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Bohr's Complementarity Principle—Its Relation to Quantum Mechanics

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Abstract. N. Bohr asserted that quantum mechanical formalism offers an adequate tool for a complementary way of describing nature. Unfortunately he did not precisely formulate the principle of complementarity and demonstrate its relation to quantum mechanics. This led to many controversies and misunderstanding particularly with respect to the complementarity between particle and wave properties. In this paper a precise formulation of the principle is intended in the spirit of Bohr, and its relation to quantum mechanical formalism is clarified. Some recent experiments (both suggested and actually performed) are discussed in the light of the present formulation and it is concluded that no experiment has succeeded to disprove Bohr's Complementarity Principle.

1 Introduction

In recent years renewed interests have been generated about Bohr's complementarity principle (BCP) as a number of authors have made critical comments about its validity in view of some new experiments—the main issue of the debate being the wave-particle duality. A plethora of varied experiments are suggested and some are actually performed

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which confront BCP or more precisely the conceptual content of the wave-particle duality. All these experiments may be classified broadly into three classes. The first group of experiments [1, 2] are claimed to provide experimental evidence in favour of mutual exclusiveness (in the sense that a single experimental set up can not give a precise information about both the complementary aspects) in wave-particle duality. In a rather novel thought experiment suggested by Scully et al. [2], it is claimed that "welcher weg information" stored in a microscopic system (potential knowledge!) can destroy interference effect but if the stored information is erased out, the interference can be restored. Moreover, in this experiment the incompatibility between "welcher weg"[1] (which path) knowledge and appearance of interference cannot be traced to any uncertainty relation. On the other hand certain "welcher weg" experiments [3–5] which made joint unsharp measurements of complementary observables have been used to argue that they yield "partial particle" and "particle wave" information and is therefore in conflict with the Bohrian notion of mutual exclusiveness (ME).

Recently Ghose et al. [6] and Vigier et al. [7] have each proposed experiments to demonstrate that quantum mechanical entities may display their particle and wave aspects at the same time, thereby contradicting the principle of complementarity. On going through the literature on BCP one feels that much of the controversy arise because the formulation of BCP is not precise. There are no guidelines to decide whether a suggested pair of properties should be considered complementary. No precise quantum mechanical principle is adduced from which the incompatibility can be derived and hence BCP seems to dangle aloof near the periphery of the quantum mechanical formalism. Bohr's assertion in respect of incompatibility between "the claim of causality" and "the space time description of a phenomenon" throws only some diffuse light on the reasons for incompatibility. Without a precise formulation of BCP it seems futile to critically examine the thought experiments or actual experiments which claim to violate BCP.

Our aim in this paper is to examine if, in the spirit of the statements made by Bohr, we can make the statement of BCP particularly about the wave-particle complementarity somewhat more precise. Also we would like to clarify the relation between BCP and the general formalism of quantum mechanics (QM). We shall then try to examine the significance of some of the recent experiments referred to before, in the light of this precise statement.

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[1] In quantum mechanics the concept of path is used in a very restricted sense, unlike the classical particle like passage. The path prior to the detection of any physical entity can be retrodicted but it is a matter of personal belief whether such an extrapolation of the past history can be ascribed any physical reality or not. We insist the use of "which state" instead of "welcher weg" as a preferable terminology to avoid confusion.

2 Bohr's statement of complementarity principle

The impossibility of integrating the quantum of action into the body of deterministic laws of classical physics corresponds to a phase of quantum theory ridden with contradictions (the old quantum theory period) which ultimately culminated in the formulation of quantum mechanics in 1925–26. The uncertainty principle was formulated in March 1927. A few months later in September 1927 Bohr enumerated the complementarity principle in his Como lecture. Heisenberg used Dirac-Jordon transformation theory to get an estimate of dispersion in x and p_x and then discussed the thought experiment for simultaneous measurement of x and p_x. Thus it is clear that his principle is a consequence of the formalism of QM. As the title of his paper (On the intuitive contents of the quantum theoretic kinematics and mechanics) suggests, the purpose of the principle was to bring out explicitly some important features of QM not quite obvious from the formalism itself. It is clear that the uncertainty principle adds nothing new to the formalism. Initially there were some suggestions of thought experiments which seemed to violate the uncertainty principle. But it was quite clear that such attempts if successful would mean violation of QM itself.

In contrast, Bohr did not clarify the relation of the complementarity principle with the formalism of QM. He only asserted that the formalism of QM supplies an adequate tool for a complementary way of describing nature. In particular the question whether the principle of complementarity is inbuilt within the formal structure of QM is left unanswered. This vagueness has been the cause of much confusion particularly in respect of wave-particle complementarity and led to diverse reactions among physicists.

Let us now see what Bohr actually said about BCP. Unfortunately in the Como Lecture [8] where he first propounded the principle, his statements created only some vague impressions about the ideas he was driving at. Much later in another article [9] he almost repeated the same statements without improving their precision. We quote below some of his significant remarks on BCP.

(1) "However far the phenomena transcend the scope of classical physical explanation, the account of all evidence must be expressed in classical terms" [10].

(2) "The very nature of quantum theory thus forces us to regard the space-time coordination and the claim of causality, the union of which characterizes the classical theories, as complementary but exclusive features of the description" [8].

(3) "... we are presented with a choice of either tracing the path of a particle or observing the interference effect which allows us to escape from the paradoxical necessity of concluding that the behaviour of an electron or a photon should depend on the presence of a slit in the diaphragm which it could be proved not to pass" [11].

(4) "An adequate tool for a complementary way of description is offered by the quantum mechanical formalism .... It must be remembered that even in the indeterminacy relation we are dealing with an implication of the formalism which defies unambiguous expression in words suited to describe classical physical pictures. Thus a sentence like "we cannot know both the position and the momentum of an atomic object" raises at once

questions as to the physical reality of two such attributes of the object which can be answered only by referring to the conditions for the unambiguous use of space time concepts, on the one hand and the dynamic conservation laws on the other hand. While the combination of these concepts into a single picture of a causal chain of events is the essence of classical mechanics, room for regularities beyond the grasp of such a description is just afforded by the circumstances that the study of the complementary phenomena demands mutually exclusive experimental arrangements," [12].

In addition to these statements we have the specific examples of incompatible complementary pairs of properties which Bohr discussed. One example is that of a particle described by a wave group. "Here the complementary character of the description appears, since the use of wave groups is necessarily accompanied by a lack of sharpness in the definition of period and wavelength, and hence also in the definition of the corresponding energy and momentum" [8]. Thus according to Bohr $\mathbf{r}$, t are complementary to $\mathbf{p}$ and $\mathscr{E}$.

Again discussing complementarity in relation to wave particle duality in Compton scattering, Bohr writes "Evidence obtained under different experimental conditions cannot be comprehended within a single picture, but must be regarded as complementary in the sense that only the totality of the phenomena exhausts the possible information about the objects. An illustrative example, of how the apparent paradoxes are removed by an examination of the experimental conditions under which the complementary phenomena appear is also given by the Compton effect.... Thus any arrangement suited to study the exchange of energy and momentum between the electron and photon must involve a latitude in the space time description of the interaction sufficient for the definition of wave number and frequency.... Conversely, any attempt of locating the collision between the photon and the electron more accurately would on account of the unavoidable interaction with the fixed scales and the clocks defining the space-time frame, exclude all closer account as regards the balance of momentum and energy" [13].

Besides these examples, Bohr [9] discussed in detail the wave-particle complementarity in relation to the well known double slit experiment.

3 Physicists reactions to BCP

Even at the Como lecture, where a galaxy of contemporary eminent physicists were present, the response to BCP was hardly enthusiastic. Wigner [14] commented that Bohr's lecture "will not induce any one of us to change his opinion about QM. In other words according to Wigner BCP adds nothing to the formalism of QM. von Neumann's reaction [14] was "well, there are many things which do not commute". Here Neumann's attention was focussed on those variables where complementarity was due to quantum mechanical principle of non-commutation. There was hardly anything basically new and surprising about it. Pauli [14] also interpreted BCP to signify complementarity between two non-commuting variables such as x and p_x. C. F. von Weizsäcker on the other hand wrote [14], "the complementarity between space-time description and the claim of causality is therefore precisely the complementarity between the description of nature in classical mechanics and in terms of the ψ-function". The vagueness of Bohr's formulation of BCP inevitably leads to many interpretations of the principle. This is true, as we see, even for those who accept BCP as correct and consider it as a key to the best possible understanding of quantum theory.

Physicists like Born, Rosenfeld, Fock and Bohm hailed BCP as a new principle of fundamental importance, but there are many who were not favourably disposed to BCP. One main reason is the vagueness of the formulation and the consequent difficulty in arriving at a clear meaning of the principle. For example in 1949 Einstein [15] complained that "despite much effort which I have expended on it, I have been unable to achieve the *sharp formulation* of Bohr's principle of complementarity" (italics ours). The same uneasiness rings as Heisenberg [16] writes:

"The concept of complementarity introduced by Bohr has encouraged the physicist to use an ambiguous rather than an unambiguous language, to use the classical concepts in a somewhat vague manner in conformity with the principle of uncertainty, to apply alternately different classical concepts which would lead to contradictions if used simultaneously. When this vague and unsystematic use of language leads into difficulties, the physicist has to withdraw into the mathematical scheme and its unambiguous correlation with experimental facts".

Note that Heisenberg is very clear in his assertion that the formalism of QM is really the ultimate arbiter in every problem concerning quantum theory. Precisely because of this (that BCP adds nothing to the formalism of QM), some physicists were led to outright rejection of BCP as a useful principle. Thus J.-M. L. Leblond [17] branded it as a "parasitical philosophical notion" and a "totally irrelevant idea for physics as soon as one accepts the specifically quantum, i.e. qualitatively nonclassical, nature of quantum theory" while A. Lande [18] called it "an ingenious attempt at talking us out of a difficult problem of theoretical physics rather than solving it by means and methods of theoretical physics itself". Most authors of text books on quantum mechanics either ignore BCP or make a cursory mention of it. J. S. Bell [19] once observed "One is tempted to suspect that the authors do not understand the Bohr philosophy sufficiently to find it helpful".

There are others who maintain that the principle as stated by Bohr is wrong. When BCP is extended to include the statement, wave properties are complementary to particle properties, it becomes necessary to know in a given experimental set up which particle properties are to be regarded as complementary to which wave properties. Classical waves and particles have many common properties. As Bacry [20] puts it " . . . there are situations where the two aspects compete to interpret a phenomenon". Rectilinear propagation with a finite velocity, reflection from a perfect reflector, are examples. About refraction of light there is ambiguity. There are both wave and corpuscular explanations, but lower velocity in the denser medium is not explained by the Newtonian version of the corpuscular theory. Does this allow us to interpret refraction with lower velocity in the denser medium as specifically a wave property? This and many similar questions may be asked to which no clear answer can be extracted from Bohr's statement of BCP. Some physicists are, in effect, asserting that any property for which a standard corpuscular theory does not exist is a wave property and vice versa. It is essentially this interpretation which has been summarised by Ghose et al. in several papers [6, 21]. According to this view point, simultaneous reflection and refraction is a specific wave property and reflection or refraction in a stochastic manner is a specific particle property. This is why Aspect et al. [1] consider anticoincidence as a signature of particle property.[2] Similarly, Ghose et al. consider existence of evanescent waves leading to transmission of light at the critical angle as a specific wave property.[3]

If we accept the interpretations of Aspect and Ghose et al. of wave-particle complementarity in BCP, then Mizobuchi and Ohtake's experiment [22] definitely contradicts ME. Bacry [20] on the other hand argues, Bohr's argument that in every single experiment either the wave aspect or the corpuscular aspect (say of a photon) will be present, can not obviously be true: "Given two experiments say A_0, an undulatory one and A_1, a corpuscular one, it could exist a continuous set of experiments $A(x)$ such that $A(0) = A_0$ and $A(1) = A_1$. Then where is the frontier in this set which separate the wave experiments from the corpuscular ones?"

From this brief review the following different attitudes to BCP emerge:

(1) That BCP is a correct principle and is important in understanding atomic phenomena. But the interpretation of BCP by different physicists do not always agree.

(2) That the statement of BCP is vague and it is difficult to draw unambiguous conclusions from it.

(3) That BCP adds nothing to the formalism of QM and hence useless.

(4) That BCP is on the whole a clearly stated principle. But the exclusiveness of wave-particle complementarity as claimed in BCP is wrong.

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[2] The anticoincidence observed in two detectors in a single-photon field has a totally different explanation in Bohm's quantum field theory [23] devoid of photon-particle picture, a photon being just an excitation of a mode. We have an example here for Bacry [20] where the two aspect seem to compete for an interpretation in the photon picture.

[3] Chiao et al. [24], on the other hand, described the phenomenon of "frustrated total internal reflection" of the purely non-classical single-photon state exploiting an analogy with particle (quantum) tunnelling through classically forbidden region. They have also proposed experiment to measure the photon tunneling time using the "particle aspect" of photon tunnelling. According to Chiao et al., there exists a classical limit associated with coherent states of the electromagnetic field in which the tunnelling phenomenon can be understood entirely as a classical wave phenomenon.

4 Precise statement of BCP

We have quoted at length, Bohr's comments on BCP. From this we hope that one will have some idea about the difficulties of extracting a precise statement. One has a feeling that Bohr was grappling with a very broad and general principle of which the uncertainty relation was a specific example. We have also given examples of reactions to BCP by different physicists and classified them into four different categories to bring into relief the main difficulties created by the conventional statement of BCP. Our aim in this section is to remove some of these difficulties and to evolve a precise statement of the complementarity principle.

It is a fact that in interpreting the experimental results in the atomic domain, physicists use only the standard formalism of QM. Nowhere it becomes necessary to invoke in addition the complementarity principle. This is a clear demonstration that the formalism of QM is complete and BCP adds nothing to it. But it is incorrect to conclude on this ground that BCP is useless. The uncertainty principle also adds nothing to the formalism. Still it is often found useful to comprehend complex situations. As Heisenberg claimed, it brings out explicitly some intuitive contents which remain normally obscured in the formalism. *The usefulness of BCP is to be interpreted in a similar way.*

Though not stated explicitly, two distinct types of complementarity are implied in BCP. One is the complementarity between a pair of variables such as x and p_x. But the wave-particle complementarity is between a pair of properties for which we do not have any specific variables. The origin of ME in the two cases are quite distinct and hence it is necessary to specify the two types of complementarity separately. We now propose the following precise statements of BCP.

4.1 *BCP—First part*

In classical physics "particle" and "wave" are the sole ultimate categories of physical entities. "Particles" are described by Newton's equation of motion and a "wave" described by a space-time field function which satisfies a differential equation—the so-called wave equation, and there is no mix-up. But the classical "particle" is described in QM by a field function (ψ) satisfying Schrödinger's equation which has a wave-equation like flavour. Similarly, classical "waves" are described by quantum field equations in QM which may be recast in a Schrödinger equation [25] for the field quanta wavefunction and we are in for the typical "wave-particle duality" in the world of quantum mechanics.

But the fact remains that as an entity a photon is very different from an electron.[4] There are limiting situations, such as classical limit, this difference becomes most sharp, the

[4] For example, even in a quantum mechanical one photon state we cannot precisely define the position coordinate of a photon—the photon wavefunction is non-local in coordinate space [25]. On the otherhand, we can always measure precisely the position of an electron.

former goes over to a classical field and the latter to a classical particle. With this introductory remark we now give the statement for the first part of BCP, which is essentially a statement about the existence of complementarity in quantum mechanics. In the formalism of QM, this intuitive content remains implicit. In BCP, it is made explicit and focussed as the essential difference between the quantum mechanical and classical description of nature.

In the quantum mechanical description of a physical entity which in the classical limit is described either as a particle or a wave, we find that pair of properties exist which are mutually exclusive in the sense that a *sharp simultaneous knowledge of both is impossible.* These would be called complementary properties. The term "properties" is used here in a general sense. Specifically, they may be a pair of variables or some properties not expressible in terms of variables.

It is important to note that in the above statement we have made ME a *defining property* of complementary pairs. Ghose et al. [21] have acknowledged ME as a "necessary" element of BCP and on going through the writings of Bohr on BCP it is evidently clear that ME is intended to be the essence of complementarity. We conclude that *if for a system mutually exclusive "properties" do not exist, the system is a purely classical one.* In every complementary pair he has discussed Bohr stressed the ME aspect. But a gap was created when he said that wave properties are complementary to particle properties and in this respect Bohr's specific statement is significant. About the photon concept he writes [26] "... any simple corpuscular picture of radiation would obviously be irreconcilable with interference effects, which present so essential an aspect of radiative phenomena and which can be described *only in terms of a wave picture.* The acuteness of the dilemma is stressed by the fact that *the interference effects offer our only means of defining the concepts of frequency and wavelength* entering into the very expressions of energy and momentum of photon" (italics ours). Note that Bohr very clearly and unambiguously pointed out that interference effects are our only means of defining the essential wave property. One can measure wavelength from refractive index by using a suitable dispersion formula. But they are not defining property of a wave. We want to emphasize that it could not have been Bohr's intention to assert that *any arbitrary wave property is complementary to any so-called particle property.* Bohr remarked "... any given application of classical concepts precludes the simultaneous use of other classical concepts which in a different connection are equally necessary for the elucidation of the phenomena" [27].

The above statement of BCP keeps one important question vague. When an experiment is performed to measure some properties and use classical concepts for the elucidation of the phenomenon, how do we ascertain which concepts are to be used and which concepts are to be precluded? For example, when we measure the precise position of an electron, we know that the concept of momentum has to be excluded. This knowledge comes from the principles of QM. Thus it is quite clear that to interpret BCP we have to use QM to find which group of classical concepts can be used for the interpretation of the results.

In the so-called wave-particle complementarity we are in difficulty. Quantum mechanical principles do not specify which wave property is incompatible with which particle property. In fact the choice of this particular term wave-particle duality/complementarity is at the root of much confusion and prompted a flat refusal from K. R. Popper [28] as he declares: "To avoid any misunderstanding, I wish to make clear that I do not believe in a dualism of particles and waves or anything even faintly resembling it". However, Bohr has clearly stated that interference phenomenon is the essential wave property and superposition of two or more states is the essential requirement for interference. If we are doing an interference experiment with double slit say, it is the which state information that is complementary to the appearance of the interference pattern. From this point of view we can complete the statement of the first part of BCP by the following statement, which we call the second part of BCP and which correlates ME to quantum mechanical principles.

4.2 *BCP—Second part*

Based on the origin of mutual exclusiveness (ME) we can distinguish between two classes of complementary properties. These are:

Class I:–Pair of variables obtained from a Fourier transform of state vector and a pair of variables which satisfy a non-commutation relation. These include pair of canonically conjugate variables.

Class II:–A property such as interference[5] which depends on the superposition of a number of states and the property associated with the "which state" (rather than "welcher weg") information form a pair of complementary properties.

In class I we have the complementary pairs $\mathbf{r}, \mathbf{p}$ and $t, \mathscr{E}$ derived from $\Psi(\mathbf{r}, t)$ and its Fourier transform $\Phi(\mathbf{p}, \mathscr{E})$. Also all pairs of non-commuting variables such as components of angular momentum J_x, J_y; J_x, J_z etc. belong to this class. Similarly the radiation field variables $\mathbf{E}$ and $\mathbf{H}$ (in quantum electrodynamics) which satisfy field commutation relation may serve as the complementary pair in the case of light beam. Existence of more esoteric complementary pair of variables ("spin, on the one hand, and the measurement of the magnetic structure, i.e. the observation of the nth partial angular momentum $(1/2)g_n^2$, on the other hand" [29]) has also been reported from experiments on elementary particles.

In class II type of complementary pairs, ME arises from collapse of wave function. In a recent paper Uffink and Hilgevoord [30] have made a quantitative analysis of this type of complementarity. They introduced a measure for the indistinguishability U, $(0 \leqslant U \leqslant 1)$, of two quantum states in a given measurement and the amount of interference I observable in the same measurement and established an inequality $U \geqslant I$, which they regarded as a "quantitative expression of Bohr's claim that one cannot distinguish between two possible paths of a particle while maintaining an interference phenomenon".

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[5] Computations with non-local potential (Bohm's quantum potential) yield particle trajectories which group together to produce a set of alternating bright and dark fringes (interference pattern) [31]. This result is, however, irrelevant in the context of class II types of complementarity between properties depending on superposed and unsuperposed states.

In a similar way, but in still simpler terms we can define I and a distinguishability parameter D (see Appendix) such that:

$$D + I = 1.$$

Thus when $D = 1$, $I = 0$ and vice versa and D and I are complementary aspects. In the case of scattering of two identical particles (i.e. indistinguishable and $D = 0$), interference of amplitude in the differential scattering cross-section is observed. But for two distinguishable particles no interference effect is observed in the differential scattering cross-section.

However, in all wave-particle complementarity measurement, time is an important parameter. We can introduce time in the above analysis and it may happen that at $t = 0$, $D = 1$ and $I = 0$ and at a subsequent instant in the interference zone $D = 0$ and $I = 1$. A double slit experiment closely resembles this situation. The essence of wave-particle duality problem is whether we can make a which state measurement at $t = 0$ (or in the non-interfering region in the vicinity of the slits) and still get interference at a subsequent instant of time t? This is ruled out in quantum mechanics because of the collapse hypothesis. We also assert that class II gives the precise expression for the so-called wave-particle complementarity.

It is clear that with this statement of BCP, ME follows as a consequence of some fundamental quantum mechanical principles. As such no experiment can violate ME, without violating quantum mechanics itself. The central motivation in this formulation has been to incorporate BCP within the framework of QM. Some physicists seem to accept this principle as independent of QM and its relation to QM remains ill defined. Any attempt to show that ME in BCP can be violated without violating QM, presumes the view that BCP is independent of QM. We do not rule out the possibility of such an interpretation of BCP. But here we show that there is an alternative interpretation which makes BCP consistent with QM.

5 Concluding remarks

The unsharp measurements [3–5] which claim to observe both interference and the path of the particle, do not really fall within the compass of BCP. Bohr himself in the Como Lecture discussed the wave packet description of a particle (where approximate knowledge of both momentum and position exists) as an illustration of the complementarity principle implying that unsharp knowledge of conjugate variables does nto violate BCP. Both class I and class II types of BCP imply sharp measurement.

The proposed neutron interferometry experiment by Vigier et al. (for a detailed account of the experiment see Refs. [7] & [23]) where one observes interference in the intensity of the detected neutrons, cannot give the so-called which-path information. As pointed out by Scully and Walther [32] and Unnerstall [33], r.-f. coils generate coherent states of photons and the passage of neutron through the coil cannot leave "which-path" information in the coil since the coherent photon distribution remains essentially unchanged by the addition of a single photon associated with spin flip. In the experiment proposed by Ghose et al. [6] no superposed state is involved and hence no interference effect is there. It belongs neither to class I nor to class II types of BCP. We conclude that this experiment is not suited to test BCP.

The analysis of Scully et al. [2] clearly shows that quantum mechanical formalism guarantees the validity of ME in experiments involving superposed states (class II types of complementarity). Whenever one has which state information the interference pattern gets washed out.

In our formulation of BCP we have correlated the mutual exclusiveness of complementary properties to some aspects of quantum mechanical formalism. As such it is obvious that any experiment which claims to prove BCP false will imply some limitations of the quantum mechanical formalism itself. This we believe is in true spirit of Bohr's original ideas.

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Appendix

Consider two orthogonal functions,

$$\psi_1 = \sum_i \alpha_i |i\rangle \quad \text{and} \quad \psi_2 = \sum_i \beta_i |i\rangle$$

where $|i\rangle$ are eigen vectors of some observable Ω with eigen values ω_i. If some eigen vectors are common in both ψ_1 and ψ_2 and others are all different, then we can rewrite:

$$\psi_1 = \sum_1^n \alpha_i |i\rangle + \sum_j \alpha_j |j\rangle \quad \text{and} \quad \psi_2 = \sum_i^n \beta_i |i\rangle + \sum_l \beta_l |l\rangle,$$

where $|j\rangle \neq |l\rangle$ for all j's and l's. It is clear that on a measurement of Ω if the outcome belongs to ω_j or ω_l we can distinguish the state. Otherwise no distinction is possible. We can, therefore, define a distinguishability parameter by

$$D = 1 - \frac{1}{2}\sum_1^n (|\alpha_i|^2 + |\beta_i|^2) = 1 - \frac{1}{2}\sum_1^n (p_i + q_i)$$

where,

$$\sum |\alpha_i|^2 + \sum |\alpha_j|^2 = \sum |\beta_i|^2 + \sum |\beta_l|^2 = 1.$$

Since $\sum_1^n p_i$ and $\sum_1^n q_i$ are both $\leqslant 1$ and $\geqslant 0$, D lies between 0 and 1. The zero value indicates complete lack of distinguishability and 1 complete distinguishability. A value 0.6 for D signifies 60% distinguishability implying that if Ω is measured N times in 60% cases we shall be able to distinguish the state and in 40% cases we fail.

Interference effect can be measured by measuring the value of Ω in the superposed state

$$\psi = (1/\sqrt{2})\,[\psi_1 + \psi_2].$$

The probability of getting a particular value ω_k is

$$\begin{aligned} P_k(\psi) &= \langle \psi | \Lambda_k | \psi \rangle; \quad \text{where } \Lambda_k \equiv |k\rangle\langle k| \\ &= \tfrac{1}{2} P_k(\psi_1) + \tfrac{1}{2} P_k(\psi_2) + I_k(\psi_1, \psi_2); \quad \text{for } k \in \{1, 2, \ldots, n\} \\ &= \tfrac{1}{2} P_k(\psi_1) + \tfrac{1}{2} P_k(\psi_2); \quad \text{for } k \,\bar{\in}\, \{1, 2, \ldots, n\} \end{aligned}$$

where,

$$P_k(\psi_1) = |\alpha_k|^2 = p_k, \qquad P_k(\psi_2) = |\beta_k|^2 = q_k$$

and,

$$I_k(\psi_1, \psi_2) = \tfrac{1}{2}(\alpha_k^* \beta_k + \alpha_k \beta_k^*).$$

Hence,

$$P_k(\psi) = \tfrac{1}{2}(p_k + q_k) + I_k(\psi_1, \psi_2); \quad \text{for } k \in \{1, 2, \ldots, n\}$$

and

$$= \tfrac{1}{2}(p_k + q_k); \quad \text{for } k \,\bar{\in}\, \{1, 2, \ldots, n\}.$$

So the total probability that the measurement will show interference is given by,

$$\begin{aligned} I(\psi) &= \sum_i^n P_k(\psi) \\ &= \frac{1}{2} \sum_i^n (p_k + q_k), \quad \text{because } \sum I_k(\psi_1, \psi_2) = 0, \end{aligned}$$

for two orthogonal ψ_1 and ψ_2.

$I(\psi)$ may be regarded as a parameter for indicating interference effect and we find,

$$D + I(\psi) = 1.$$

Thus when $D = 1$, $I(\psi) = 0$ and vice versa. So D and $I(\psi)$ are complementary aspects.

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