}^0 - X''^0 = R^2 \cdot H\sum_i[(X'^i)^2 - (\dot{X}^i)^2] \tag{2.4b}$$

where $H \equiv R^{-1}dR/dX^0 = R^{-1}dR/dt$.

The constraints (2.3b,c) become:

$$(\dot{X}^0)^2 + (X'^0)^2 = R^2 \sum_i [(X'^i)^2 + (\dot{X}^i)^2] \tag{2.5a}$$

$$X'^0 \dot{X}^0 = R^2 \dot{X}^i X'^i \tag{2.5b}$$

Equations (2.4a,b), (2.5a,b) are a system of non-linear partial differential equations which, as such, does not look particularly easy to deal with.

Exact solutions appear out of question for any non-trivial $R(t)$. The only hope appears to be finding a small parameter in which to expand the exact solution.

A few years ago, De Vega and Sánchez [DS] [3] proposed an expansion around the point (massive in general) particle motion. The latter is obtained by solving a system of **ordinary** differential equations. It is exactly known in the implicit form:

$$\begin{aligned} \alpha' \tau &= \int_{\eta(0)}^{\eta(\tau)} d\eta' R(\eta') \, (m^2 + p^2/R^2)^{-1/2} \\ x^i(\tau) &= x^i(0) + \alpha' p^i(0) \int_0^\tau d\tau' \; R^{-2}(\eta(\tau')) \end{aligned} \tag{2.6}$$

where $\eta(\tau)$ is the conformal time defined in (1.7) and m is the point-particle mass. Equations (2.6) represent indeed a **general solution** since they contain the correct number $(2(D-1))$ of arbitrary constants $(x^i(0), p^i(0))$.

A simplification occurs at large R, since $p/R \to 0$ (red shift!):

$$\alpha' \tau = \frac{1}{m} \int_{\eta(0)}^{\eta(\tau)} d\eta' \cdot R(\eta') = \frac{1}{m}(t - t_0) \tag{2.7}$$

where (1.7) has been used. At large R, world sheet and **cosmic** times become proportional.

Coming to the string, one can try the expansion [3]:

$$X^\mu(\sigma, \tau) = x^\mu(\tau) + Y^\mu(\sigma, \tau) \tag{2.8}$$

treating Y^μ as a perturbation. The physical idea behind this expansion is that a small-enough string (in a sense to be specified) should follow closely the point-particle geodetic.

It is not too hard to study the problem up to second order in Y^μ. For the components Y^i orthogonal to x^i the equations are particularly simple:

$$\ddot{Y}^i - Y''^i + 2H\dot{Y}^i = 0 \tag{2.9}$$

Introducing $\chi^i = RY^i = \sum_n \chi_n^i e^{in\sigma}$ one easily gets:

$$\ddot{\chi}_n^i = [-n^2 + \ddot{R}/R]\chi_n^i \tag{2.10}$$

For decelerated (i.e., non-inflationary) expansions the behaviour of χ_n^i is oscillatory with constant amplitude and frequencies approaching $\pm n$ for $R \to \infty$ ($\ddot{R}/R \to 0$).Thus, for non-inflationary metrics, the following physical picture holds: the string's motion is close to the point motion with a **proper** oscillation amplitude $\chi = RX$ remaining constant as R grows. In other words, if we consider two non-interacting strings evolving in this class of FRW metrics, their distance grows like R while each one's size stays fixed: each string "sees" an expansion of the Universe if it measures distances in its own size's units. We shall refer to regimes of this kind as "stable" regimes.

Let us now change the sign of $\ddot{R}$ from negative to positive, corresponding to an inflationary situation. Recalling (2.7) we find:

$$\ddot{\chi}_n = [-n^2 + (\alpha' m)^2 R^{-1}\frac{d^2R}{dt^2}]\chi_n \tag{2.11}$$

The crucial parameter is obviously

$$\gamma^2(t) \equiv (\alpha' m)^2 R^{-1}\ \frac{d^2R}{dt^2} \tag{2.12}$$

where we recognize in $\alpha' m$ the size of the string and in $R^{-1}\frac{d^2R}{dt^2}$ the curvature of the FRW metric (up to some factor). If the string size is small compared to the curvature radius the ensuing regime is again of the stable kind. However, if the string size becomes of the same order or larger than the curvature radius (H^{-1} for de Sitter inflation), imaginary frequencies develop [3] which have to be interpreted [4] as the outset of Jeans-like instabilities.

Indeed, by introducing a Jeans frequency

$$n_J = \gamma = (\alpha' m)\cdot (R^{-1}\frac{d^2R}{dt^2})^{+1/2} \tag{2.13}$$

we see that, for $n < n_J$, χ_n starts to grow exponentially. The DS expansion can be shown to break down in this case, while an analysis of higher (i.e., second) order corrections strongly suggests [4] that the instability ... is contagious, i.e., it propagates from the low frequencies to all frequencies.

This latter feature can be understood as follows. If $1 < n_J < 2$, the $n = 1$ mode is unstable and the associated amplitude χ_1 grows. Through the constraints (2.5a,b) this entails [4] a growth in m (so that m ceases to be a fixed parameter for the geodetic). According to (2.13), n_J grows as well (take for simplicity de Sitter inflation for which $R^{-1}d^2R/dt^2$ is a constant) and eventually becomes larger than 2. Now χ_2 becomes unstable giving an extra boost to m (and to n_J) and making χ_3 unstable at a later time, and so on, till all modes have become unstable. The situation is sketched in Fig. 1 where the qualitative behaviour of $\gamma(t)$ is shown as a function of t. If $\gamma(t) < 1$, no instability occurs (this distinguishes strings from other systems which have a continuous spectrum of frequencies and which inevitably have instabilities in the lowest modes). If $\gamma(t) \geq 1$ at some initial time then, most likely, γ will grow with t faster and faster. Thus strings which were not so different in size at some initial time may evolve along completely different patterns at late times (chaos, bifurcation?).

A quantitative study of this unstable regime looks prohibitive at first sight. Fortunately, however, a nice simplification occurs in the "extremely unstable" limit, $n \ll n_J$. In this case, Eq. (2.11) gives simply

$$\chi_n = const.R \quad \text{i.e.,} \quad Y = R^{-1}\chi = \text{const.} \tag{2.14}$$

In this regime string sizes grow as fast as the expansion rate of the Universe! Equation (2.14) gives the crucial clue to the construction of a new expansion. If $X^i \to$ const., then $\dot{X}^i \ll X'^i$. Why not try to see what happens if we neglect $\dot{X}^i$ relative to X'^i? Equations (2.4b) and (2.5a) become:

$$\ddot{X}^0 - X''^0 = R^2 \cdot H \sum_i (X'^i)^2 \tag{2.15a}$$

$$\dot{X}^{02} + X'^{02} = R^2 \sum_i (X'^i)^2 \tag{2.15b}$$

Let us try further the ansatz $X'^0 \ll \dot{X}^0$. In this case we obtain

$$\ddot{X}^0 = H\dot{X}^{02}; \quad \dot{X}^{02} = R^2 \sum_i (X'^i)^2 \tag{2.16}$$

whose solution is

$$\dot{X}^0 = R\ L(\sigma); \quad L^2(\sigma) = \sum_i (X'^i)^2 \tag{2.17}$$

i.e., recalling (1.7)

$$\eta = L(\sigma)\tau; \quad L^2(\sigma) = \sum_i (X'^i)^2 \tag{2.18}$$

Thus in the regime of extreme instability world sheet and **conformal** (rather than cosmic) time become proportional. Given the fact that, for inflationary metrics, $\eta \to 0^-$ for $R \to \infty$, the large R limit becomes the $\tau \to 0$ limit and equations of motion and constraints can be solved in the form of a small τ expansion [5]. The leading terms of the expansion were given in Ref. [5] while a number of sub-leading corrections have been computed since [6]. One finds the following expansions to hold:

$$\begin{aligned} X^i = \; & A_0^i + \tau^2 A_1^i + \tau^4 A_2^i \\ & + B_0^i \tau^{2\alpha+1} + B_1^i \tau^{2\alpha+3} \\ & + C_0^i \tau^{4\alpha+2} + \\ \eta = \; & \eta_0 \tau + \eta_1 \tau^3 + \eta_2 \tau^5 + \ldots \\ & + \lambda_0 \tau^{2\alpha+2} + \lambda_1 \tau^{2\alpha+4} + \ldots \\ & + \zeta_0 \tau^{4\alpha+1} + \ldots \end{aligned} \tag{2.19}$$

where the parameter α characterizes the metric as in Eq. (1.8) and $A_0^i(\sigma), B_0^i(\sigma)$ are arbitrary (periodic) functions of σ subject to the constraint

$$A_0'^i \, B_0^i = 0 \tag{2.20}$$

All other coefficients appearing in (2.19) are determined in terms of the above A_0^i, B_0^i. At this point the solution (2.19) would appear to depend upon $(2D - 3)$ arbitrary functions, which is one too many (as we know from free strings). The point is that we have not yet fully exploited σ-representation invariance which allows us to add one further constraint, e.g.,

$$\eta_0 = (A_0'^i A_0'^i)^{1/2} = \text{const.}, \text{ i.e., } A_0'^i A_0''^i = 0 \tag{2.21}$$

In this special "gauge" the coefficients defined in (2.19) are given in Table 1 (taken from Ref. [6]). In Table 2 (from Ref. [5]), we give instead the regimes ("stable" or "unstable") allowed by various FRW metrics (expressed in terms of conformal or cosmic time). Perhaps the most interesting feature of the table is that (while decelerating or superinflationary cosmology are only consistent with one kind of regime) intermediate inflationary metrics can allow both regimes.

Consider for instance power-law inflation, assuming an initial value for γ (see Eq. (2.12)) $\gamma(t_0) > 1$. In these metrics $R^{-1}\frac{d^2R}{dt^2}$ is a **decreasing** function of time and thus pulls down γ while the instability pushes γ up by increasing m.The morale is that some strings make it (in the sense of becoming unstable and blowing up) and some do not (they remain small for ever). We thus understand better the meaning of Fig 1.

3. The other way around

We shall now try to answer the following question: are highly unstable strings a suitable **source** for inflation?

In order to answer this question we have first to find the equation of state obeyed by a fluid made up of unstable strings. This is easy to do in the perfect-fluid approximation (non-interacting strings). The energy momentum tensor of such a system is given, in general, by the variation of the action with respect to the external metric and, in analogy with point particles, reads:

$$\sqrt{-g}T^{\mu\nu}(x) = \frac{1}{4\pi\alpha'}\sum_{K=1}^{N}\int d\sigma d\tau\ \delta(x - X_K(\sigma,\tau))\ (\dot{X}_K^{\mu}\dot{X}_K^{\nu} - X_K'^{\mu}X_K'^{\nu}) \tag{3.1}$$

where K labels the strings. Using the properties of the limit of high instability, $\dot{X} \gg X'^0, X'^i \gg \dot{X}^i$ we find

$$\begin{aligned} T^{00} &\sim \text{const.} \int (\dot{X}^0)^2 \\ T^{ij} &\sim \text{const.} \sum (X'^i X'^j)\ (-1) \end{aligned} \tag{3.2}$$

with the **same** proportionality constant and a (crucial!) minus sign coming from the relative minus sign appearing on the right-hand side of (3.1). However, the constraint (2.5a) reads:

$$(\dot{X}^0)^2 \sim R^2(X'^i)^2 \tag{3.3}$$

so that

$$(D-1)p = -T_i^i = R^2T^{ii} = -T_0^0 = -\rho \tag{3.4}$$

Inserting the more precise solution of Table 1, one finds:

$$\rho + (D-1)p = 0(\max(\tau^2, \tau^{4\alpha})) > 0 \tag{3.5}$$

In other words, p approaches the limiting value $-\rho/(D-1)$ from above.

In D space-time dimensions, the Einstein equations read:

$$(D-1)R^{-1}\frac{d^2R}{dt^2} = -\frac{8\pi G}{(D-2)}\left(\rho(D-3)+(D-1)p\right) \tag{3.6}$$

Inserting (3.5) we obtain

$$(D-1)\,(D-2)R^{-1}\frac{d^2R}{dt^2} = -8\pi G\rho(D-4) \tag{3.7}$$

which clearly yields $d^2R/dt^2 \leq 0$ for $D \geq 4$.

The (unfortunate) conclusion is that unstable strings are not able to self-sustain inflation in the perfect-fluid approximation!

Before giving up on the interesting idea of self-sustained inflation, we should mention two possible ways out. The first one [2] adds to the perfect fluid equation a state, a viscosity term, supposedly representing quantum-string-creation in a background having a finite Hawking temperature. As discussed by Barrow [7] everything becomes now possible, but there is a lot of arbitrariness.

A second, more appealing possibility has been investigated recently [8]. Here one gives up complete isotropy of the FRW metric and looks instead for a self-consistent scenario in which three dimensions undergo inflation while n others contract.

It was shown in Ref. [8] that, for $n \geq 10$, solutions of both string and Einstein equations exist and that they correspond to superinflation of three spacial dimensions. Since we know that string theory likes to live in many (10, 26) dimensions this scenario looks appealing. The initial conditions needed in order to achieve a sufficient amount of inflation turn out to be rather stringent but not, perhaps, completely unreasonable. Certainly the idea of a string-driven "Compactflation" should be pursued.

Incidentally, in this asymptotic scenario, the scale factor, for the internal dimensions shrinks to zero at late times so that our $R \to \infty$ solutions have to be (and have been) generalized [8] to the $R \to 0$ case.

In so doing, one uncovers an amusing "duality-relation" between soltuions for a FRW scale factor R and the dual scale factor R^{-1}. This scale factor duality (SFD) appears as an interesting extension of the usual $R \to \lambda_S^2/R$ duality where R stands now for a compactification radius and

$$\lambda_S \equiv \sqrt{2\alpha'\hbar} \tag{3.8}$$

in the fundamental quantum length parameter of string theory. It can be shown, however [9], that, in order to preserve SFD at the level of Einstein's equations, these have to be modified by additional dilaton-dependent terms in the way prescribed by the string's effective action.

To conclude, the game of strings and inflation is still wide-open: there are certainly many indications that a lot of challenging questions are lying ahead. But, mostly, I have found the topic of strings in inflation and of inflation with strings particularly amusing and thus suitable for wishing Henri and Raoul of many more years of productive and amusing research.

TABLE 1

Explicit results for the first few coefficients of the expansion (2.19)

A_0, B_0 arbitrary subject to $A_0' A_0'' = A_0' B_0 = 0$

$$\eta_0 = \mathrm{L} \equiv \sqrt{\mathrm{A}_0' \mathrm{A}_0'}$$

$$\eta_1 = \frac{\alpha}{3\mathrm{L}} \frac{(\mathrm{A}_0'' \mathrm{A}_0'')}{(1-2\alpha)^2}$$

$$\lambda_0 = \frac{2\alpha}{\mathrm{L}} \frac{(\mathrm{A}_0' \mathrm{B}_0')}{(\alpha+1)(2\alpha-1)}$$

$$\lambda_1 = \frac{1}{2L(\alpha+2)} \left[(A_0' B_1' + A_1' B_0') + 2(2\alpha+3)(A_1 B_1) + 4(2\alpha+1)(A_2 B_0) - 6(\alpha+1)\eta_1 \lambda_0 \right]$$

$$\zeta_0 = \frac{(2\alpha+1)^2}{2(4\alpha+1)} \frac{(B_0 B_0)}{L}$$

$$\mathrm{A}_1 = \frac{\mathrm{A}_0''}{2(1-2\alpha)}$$

$$\mathrm{A}_2 = \frac{1}{(3-2\alpha)(1-2\alpha)} \left[\frac{\mathrm{A}_0''}{8} - \frac{\alpha^2}{3(1-2\alpha)\mathrm{L}^2} \left[(\mathrm{A}_0'' \mathrm{A}_0''') \mathrm{A}_0' - \frac{(\mathrm{A}_0'' \mathrm{A}_0'') \mathrm{A}_0''}{(1-2\alpha)} \right] \right]$$

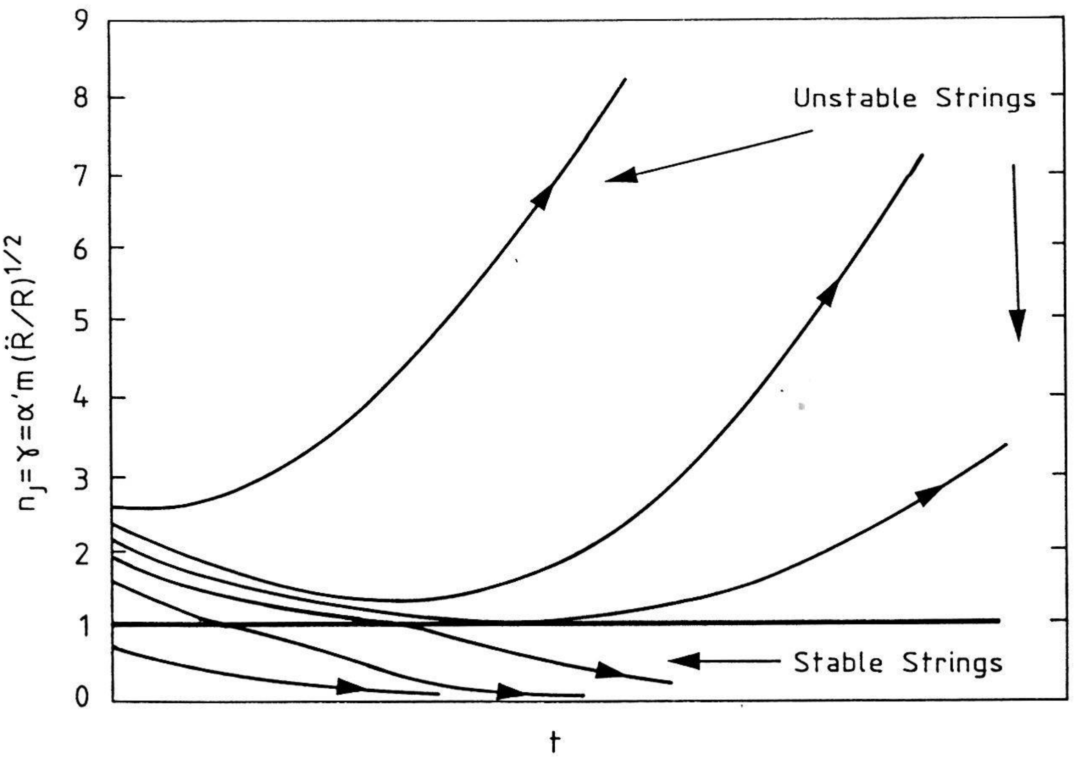
$$\mathrm{B}_1 = \frac{1}{(2\alpha+3)\mathrm{L}^2} \left[\frac{2\alpha^2(2\alpha+1)}{(2\alpha-1)^2} \left(\frac{1}{3}(A_0'' A_0'') B_0 - \frac{(A_0' B_0') A_0''}{(\alpha+1)} \right) - \frac{2\alpha^2}{(\alpha+1)(2\alpha-1)} (\mathrm{A}_0' \mathrm{B}_0')' \mathrm{A}_0' + \frac{1}{2} \mathrm{L}^2 \mathrm{B}_0'' \right]$$

$$\mathrm{C}_0 = \frac{\alpha}{\mathrm{L}^2} \left[\frac{2\alpha}{(\alpha+1)(2\alpha-1)} (\mathrm{A}_0' \mathrm{B}_0') \mathrm{B}_0 + \frac{2\alpha}{(4\alpha+1)(1-2\alpha)} (\mathrm{B}_0 \mathrm{B}_0) \mathrm{A}_0'' - \frac{1}{(4\alpha+1)} (\mathrm{B}_0 \mathrm{B}_0') \mathrm{A}_0' \right]$$

TABLE 2

A classification of spatially flat FRW backgrounds, according to their asymptotic compatibility with stable and unstable string configurations.

	$R(\eta)$	$R(t)$	Allowed Regimes
flat	const	const	stable
standard	$\eta^{\alpha}, \quad \alpha > 0$	$t^{\beta}, \quad 0 < \beta < 1$	stable
linear	$\exp(K\eta)$	$Kt, \quad KL < 1$	stable
power-law	$\eta^{-\alpha}, \quad \alpha > 1$	$t^{\beta}, \quad \beta > 1$	stable and unstable
de Sitter	$-(H\eta)^{-1}$	$e^{Ht}, \quad \alpha\prime MH < 1$	stable and unstable
de Sitter	$-(H\eta)^{-1}$	$e^{Ht}, \quad \alpha\prime MH > 1$	unstable
super	$\eta^{-\alpha}, \quad \alpha < 1$	$t^{-\beta}, \quad \beta > 0$	unstable



"Bifurcation" in the evolution of strings in an inflationary epoch (with $\dot{H} < 0$). "Unstable" strings manage to stay above $\gamma = 1$ all the time and blow up. "Stable" strings fall below $\gamma = 1$ and then stay small forever.

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