# N and spectra and the effect of negative energy component of Bethe-Salpeter amplitude 

Autor(en): Yu-bing, Dong<br>Objekttyp: Article<br>Zeitschrift: Helvetica Physica Acta

Band (Jahr): 68 (1995)
Heft 2

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\text { PDF erstellt am: } \quad 03.06 .2024
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Persistenter Link: https://doi.org/10.5169/seals-116733

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# N and $\Delta$ Spectra and the Effect of Negative Energy Component of Bethe-Salpeter Amplitude 

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Abstract. By mean of the instantaneous Bethe-Salpeter equation, the effect of negative energy component of Dirac spinor on the one-gluon exchange interaction is studied. Through the calculation of N and $\Delta$ baryon spectra, one sees that the effect changes the potential parameters significantly, however leaves the global structures of the spectra almost untouched.

## 1 Introduction

Although the applications of non-relativistic quark potential model to heavy quarkonium and baryon $(\mathrm{N}, \Delta, \Sigma, \Lambda, \Xi$ and $\Omega)$ spectrum, are successful ${ }^{[1]}$, it's always an interesting issue to study the differences between the non-relativistic Schrödinger equation and the relativistic covariant Bethe-Salpeter(B-S) equation when one investigates light quark systems, such as N and $\Delta$ baryons. It is necessary to know how many relativistic effects that Schrödinger equation neglects in the calculations of N and $\Delta$ systems? It is commonly believed that the B-S equation differs from Schrödinger equation in the two main aspects: Firstly, the B-S amplitude of bound states depends on four-dimensional momentum ( $\vec{P}, P_{0}$ ), other than only on $\vec{P}$ as the Schrödinger wave function acts. Secondly, the B-S amplitudes for fermions which are represented in Dirac spinor space, are unlike the Schrödinger wave functions in Pauli space. The relativistic and covariant features of the system require that the four-component Dirac spinor should be taken into account. Therefore, the effect comes from the negative en-
ergy component(NEC) of the B-S amplitude should be involved in as an effective interaction.

In this paper, the study of the effect of the NEC of the B-S amplitude in Dirac spinor space is shown. Because it is known that the relativistic B-S equation is a coupled equation, and it is very difficult to be solved exactly, here, the method of ref.[2] is employed to reduce the coupled B-S equation to a Pauli-Schrödinger(P-S) equation in Pauli space. As a result, the B-S amplitudes in Dirac spinor space are transformed to the Schrödinger wave functions in Pauli spinor space, and the effect of the NEC of the B-S amplitude is accounted for in terms of an effective interaction between the positive energy states.

In Sec. 2, we shall briefly outline and discuss the reduction of the instantaneous B-S equation and the effect of the negative energy component of the B-S amplitude. In the last section, discussions and concluding remarks about the effect on the baryon spectra calculation are drawn.

## 2 Reduction of B-S Equation and the Explicit Effect of NEC of B-S Amplitude

For a two quarks system, if the two quarks have equal quark masses $m$, the basic relativistic B-S equation in momentum space is ${ }^{[3]}$

$$
\begin{equation*}
\chi_{P \xi}(q)=S_{f}\left(\frac{1}{2} P+q\right) S_{f}\left(\frac{1}{2} P-q\right) \int \frac{d^{4} k}{(2 \pi)^{4}} K(P, q, k) \chi_{P \xi}(k), \tag{2.1}
\end{equation*}
$$

where, $\chi_{P \xi}(q)$ is the B-S amplitude, $S_{f}$ is the fermion propagator. P and q are the total and relative momenta of the two quark system. In the ladder approximation, the lowest irreducible B-S kernel is one-gluon exchange

$$
\begin{equation*}
K(P, q, k)=\frac{\alpha_{s}}{4 \pi}\left(\frac{\lambda_{1}^{c}}{2}\right)\left(\frac{\lambda_{2}^{c}}{2}\right) \gamma_{1 \mu} \gamma_{2}^{\mu} \Delta_{f}^{s}(q-k) \tag{2.2}
\end{equation*}
$$

In the instantaneous approximation, eq.(1) becomes ${ }^{[3]}$

$$
\begin{equation*}
\chi_{P \xi}(\vec{q})=S(P, \vec{q}) \int \frac{d^{3} k}{(2 \pi)^{3}} K(P, \vec{q}, \vec{k}) \chi_{P \xi}(\vec{k}) \tag{2.3}
\end{equation*}
$$

where

$$
\begin{equation*}
\chi_{P \xi}(\vec{q})=\int \frac{d q_{0}}{2 \pi} \chi_{P \xi}(q) \tag{2.4}
\end{equation*}
$$

is the instantaneous B-S amplitude, and

$$
\begin{equation*}
S(P, \vec{q})=i\left[\frac{\Lambda^{+}\left(\frac{\vec{P}}{2}+\vec{q}\right) \Lambda^{+}\left(\frac{\vec{P}}{2}-\vec{q}\right)}{P_{0}-\omega\left(\frac{\vec{P}}{2}+\vec{q}\right)-\omega\left(\frac{\vec{P}}{2}-\vec{q}\right)}+\frac{\Lambda^{-}\left(\frac{\vec{P}}{2}+\vec{q}\right) \Lambda^{-}\left(\frac{\vec{P}}{2}-\vec{q}\right)}{P_{0}+\omega\left(\frac{\vec{P}}{2}+\vec{q}\right)+\omega\left(\frac{\vec{P}}{2}-\vec{q}\right)}\right] \gamma_{0}^{1} \gamma_{0}^{2} \tag{2.5}
\end{equation*}
$$

In the above equation, $\omega(\vec{p})$ is the free-particle energy, $\Lambda^{+}(\vec{p})$ and $\Lambda^{-}(\vec{p})$ are positive and negative -energy state projection operators, respectively. They are defined as

$$
\begin{equation*}
\Lambda^{+}(\vec{p})=U(\vec{p}) U^{+}(\vec{p}), \quad \Lambda^{-}(\vec{p})=V(\vec{p}) V^{+}(\vec{p}) \tag{2.6}
\end{equation*}
$$

$U(\vec{p})$ and $V(\vec{p})$ are the positive and negative-energy Dirac spinor wave functions, respectively,

$$
U(\vec{p})=\left(\frac{\omega+m}{2 \omega}\right)^{1 / 2}\left(\begin{array}{c}
1  \tag{2.7}\\
\frac{\sigma}{\sigma} \cdot \vec{p} \\
\omega+m
\end{array}\right), \quad V(\vec{p})=\left(\frac{\omega+m}{2 \omega}\right)^{1 / 2}\binom{-\frac{\vec{\sigma} \cdot \vec{p}}{\omega+m}}{1} .
$$

Notice that eq.(1), as a coupled equation, is represented in Dirac spinor space, and it may be transformed to an equivalent P-S equation in Pauli space as shown in the appendix B of ref.[2]. This P-S equation is

$$
\begin{equation*}
\left[P_{0}-\omega\left(\frac{\vec{P}}{2}+\vec{q}\right)-\omega\left(\frac{\vec{P}}{2}-\vec{q}\right)\right] \Phi(P, \vec{q})=\int \frac{d^{3} k}{(2 \pi)^{3}} V(P, \vec{q}, \vec{k}) \Phi(P, \vec{k}) \tag{2.8}
\end{equation*}
$$

Where,

$$
\begin{equation*}
\Phi(P, \vec{q})=U^{+}\left(\frac{\vec{P}}{2}+\vec{q}\right) U^{+}\left(\frac{\vec{P}}{2}-\vec{q}\right) \chi_{P \xi}(\vec{q}) \tag{2.9}
\end{equation*}
$$

is the P-S positive-energy wave function, and

$$
\begin{align*}
V(P, \vec{q}, \vec{k})= & H_{++,++}(P, \vec{q}, \vec{k})-\int \frac{d^{3} l}{(2 \pi)^{3}} \frac{H_{++,--}(P, \vec{q}, \vec{l}) H_{--,++}(P, \vec{l}, \vec{k})}{P_{0}+\omega_{1}(\vec{l})+\omega_{2}(\vec{l})}  \tag{2.10}\\
& +\int \frac{d^{3} l_{1}}{(2 \pi)^{3}} \frac{d^{3} l_{2}}{(2 \pi)^{3}} \frac{H_{++,--}\left(P, \vec{q}, \vec{l}_{1}\right) H_{--,--}\left(P, \vec{l}_{1}, \vec{l}_{2}\right) H_{--,++}\left(P, \overrightarrow{l_{2}}, \vec{k}\right)}{\left[P_{0}+\omega_{1}\left(\vec{l}_{1}\right)+\omega_{2}\left(\vec{l}_{1}\right)\right]\left[P_{0}+\omega_{1}\left(\vec{l}_{2}\right)+\omega_{2}\left(\vec{l}_{2}\right)\right]}+\ldots \ldots
\end{align*}
$$

is the effective interaction hamiltonian. In the above equation

$$
\begin{equation*}
H_{a b, c d}(P, \vec{q}, \vec{k})=W_{a}^{+}\left(\frac{\vec{P}}{2}+\vec{q}\right) W_{b}^{+}\left(\frac{\vec{P}}{2}-\vec{q}\right) H(P, \vec{q}, \vec{k}) W_{c}\left(\frac{\vec{P}}{2}+\vec{k}\right) W_{d}\left(\frac{\vec{P}}{2}-\vec{k}\right) \tag{2.11}
\end{equation*}
$$

where $\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}=+,-$,

$$
\begin{equation*}
H(P, \vec{q}, \vec{k})=i \gamma_{1}^{0} \gamma_{2}^{0} K(P, \vec{q}, \vec{k}) \tag{2.12}
\end{equation*}
$$

and

$$
W_{a}(\vec{p})=\left\{\begin{array}{lll}
U(\vec{p}) & \text { if } & a=+  \tag{2.13}\\
V(\vec{p}) & \text { if } & a=-
\end{array}\right.
$$

It should be pointed out that the above effective interaction is the energy-dependent and nonlocalized. It is clear that the first one in the right hand side of eq.(10) serves as the conventional one-gluon exchange potential, where only the positive component (larger component) of the B-S amplitude is considered. The second and all other higher order terms in
eq.(10) describe the contributions arising from the NEC(small component) of the B-S amplitude $\chi_{P \xi}(\vec{q})$ to the P-S equation satisfied by the positive-energy P-S wave function $\Phi(P, \vec{q})$. It should be pointed out that because the B-S kernel here is only the one-gluon exchange as displayed in eq.(2), the effective interaction in eq.(10) just involves the corrections of the NEC of the B-S amplitude to the one-gluon exchange interaction.

To localize the second and all other higher order terms in the right hand side of eq.(10), we use the following approximation for their denominators:

$$
P_{0}+\omega_{1}(\vec{q})+\omega_{2}(\vec{k}) \sim 4 m
$$

To notice the fact that $H_{++,++}=H_{--,--}$, we therefore write eq.(8) in non-relativistic approximation as

$$
\begin{equation*}
\left[-\frac{\nabla^{2}}{2 \mu}+V(r)\right] \Phi(r)=E \Phi(r) \tag{2.14}
\end{equation*}
$$

where,

$$
\begin{equation*}
V(r)=V_{\text {oge }}(r)+V_{\text {conf }}(r)+V_{\text {neg }}(r)+B_{0} . \tag{2.15}
\end{equation*}
$$

In the above equation, $B_{0}$ is zero-point energy, $V_{o g e}$, which stems from the first term $H_{++,++}$ in R.H.S. of eq.(10), stands for the conventional instantaneous one-gluon exchange,

$$
\begin{equation*}
V_{o g e}\left(r_{i j}\right)=-\frac{2 \alpha_{s}}{3 r_{i j}}+V_{r e}\left(r_{i j}\right) \tag{2.16}
\end{equation*}
$$

where, the color factor $\frac{\lambda_{1}^{c} \lambda_{2}^{c}}{4}$ is taken to be $-3 / 2$ for baryons. In the above equation, $V_{r e}\left(r_{i j}\right)$ contains the spin-dependent and -independent relativistic corrections ${ }^{[4]}$ in non-relativistic approximation $p^{2} / m^{2}$.

$$
\begin{equation*}
V_{r e}\left(r_{i j}\right)=V_{r e}^{S D}\left(r_{i j}\right)+V_{r e}^{S I}\left(r_{i j}\right), \tag{2.17}
\end{equation*}
$$

where

$$
\begin{align*}
V_{r e}^{S D}\left(r_{i j}\right) & =\frac{2 \alpha_{s}}{3 m^{2}}\left[\frac{8}{3} \vec{S}_{i} \cdot \vec{S}_{j} \delta^{3}\left(\vec{r}_{i j}\right)+\frac{1}{r_{i j}^{3}}\left(\frac{3 \vec{S}_{i} \cdot \vec{r}_{i j} \vec{S}_{j} \cdot \vec{r}_{i j}}{r_{i j}^{2}}-\vec{S}_{i} \cdot \vec{S}_{j}\right)\right],  \tag{2.18}\\
V_{r e}^{S I}\left(r_{i j}\right) & =\frac{2 \alpha_{s} \pi}{3 m^{2}} \delta^{3}\left(r_{i j}\right)+\frac{\alpha_{s}}{3 m^{2}}\left(\frac{1}{r_{i j}} \vec{p}_{i} \cdot \vec{p}_{j}+\frac{1}{r_{i j}^{3}} \vec{r}_{i j}\left(\vec{r}_{i j} \cdot \vec{p}_{i}\right) \cdot \vec{p}_{j}\right) . \tag{2.19}
\end{align*}
$$

In the above equation, $p_{i}$ and $p_{j}$ are momentum of i -th and j -th quarks. In eq.(15), $V_{\text {conf }}$ is the scalar phenomenological confinement. $V_{\text {neg }}$ represents the contribution of the NEC of the B-S amplitude, which is involved in the second and all the other terms in the R.H.S. of eq.(10). For the vector coupling one-gluon exchange interaction, since we have

$$
\begin{equation*}
H_{++,--}(P, \vec{q}, \vec{l})=\frac{\alpha_{s}}{4 \pi}\left(\frac{\lambda_{1}^{c} \lambda_{2}^{c}}{4}\right) \Delta_{f}^{s}(\vec{q}-\vec{l})\left(-\overrightarrow{\sigma_{1}} \cdot \overrightarrow{\sigma_{2}}\right)+O\left(\frac{p^{2}}{m^{2}}\right) \tag{2.20}
\end{equation*}
$$

and the denominators in the second and all the other terms in R.H.S. of eq.(10) are in inverse proportion to the quark mass, thus the explicit expression for $V_{n e g}$ attributed from the leading term of $H_{++,--}$in the second and all the other terms of R. H. S. of eq.(10), is written as

$$
\begin{equation*}
V_{n e g}\left(r_{i j}\right)=\sum_{n=1}^{\infty}-\frac{\left(3-2 \vec{\sigma}_{1} \cdot \vec{\sigma}_{2}\right)}{(4 m)^{n}}\left(\frac{2 \alpha_{s}}{3 r_{i j}}\right)^{n+1} \tag{2.21}
\end{equation*}
$$

Clearly, if

$$
\begin{equation*}
\left|\frac{1}{4 m} \frac{2 \alpha_{s}}{3 r_{i j}}\right|<1 \tag{2.22}
\end{equation*}
$$

$V_{n e g}$ converges, and can be simply written as

$$
\begin{equation*}
V_{n e g}\left(r_{i j}\right)=-\left(3-2 \vec{\sigma}_{1} \cdot \vec{\sigma}_{2}\right) \frac{\left(-\frac{2 \alpha_{s}}{3 r}\right)^{2}}{4 m-\frac{2 \alpha_{s}}{3 r}} . \tag{2.23}
\end{equation*}
$$

If one chooses $\mathrm{m}=0.3 \mathrm{GeV}$ for u or d quark mass, and the strong coupling constant $\alpha_{s}=0.8$ as shown later, the corresponding convergence condition eq.(22) requires r larger than 0.088 fm . For the N and $\Delta$ systems, for they consist of light quarks( u and d ), and their mean square root radii are at least larger than 0.45 fm , the above convergence condition eq.(22) is well satisfied in this case. Because the one-gluon exchange usually stands for the short range interaction, the matrix elements is small enough to be ignored in the range of $r \leq 0.088 \mathrm{fm}$. In the following, we only consider the first term of $V_{\text {neg }}$, and neglect all other $\alpha_{s}$ higherorder terms in non-relativistic approximation $p^{2} / m^{2}$, in order to avoid higher order singular behavior of those terms at $\mathrm{r}=0$. This selection is reasonable, because other higher order terms are trivial for the matrix elements of Hamiltonian. The non-relativistic $\left(p^{2} / m^{2}\right)$ explicit form of the first term in coordinate space, is

$$
\begin{equation*}
V_{n e g}^{0}\left(r_{i j}\right)=-\left(3-2 \vec{\sigma}_{1} \cdot \vec{\sigma}_{2}\right) \frac{\alpha_{s}^{2}}{9 m r_{i j}^{2}} . \tag{2.24}
\end{equation*}
$$

If one ignores all the three-body interactions, and assumes the total hamiltonian of the baryon system is the sum of the two-body interactions between the three valence quarks, the application of above two-body interaction $V(r)$ to baryon systems, will enable one to obtain the total hamiltonian for N and $\Delta$ systems.

$$
\begin{equation*}
H=H_{0}+\sum_{i<j}^{3} H_{r e}\left(r_{i j}\right)+B_{0} \tag{2.25}
\end{equation*}
$$

where,

$$
\begin{equation*}
H_{0}=\frac{\vec{p}_{\rho}^{2}}{2 m}+\frac{\vec{p}_{\lambda}^{2}}{2 m}+\sum_{i<j}^{3}\left(\lambda r_{i j}-\frac{2 \alpha_{s}}{3 r_{i j}}\right) \tag{2.26}
\end{equation*}
$$

The first two terms in eq.(26) are the kinetic energy operators. $\vec{p}_{\rho}$ and $\vec{p}_{\lambda}$ are the momentum operators corresponding to the internal coordinates $\vec{\rho}$ and $\vec{\lambda}$ which are defined as

$$
\begin{equation*}
\vec{\rho}=\frac{1}{\sqrt{2}}\left(\vec{r}_{1}-\vec{r}_{2}\right), \quad \vec{\lambda}=\frac{1}{\sqrt{6}}\left(\vec{r}_{1}+\vec{r}_{2}-2 \vec{r}_{3}\right) . \tag{2.27}
\end{equation*}
$$

In eq.(25), $H_{r e}\left(r_{i j}\right)$ is the relativistic corrections of Hamiltonian. In order to stress the effects of NEC of the B-S amplitude and the relativistic spin-independent interaction, we shall calculate the baryon spectra with the following different three choices of $H_{r e}\left(r_{i j}\right)$ :

$$
\begin{array}{rr}
H_{r e}\left(r_{i j}\right)=V_{r e}^{S D}\left(r_{i j}\right) & (\text { case } I)  \tag{2.28}\\
H_{r e}\left(r_{i j}\right)=V_{r e}^{S D}\left(r_{i j}\right)+V_{n e g}^{0}\left(r_{i j}\right) & (\text { case } I I) \\
\left.H_{r e}\left(r_{i j}\right)=V_{r e}^{S D}\left(r_{i j}\right)+V_{r e}^{S I}\left(r_{i j}\right)+V_{n e g}^{0}\left(r_{i j}\right)\right] . & (\text { case III) }
\end{array}
$$

## 3 Discussion and Concluding Remarks

It should be emphasized that the confinement we adopt is a phenomenological scalar linear confinement as usual ${ }^{[5]}$. Although we know that a scalar confining potential has the problem of leading to essentially unstable solutions in the case of the instantaneous approximation, because of the unboundedness from below of the effective interaction ${ }^{[6]}$, we only focus our attention to the effect of the negative energy component on the one-gluon exchange interaction, and do not attempt to discuss the problem in the text. So, in the following, we do not try to solve either the instantaneous B-S equation (8) or the reduced Schrödinger equation (14) exactly, since we know that it is very difficult to solve them for three-body systems. We shall expand the wave function of eq.(14) with respect to the harmonic oscillator wave functions in a certain configuration space, such as in $N \leq 4$ harmonic oscillator space. The corresponding wave functions are obtained by solving the eigen-equation of $H_{0}$, and the energy levels are yielded by adding the effects of the relativistic corrections perturbatively. Thus one can avoid the problems of the unstable solutions in the case of the instantaneous approximation of the B-S equation caused by the phenomenological scalar confinement as usual ${ }^{[5]}$ and the not defined hamiltonian caused by the spin-spin contact terms $\delta^{3}(\vec{r})$ interaction ${ }^{[6,7]}$, simultaneously like other works ${ }^{[5,8]}$.

To calculate baryon spectrum, we select $m_{u, d}=0.3 \mathrm{GeV}$ and determine other potential parameters according to the following physical conditions.

$$
\begin{align*}
M_{\Delta}-M_{N} & =0.293 \mathrm{GeV}  \tag{3.1}\\
\frac{\partial M_{N}}{\partial b} & =0  \tag{3.2}\\
\frac{1}{2}\left(M_{\Delta}+M_{N}\right) & =1.085 \mathrm{GeV} \tag{3.3}
\end{align*}
$$

where, the second condition is stability condition of proton with respect to harmonic oscillator width constant $b$. Here, we select $b=0.5 \mathrm{fm}$. At first, we consider the lowest approximation
for the baryon wave functions, namely, the pure harmonic oscillator wave functions are taken as the wave functions of baryons. The parameters determined in this case are listed in the Table 1. The three sets of parameters shown in the table are corresponding to the three choices of $H_{r e}\left(r_{i j}\right)$ as shown in eq.(28).
Table 1 indicates that the effect of the NEC of the B-S amplitude is significant to parameters,

|  | $\alpha_{s}$ | $\lambda\left(\mathrm{GeV}^{2}\right)$ | $m_{q}(\mathrm{GeV})$ | $b(\mathrm{fm})$ |
| :---: | :---: | :---: | :---: | :---: |
| $\operatorname{set}(\mathrm{I})$ | 0.808 | 0.0498 | 0.3 | 0.5 |
| $\operatorname{set}(\mathrm{II})$ | 0.448 | 0.0560 | 0.3 | 0.5 |
| $\operatorname{set}(\mathrm{II})$ | 0.448 | 0.0283 | 0.3 | 0.5 |

Table 1: Potential parameters.
especially to the coupling constant $\alpha_{s}$. It may reduce the $\alpha_{s}$ to be a more reasonable value. This fact is easily to be understood. Because the effect supplies a short-range attractive interaction, which compensates a part of the instantaneous one-gluon exchange potential, as a result, $\alpha_{s}$ is reduced under the condition eq.(29). With the consideration of the effect, the coupling constant decreases from 0.811 to 0.441 , which is more closer to the situation of the calculations of meson spectra ${ }^{[7,8]}$. This decreasing makes the ladder approximation reasonable. In addition, the contribution of the NEC of the B-S amplitude also plays a role on the strength of linear confinement (about 10\%). This result is comprehensive too, since the effect gives a strong attractive interaction, it strongly attracts the wave functions to the origin, such the strength of the linear confinement must be enhanced so as to fit data.

Further more, Table 1 also implies that the spin-independent interaction is substantial for the determination of the confinement strength. The consideration of the interaction reduces the confinement strength a lot. This feature is easily understood, because the relativistic effects must be essential for N and $\Delta$ systems. However, the spin-independent interaction does not affect the determination of the strong coupling constant, because $\alpha_{s}$ is obtained only by the spin-dependent interactions as shown in eq.(29).

Secondly, in order to calculate baryon wave functions more exactly, we expand the wave function of baryons in $N \leq 4$ harmonic oscillator wave function space. So, the effect of configuration space mixing is considered. We use the three sets of parameters in Table 1 to calculate baryons spectrum for the three cases. Table 2 shows our calculation results for N and $\Delta$ spectrum. The values in the second, third and fourth columns are the calculated spectra with the parameters set(I), and set(II), set(III) in pure harmonic oscillator space, respectively. The values in the fifth, sixth and seventh columns are the calculated spectra with the parameters set(I), set(II), and set(III) in the Table 1 in $N \leq 4$ harmonic oscillator wave function space, respectively.

To compare the results in the second and the third columns or in the fifth and the sixth columns of Table 2, one sees that the effect of the NEC of the B-S amplitude may not

|  | In pure harmonic space |  |  | In $N \leq 4$ mixing harmonic space |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Set(I) | Set(II) | Set(III) | Set(I) | Set(II) | Set(III) | Data ${ }^{[15]}$ |
| $\Delta 1^{+} / 2$ | 1.981 | 1.904 | 1.854 | 1.898 | 1.867 | 1.832 | 1.870-1.920 |
|  | 1.982 | 1.985 | 1.935 | 1.923 | 1.899 | 1.841 |  |
| $\Delta 3^{+} / 2$ | 1.236 | 1.240 | 1.201 | 1.230 | 1.228 | 1.200 | 1.230-1.234 |
|  | 1.981 | 1.946 | 1.847 | 1.904 | 1.878 | 1.710 | 1.550-1.700 |
|  | 2.012 | 1.972 | 1.914 | 1.934 | 1.891 | 1.804 | 1.900-1.970 |
|  | 2.073 | 2.011 | 1.974 | 1.942 | 1.911 | 1.871 |  |
| $\Delta 5^{+} / 2$ | 1.980 | 1.946 | 1.877 | 1.930 | 1.881 | 1.828 | 1.870-1.920 |
|  | 2.002 | 2.012 | 1.959 | 1.942 | 1.917 | 1.864 |  |
| $\Delta 7^{+} / 2$ | 1.973 | 1.936 | 1.906 | 1.917 | 1.907 | 1.895 | 1.940-1.960 |
| $\Delta 1^{-} / 2$ | 1.608 | 1.638 | 1.531 | 1.600 | 1.625 | 1.546 | 1.615-1.675 |
|  | 1.868 | 1.886 | 1.833 | 1.851 | 1.870 | 1.813 | 1.880-1.950 |
| $\Delta 3^{-} / 2$ | 1.668 | 1.686 | 1.606 | 1.651 | 1.670 | 1.611 | 1.670-1.770 |
|  | 1.892 | 1.930 | 1.891 | 1.921 | 1.970 | 1.915 |  |
| $\Delta 5^{-} / 2$ | 2.034 | 2.095 | 2.011 | 2.123 | 2.150 | 2.053 | 1.920-1.970 |
|  | 2.334 | 2.295 | 2.206 | 2.382 | 2.350 | 2.304 | $\Delta(2.350)$ |
| $\Delta 7^{-} / 2$ | 2.264 | 2.211 | 2.153 | 2.338 | 2.271 | 2.218 | $\Delta(2.200)$ |
| $\Delta 9^{-} / 2$ | 2.244 | 2.243 | 2.174 | 2.246 | 2.234 | 2.206 | $\Delta(2.400)$ |
| $N 1^{+} / 2$ | 0.94 | 0.94 | 0.94 | 0.94 | 0.94 | 0.94 | 0.94 |
|  | 1.619 | 1.631 | 1.510 | 1.559 | 1.548 | 1.472 | 1.430-1.470 |
|  | 1.799 | 1.803 | 1.740 | 1.786 | 1.764 | 1.693 | 1.680-1.740 |
|  | 1.914 | 1.974 | 1.900 | 1.884 | 1.888 | 1.821 | - |
|  | 1.979 | 2.005 | 1.915 | 1.972 | 1.932 | 1.893 |  |
| $N 3^{+} / 2$ | 1.835 | 1.837 | 1.732 | 1.760 | 1.769 | 1.649 | 1.650-1.750 |
|  | 1.907 | 1.906 | 1.854 | 1.885 | 1.866 | 1.817 | - |
|  | 1.979 | 1.974 | 1.906 | 1.929 | 1.899 | 1.826 | - |
|  | 2.000 | 1.996 | 1.921 | 1.940 | 1.907 | 1.854 | - |
|  | 2.028 | 2.011 | 1.981 | 1.974 | 1.938 | 1.884 |  |
| $N 5^{+} / 2$ | 1.835 | 1.737 | 1.698 | 1.763 | 1.670 | 1.632 | 1.675-1.690 |
|  | 1.907 | 1.705 | 1.683 | 1.930 | 1.871 | 1.759 | - |
|  | 2.027 | 1.837 | 1.814 | 1.964 | 1.941 | 1.927 |  |
| $N 7^{+} / 2$ | 1.961 | 2.000 | 1.976 | 1.881 | 1.909 | 1.892 | N (1.990) |
|  | 2.568 | 2.529 | 2.510 | 2.598 | 2.604 | 2.555 |  |
| $N 1^{-} / 2$ | 1.462 | 1.457 | 1.316 | 1.460 | 1.453 | 1.338 | 1.520-1.555 |
|  | 1.635 | 1.690 | 1.627 | 1.660 | 1.669 | 1.609 | 1.640-1.680 |
| $N 3^{-} / 2$ | 1.462 | 1.457 | 1.396 | 1.460 | 1.450 | 1.406 | 1.515-1.530 |
|  | 1.666 | 1.663 | 1.623 | 1.734 | 1.708 | 1.641 | 1.650-1.750 |
| $N 5^{-} / 2$ | 1.593 | 1.622 | 1.589 | 1.594 | 1.585 | 1.544 | 1.670-1.685 |
| $N 7^{-} / 2$ | 2.225 | 2.230 | 2.186 | 2.263 | 2.261 | 2.194 | 2.110-2.200 |
| $N 9^{-} / 2$ | 2.246 | 2.283 | 2.213 | 2.109 | 2.239 | 2.189 | 2.170-2.310 |

Table 2: Baryon spectra(in GeV ) with the three sets of parameters and in the two kinds of wave function approximations.
change the spectra remarkably. Although some energy level is changed about 80 MeV , such as $\Delta_{1^{+} / 2}^{*}(1.850-1.980)$, the total global structure of spectra is almost untouched. This conclusion is manifestly, because the effect of the NEC of the B-S amplitude is a short-range-interaction, it may provide a small effect on excited-state. In comparison with the calculated results in the columns 2 and 5 or in the columns 3 and 6 of Table 2, one may easily find that the influence of configuration mixing is about $70-90 \mathrm{MeV}$ for Roper resonance $N_{1^{+} / 2}^{*}(1.430-1.470)$ and $\Delta_{3^{+} / 2}^{*}(1.550-1.700)$, respectively, which suppresses the first radial excited-state and improves the theoretical results.

Moreover, from our calculation, we see that the relativistic corrections, the spin-dependent, spin-independent interactions i.e., are all substantial for the description of N and $\Delta$ systems. The spin-dependent interactions ( $V_{r e}^{S D}$, and $V_{n e g}^{0}$ ) enable us to get the spindependent mass splittings of N and $\Delta$ baryons, such as $M_{\Delta}-M_{N}$. The inclusion of the tensor force is of importance too, because it could introduce the mixture between the states in the different harmonic wave functions with the different angular momentum. In addition, the spin-independent $V_{r e}^{S I}$ interaction is also necessary to be considered for the systems, since it affects the spin-average energy levels and suppresses them a lot, especially for the low-lying states. To compare the results in the columns 3 , and 4 or in the columns 6 and 7 , one sees that the spin-independent interaction cannot be ignored. Its effect on the low-lying state, such as $N_{1^{-} / 2}(1.520-1.555)$ and $N_{1^{+} / 2}^{*}(1.430-1.470)$, are about 120 and 170 MeV . Moreover, we find that the consideration of the spin-independent interaction could push the baryon wave function to the origin, and make the distribution of the wave function smaller, as a result, it reduces the energy level. This feature of the interaction is helpful for one to understand the Roper resonance state $N_{1^{+} / 2}^{*}(1.430-1.470)$ and $\Delta_{3^{+} / 2}^{*}(1.550-1.770)$ for it suppresses the mass of Roper resonance from 1.548 MeV to 1.472 GeV .

According to the above analyses, one finds that the effect of the negative energy component of the B-S amplitude on the one-gluon-exchange plays a substantial role on the determination of the coupling constant $\alpha_{s}$. It provides a short-range-attractive interaction, and strengths the singular behavior of the interaction at short distance. It decreases the coupling constant a lot and makes ladder approximation more reasonable. Meanwhile, It also enlarges the strength constant of the linear confinement. But its influence on the whole structure of spectra is trivial. This conclusion is similar to that of ref.[9]. In P. C. Tiemeiger and J. A. Tjon's work ${ }^{[9]}$, quasi-potential approximations, namely, the Blankerbecler-Sugar-Logunov- Tavkhe and equal-time approximations were adopted for fermion propagator in the B-S equation, respectively, and they examined the NEC effect on meson spectra. They achieved the similar conclusion. Here, unlike ref.[9], we use instantaneous approximation, and investigate the effect on baryon spectra calculation. However, both calculations obtain the same conclusion for this effect.

Finally, one sees that the consideration of the correction from the NEC of the B-S am-
plitude on the one-gluon-exchange interaction is not enough in explanation of the baryon spectra, if one attempts to solve the difficulty in description of baryon spectrum, such as explaining of Roper resonance $N_{1^{+} / 2}^{*}(1.430-1.470)$ and $\Delta_{3^{+} / 2}^{*}(1.550-1.700)^{[4,10,11]}$. In order to improve our calculation, we need to take the spinor coupling structure of the phenomenological confining potential as the admixture of the scalar coupling and other couplings, such as vector-vector coupling confinement ${ }^{[12]}$ or other kind of confinement ${ }^{[6,11]}$. It may be expected that if the effect of the NEC of the B-S amplitude on the confinement is considered, one may obtain a confinement, which is much lower than the usual linear confinement at large distance. This feature of the confinement is similar to lattice gauge calculation ${ }^{[13]}$. By utilizing the confinement with the color screening effect, one may easily suppress the first radial-excited energy level a lot, and obtain some improvements on the calculations of baryon spectra, and heavy quarkonium ${ }^{[14]}$. Regarding the spin-independent relativistic correction, one finds that although it suppresses the Roper resonance a lot, it also reduces the energy level of $N_{1^{-/ 2}}(1.520-1.555)$ about 100 MeV . Therefore, a smaller harmonic oscillator width b is needed in order to fit the data. This conclusion is reasonable, since the interaction pushes the baryon wave function to the origin, so the corresponding radii of baryons must be smaller.

To conclude this paper, one sees that the relativistic effects, such as the effect of the negative energy component of the B-S amplitude could not be ignored at least in the determination of the parameters. In the non-relativistic calculations, the effective parameters are chosen to describe the effective interaction. However, if the effect of the negative energy component of the B-S amplitude, as a kind of relativistic correction, is taken into account, one may at least expect that one has gotten a set of parameters which are much closer to the really nature world.

## References

[1] S. N. Mukkerjee, R. Nag, S. Sanyal et al., Phys. Rep. 231 (1993), 201; N. Isgur and G. Karl, Phys. Lett. 72B (1977), 109; 74B (1978), 353; Phys. Rev. D18 (1978), 4187; D19 (1979), 2653; D. Gromes, Nucl. Phys. B130 (19977), 18; D. Gromes and I. O. Stamatescu, Nucl. Phys. B112 (1976), 213; A. J. G. Hey et al., Phys. Rep. 96 (1983), 71; E. Eichten et al., Phys. Rev. Lett. 34 (1975), 369; 36 (1976), 500; H. De Rujula et al., Phys. Rev. D12 (1975), 147.
[2] Junchen Su, Zuoqun Chen and Shishu Wu, Nucl. Phys. A524 (1991),615.
[3] C. Itzykson and R. Zuber, "Quantum Field Theory", (McGraw-Hill, New York, 1980).
[4] Y. B. Dong, Y. W. Yu, Z. Y. Zhang and P. N. Shen, Phys. Rev. D49 (1994), 1642.
[5] S. Capstick and N. Isgur, Phys. Rev. D34 (1986), 2809; D. Gromes and I. O. Stamatescu, Z. Phys. C3 (1979), 43, F. F. Schöberl, Z. Phys. C15 (1982), 261; D. Flamm and F.

Schöberl, Introduction to the Quark Model of Elementary Particles, Volume 1. New York: Gordon and Breach 1986.
[6] J. Parramore and J. Pickarewicz, Nucl. Phys. A585 (1995), 705.
[7] C. Quigg and J. L. Rosner, Phys. Rep. 56 (1979), 167; W. Lucha, F. F. Schöberl and D. Gromes, Phys. Rep. 200 (1991), 127(and references cited therein).
[8] D. P. Stanley and D. Robson, Phys. Rev. D21 (1980), 3180; S. Godfrey and N. Isgur, Phys. Rev. D32 (1985), 189; S. Ono and F. F. Schöberl, Phys. Lett. B118 (1982), 419.
[9] P. C. Tiemeiger and J. A. Tjon, Phys. Rev. C48 (1991), 896.
[10] H. Leeb, H. Fiedeldey, E. J. O. Gavin, S. A. Sofianos, and R. Lipperheide, Few-Body Systems, 12 (1992),55.
[11] Dong Yu-bing, Su Jun-chen and Wu Shi-shu, J. Phys. G 20 (1994), 73.
[12] S. Deoghuria and S. Chakrabarty, Z. Phys. 53 (1992), 293.
[13] E. Laermann, F. Langhammer, I. Schmitt, et al., Phys. Lett. B173(1986), 457;R. Gupta et al., Phys. Rev. D44 (1991), 3237;W. Sakuler et al., Phys. Lett. B276 (1992),155.
[14] C. S. Kalman and B. Tran, Nuovo Cimento, 102A (1989), 835; Z.Y. Zhang, Y. W. Yu, P. N. Shen, X. Y. Shen and Y. B. Dong, Nucl. Phys. A561 (1993), 595; Y. B. Dong, Y. W. Yu, Z. Y. Zhang and P. N. Shen, Phys. Rev. D49 (1994), 1642
[15] Particle Data Group; Review of Particle Properties: Phys. Rev. D45 11-II (1992), 1.

