Helv Phys Acta
Vol. 65 (1992)
0018-0238/92/030411-49$1.50+0.20/0

Nonperturbatively Renormalizable Quantum Field Theories in 2+1 Dimensions

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Abstact. Recently several 2+1 dimensional models were found which are non-renormalizable in weak coupling expansion but renormalizable in $1/N$. We show that the IR fixed points of Yukawa theories and scalar QED_3 (Landau-Ginzburg) are some of these theories. The critical 4-Fermi interaction model is the IR stable fixed point of Yukawa theories while the CP^{N-1} model is the IR fixed point of N-component scalar QED_3.

As is well known, there is a close relation between renormalizable relativistic quantum field theories (QFT) in 2+1 dimensions and second order phase transitions in 3D. For (rotationaly invariant) statistical systems universal critical properties are determined by an infrared (IR) fixed point of the corresponding QFT. For example, critical exponents of Heisenberg (anti)ferromagnets are described by the O(3) symmetric ϕ_3^4 [1] theory

$$S_E[\phi] = \frac{1}{2}(\partial\phi_i)^2 - \frac{\mu^2}{2}\phi_i^2 + \frac{\lambda}{4!}\phi_i^4 . \tag{1}$$

The IR fixed point of ϕ^4 is the conformally invariant critical σ-model. Note that although the nonlinear σ-model is non renormalizable in weak coupling perturbation theory it is renormalizable nonperturbatively.

Recently several new classes of nonperturbatively renormalizable QFT's in 2+1 dimensions were found using the $1/N$ expansion. These include four-Fermi interaction models like the Z_2 chiral invariant Gross-Neveu (GN) model [2]

$$\mathcal{L} = \bar{\psi}_i \partial\!\!\!/ \psi_i + \frac{g^2}{2N}(\bar{\psi}_i\psi_i)^2 . \tag{2}$$

We show that the critical GN model is the IR fixed point of the superrenormalizable Yukawa theories

$$\mathcal{L} = \frac{1}{2}(\partial\sigma)^2 + \bar{\psi}_i \partial\!\!\!/ \psi_i + \lambda_0\sigma\bar{\psi}_i\psi_i + \frac{m_0^2}{2}\sigma^2 \tag{3}$$

and therefore determines its universality class. At the phase transition point the chiral Z_2 breaks down spontaneously. The renormalized coupling of the Yukawa model is determined by (we use notations of [1])

$$\sqrt{m}\lambda = \Gamma_{1,2}(p_1 = 0, p_2 = 0, p_3 = 0) \tag{4}$$

where m is the boson's mass, $\Gamma_b^2(m) = 0$. To fix the field's normalizations we choose to impose $\frac{\partial\Gamma_b^2}{\partial p^2}\Big|_{p^2=m^2} = 1$, $\frac{\partial\Gamma_f^2}{\partial \not p}\Big|_{p^2=0} = 1$. To leading order in $1/N$ the corresponding Callan-Symanzick β function is

$$\beta(\lambda) = -\frac{1}{2}\lambda + \frac{1}{16}\lambda^3 \tag{5}$$

It has a finite IR fixed point at $\lambda^* = 2\sqrt{2}$. The connection between the bare and renormalized couplings is determined by the RG equation

$$\left(1 + \beta(\lambda)\frac{\partial}{\partial\lambda}\right)\frac{\lambda_0^2}{m} = 0 \tag{6}$$

The solution is

$$\lambda_0(\lambda) = \sqrt{m}\lambda \exp\left\{-\frac{1}{2}\int_0^\lambda \left[\frac{1}{\beta(x)} + \frac{1}{x}\right] dx\right\} \tag{7}$$

As $\lambda \to \lambda^*$, the bare coupling λ_0 diverges $\lambda_0 \to \infty$. In this limit the Yukawa action becomes the GN action eq.(2).

This is seen explicitly by calculating the dynanical dimension of the kinetic term for σ: $[(\partial\sigma)^2] = 4$ [3]. It becomes irrelevant for $d = 3$. On the other hand some irrelevant operators at weak coupling become relevant or marginal near the fixed point. The dimension four $(\bar{\psi}\psi)^2$ operator aquires scaling dimension two and thus becomes relevant , while the dimension six operator $(\bar{\psi}\psi)^3$ has in the leading order in $1/N$ scaling dimension three and thus becomes marginal. In this case more careful analysis is needed. The next to leading order in $1/N$ shows that in fact it becomes relevant. Similar phenomenon cccurs in abelian gauge theories . In scalar QED the IR fixed point is the nonperturbatively renormalizable CP^{N-1} model [4]. It is interesting to note that the vortex operator V that creates Nielsen-Olesen vortex [5] becomes relevant at $N < 8$.

To summarize we found that recently discovered nonperturbatively renormalizable 2+1 dimensional QFT contain IR fixed points describing new universsality class. It is interesting to know wether these universality classes are realized in real materials.

The work of A.K and B.R was supported by NSERC of Canada.

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