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# What Particles Are Described by the Weinberg Theory?

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*Abstract.* The main goal of the paper is to study the origins of a contradiction between the Weinberg theorem $B - A = \lambda$ and the 'longitudity' of an antisymmetric tensor field (and of a Weinberg field which is equivalent to it), transformed on the $(1,0) \oplus (0,1)$ Lorentz group representation. On the basis of analysis of dynamical invariants in the Fock space situation has been partly clarified.

The interest in the $2(2j+1)$ component model [1, 2, 3, 4] and in the antisymmetric tensor fields [5, 6] has grown in the last years. Antisymmetric tensor fields are of importance for physical applications [7]. Moreover, they are an object of continuous and renewed interests due to their connection with topological field theories [8].

However, many points are unclear to understand at the moment. The most intrigued thing, in my opinion, is the following contradiction [3, 4]: the $j = 1$ antisymmetric tensor field is shown to possess the longitudinal components only [6, 9, 10], the helicity is equal to $\lambda = 0$. In the meantime, they transform according to the $(1,0) + (0,1)$ representation of the Lorentz group (like a Weinberg bispinor[2]). How is the Weinberg theorem, $B - A = \lambda$, ref. [11], for the $(A, B)$ representation to be treated in this case? Moreover, does this fact signify that we must abandon the Correspondence Principle (in the classical physics we have become accustomed that antisymmetric tensor field has only transversal components) ?

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[2]See for the mapping between antisymmetric tensor field equations and Weinberg equations ref. [3e,4].

In the beginning let me reproduce the previous results. In ref. [3e] I worked with the Lagrangian density (see also [12, 13]):

$$\mathcal{L}^W = -\partial_\mu \overline{\psi} \gamma_{\mu\nu} \partial_\nu \psi - m^2 \overline{\psi} \psi \quad , \tag{1}$$

$\gamma_{\mu\nu}$ are the Barut-Muzinich-Williams matrices which are chosen to be Hermitian. In a massless limit, implying an interpretation of the 6-spinor as[3]

$$\begin{cases} \chi = \vec{E} + i\vec{B} & , \\ \phi = \vec{E} - i\vec{B} & , \end{cases} \tag{2}$$

$\psi = \mathrm{column}(\chi \quad \phi)$, $\vec{E}$ and $\vec{B}$ are the real 3-vectors, the Lagrangian (1) can be re-written in the following way:

$$\mathcal{L}^W = -(\partial_\mu F_{\nu\alpha})(\partial_\mu F_{\nu\alpha}) + 2(\partial_\mu F_{\mu\alpha})(\partial_\nu F_{\nu\alpha}) + 2(\partial_\mu F_{\nu\alpha})(\partial_\nu F_{\mu\alpha}) \quad . \tag{3}$$

This form of the Lagrangian leads to the Euler-Lagrange equation

$$\Box F_{\alpha\beta} - 2(\partial_\beta F_{\alpha\mu,\mu} - \partial_\alpha F_{\beta\mu,\mu}) = 0 \quad , \tag{4}$$

where $\Box = \partial_\nu \partial_\nu$.

The Lagrangian (3) is found out there to be equivalent to the Lagrangian of a free massless skew-symmetric field, given by Hayashi in ref. [9]:[4]

$$\mathcal{L}^H = \frac{1}{8} F_k F_k \quad , \tag{5}$$

with $F_k = i\epsilon_{kjmn} F_{jm,n}$. It is re-written in:

$$\begin{aligned} \mathcal{L}^H &= -\frac{1}{4}(\partial_\mu F_{\nu\alpha})(\partial_\mu F_{\nu\alpha}) + \frac{1}{2}(\partial_\mu F_{\nu\alpha})(\partial_\nu F_{\mu\alpha}) = \\ &= \frac{1}{4}\mathcal{L}^W - \frac{1}{2}(\partial_\mu F_{\mu\alpha})(\partial_\nu F_{\nu\alpha}) \quad , \end{aligned} \tag{6}$$

what proves the statement made above if take into account the generalized Lorentz condition, ref. [9]. After applying the Fermi method *mutatis mutandis* as in ref. [9] we achieved the result that the Lagrangians (1) and (5) describe massless particles possessing longitudinal physical components only. Transversal components are removed by means of the "gauge" transformation

$$F_{\mu\nu} \to F_{\mu\nu} + A_{[\mu\nu]} = F_{\mu\nu} + \partial_\nu \Lambda_\mu - \partial_\mu \Lambda_\nu \tag{7}$$

(or by the transformation similar to the above but applied to the Weinberg bivector). This fact is very surprising from a viewpoint of the Weinberg theorem about a connection between

[3] One can also choose

$$\psi^{(2)} = \begin{pmatrix} \vec{E} + i\vec{B} \\ -\vec{E} + i\vec{B} \end{pmatrix} = \gamma_5 \psi \quad .$$

Since $\overline{\psi}^{(2)} = -\overline{\psi}\gamma_5$ the formula (3) is not changed.

[4] See also for describing closed strings on the ground of this Lagrangian in ref. [10].

the helicity $\lambda$ and the Lorentz group representation $(A, B)$ which field operators transform on: $B - A = \lambda$.

Here I am going to clarify this question. In ref. [4] the concept of the Weinberg field as a system of degenerate states has been proposed. Unfortunately, the consistent description of the Weinberg "doubles" [4] and/or of the antisymmetric tensor fields $F_{\mu\nu}$ and its dual $\tilde{F}_{\mu\nu}$ as the parts of a degenerate doublet are absent in the literature (to my knowledge). Many works deal with the dual theories, *e.g.* [14], but do not contain quantization topics.

Firstly, we need to choose the appropriate Lagrangian. In the case of the use of the pseudoeuclidean metric (when $\gamma_{0i}$ is chosen to be anti-Hermitian) it is possible to write the Lagrangian following F. D. Santos and H. Van Dam, ref. [13] (see also ref. [3b], where this Lagrangian has been obtained independently):

$$\mathcal{L} = \partial^\mu \overline{\psi} \gamma_{\mu\nu} \partial^\nu \psi - m^2 \overline{\psi} \psi \quad . \tag{8}$$

One can use the Lagrangian which is similar to Eq. (8):[5]

$$\mathcal{L}^{(1)} = -\partial_\mu \overline{\psi}_1 \gamma_{\mu\nu} \partial_\nu \psi_1 - \partial_\mu \overline{\psi}_2 \gamma_{\mu\nu} \partial_\nu \psi_2 - m^2 \overline{\psi}_1 \psi_1 + m^2 \overline{\psi}_2 \psi_2 . \tag{9}$$

in the Euclidean metric. The only inconvenience to be taken in mind where it is necessary is that we need to imply that $\partial_\mu^\dagger = (\vec{\nabla}, -\partial/\partial x_4)$, provided that $\partial_\mu = (\vec{\nabla}, \partial/\partial x_4)$, ref. [15].

The Lagrangian (9) leads to the equations:[6]

$$(\gamma_{\mu\nu} p_\mu p_\nu + m^2) \psi_1 = 0 \quad , \tag{10}$$
$$(\gamma_{\mu\nu} p_\mu p_\nu - m^2) \psi_2 = 0 \quad , \tag{11}$$

which possess solutions with a correct physical dispersion. The second equation coincides with the Ahluwalia *et al.* equation for $v$ spinors (Eq. (13) ref. [1b]) or with Eq. (12) of ref. [16c].[7]

If accept the concept of the Weinberg field as a set of degenerate states, one has to allow for possible transitions $\psi_1 \leftrightarrow \psi_2$ (or $F_{\mu\nu} \leftrightarrow \tilde{F}_{\mu\nu}$). Therefore, one can propose the Lagrangian with the following dynamical part:

$$\mathcal{L}^{(2)} = -\partial_\mu \overline{\psi}_1 \gamma_{\mu\nu} \partial_\nu \psi_2 - \partial_\mu \overline{\psi}_2 \gamma_{\mu\nu} \partial_\nu \psi_1 \quad . \tag{12}$$

But this form appears not to admit a mass term in a usual manner. Prof. Sachs proposed to consider inertial mass as a varying parameter. In the present framework we are going to go further: to consider the possibility of the *complex* mass parameter. This may be understood

[5]In refs. [3b,13] the possibility of appearance of the "doubles" has not been considered (neither in any other paper on the $2(2j+1)$ formalism, to my knowledge).

[6]Problems related with the Wick propagator and the Feynman-Dyson propagator are considered in the approaching publication in the framework of the proposed model, see also for the discussion of these topics on the page B1324 in ref. [11a] and ref. [2].

[7]For the discussion of differences between ref. [1] and our model see ref. [4] and what follows.

by means of the definition of the mass as the normalization coefficient. The field operator may be defined in different forms (the commutation relations as well) and different global phase factors between fields $\psi_1$ and $\psi_2$ can lead to the normalization changes. If consider $m^2$ as a pure imaginary quantity ($m = (1 \pm i)\tilde{m}$), *i. e.* in quantum theory as an anti-Hermitian operator, one can write the mass terms for the Lagrangian (12). However, the problem of origin of the mass term will be analyzed more carefully elsewhere.

At this moment I have to treat the question of solutions in the momentum space. The explicit form of Hammer-Tucker bispinors, ref. [17] (see also refs. [3, 18]), is

$$u_1^\sigma(\vec{p}) = v_1^\sigma(\vec{p}) = \frac{1}{\sqrt{2}} \begin{pmatrix} \left[1 + \frac{(\vec{J}\vec{p})}{m} + \frac{(\vec{J}\vec{p})^2}{m(E+m)}\right] \xi_\sigma \\ \left[1 - \frac{(\vec{J}\vec{p})}{m} + \frac{(\vec{J}\vec{p})^2}{m(E+m)}\right] \xi_\sigma \end{pmatrix} \quad , \tag{13}$$

for the equation (10); and

$$u_2^\sigma(\vec{p}) = v_2^\sigma(\vec{p}) = \frac{1}{\sqrt{2}} \begin{pmatrix} \left[1 + \frac{(\vec{J}\vec{p})}{m} + \frac{(\vec{J}\vec{p})^2}{m(E+m)}\right] \xi_\sigma \\ \left[-1 + \frac{(\vec{J}\vec{p})}{m} - \frac{(\vec{J}\vec{p})^2}{m(E+m)}\right] \xi_\sigma \end{pmatrix} \quad , \tag{14}$$

for the equation (11). The bispinor normalization in the cited papers is chosen as $\overline{u}_1^\sigma(\vec{p})u_1^\sigma(\vec{p}) = \overline{v}_1^\sigma(\vec{p})v_1^\sigma(\vec{p}) = -\overline{u}_2^\sigma(\vec{p})u_2^\sigma(\vec{p}) = -\overline{v}_2^\sigma(\vec{p})v_2^\sigma(\vec{p}) = 1$, what corresponds to a normalization of the "Pauli spinors" to $\xi_\sigma^* \xi_{\sigma'} = \delta_{\sigma\sigma'}$.

Using the plane-wave expansion, *e.g.*, ref. [1a,formula (8)] or Eqs. (26,27) of the present paper, it is easy to convince ourselves that both $u_i^\sigma$ and $v_i^\sigma$ satisfy the equations

$$\left[-\gamma_{44}E^2 + 2iE\gamma_{4i}\vec{p}_i + \gamma_{ij}\vec{p}_i\vec{p}_j + m^2\right] u_1^\sigma(\vec{p}) = 0 \quad (\text{or } v_1^\sigma(\vec{p})) \quad , \tag{15}$$

and

$$\left[-\gamma_{44}E^2 + 2iE\gamma_{4i}\vec{p}_i + \gamma_{ij}\vec{p}_i\vec{p}_j - m^2\right] u_2^\sigma(\vec{p}) = 0 \quad (\text{or } v_2^\sigma(\vec{p})) \quad , \tag{16}$$

respectively.

Bispinors of Ahluwalia *et al.*, ref. [2], can be written in a more compact form:

$$u_{AJG}^\sigma(\vec{p}) = \begin{pmatrix} \left[m + \frac{(\vec{J}\vec{p})^2}{E+m}\right] \xi_\sigma \\ (\vec{J}\vec{p})\xi_\sigma \end{pmatrix} \quad , \quad v_{AJG}^\sigma(\vec{p}) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} u_{AJG}^\sigma(\vec{p}) \quad . \tag{17}$$

They coincide with the Hammer-Tucker-Novozhilov bispinors within a normalization and an unitary transformation by $\mathcal{U}$ matrix:

$$u_{AJG}^\sigma(\vec{p}) = m \quad \cdot \mathcal{U} u_1^\sigma(\vec{p}) = \frac{m}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} u_1^\sigma(\vec{p}) \quad , \tag{18}$$

$$v_{AJG}^\sigma(\vec{p}) = m \quad \cdot \mathcal{U} v_2^\sigma(\vec{p}) = \frac{m}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} v_2^\sigma(\vec{p}) \quad . \tag{19}$$

In the case of a choice $u_1^\sigma$ and $v_2^\sigma$ as physical bispinors[8] we come to the Bargmann-Wightman-Wigner-type (BWW) quantum field model proposed by Ahluwalia *et al.* Of course, following to the same logic one can choose $u_2^\sigma$ and $v_1^\sigma$ bispinors and come to the reformulation of the BWW theory. Though in this case parities of a boson and its antiboson are opposite, we have $-1$ for $u$ bispinor and $+1$ for $v$ bispinor, *i.e.* different in the sign from the model of Ahluwalia *et al.*, ref. [1].

Now let me repeat the quantization procedure of ref. [9], however, it will be applied to the Weinberg field. Let me trace the contributions of $\mathcal{L}^{(1)}$ and $\mathcal{L}^{(2)}$ to dynamical invariants separately .

From the definitions [15]:

$$\mathcal{T}_{\mu\nu} = -\sum_i \left\{ \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)} \partial_\nu \phi_i + \partial_\nu \overline{\phi}_i \frac{\partial \mathcal{L}}{\partial(\partial_\mu \overline{\phi}_i)} \right\} + \mathcal{L}\delta_{\mu\nu} \quad , \tag{20}$$

$$P_\mu = \int \mathcal{P}_\mu(x) d^3x = -i \int \mathcal{T}_{4\mu} d^3x \tag{21}$$

one can find the energy-momentum tensor

$$\begin{aligned}\mathcal{T}_{\mu\nu}^{(1)} &= \partial_\alpha \overline{\psi}_1 \gamma_{\alpha\mu} \partial_\nu \psi_1 + \partial_\nu \overline{\psi}_1 \gamma_{\mu\alpha} \partial_\alpha \psi_1 + \\ &+ \partial_\alpha \overline{\psi}_2 \gamma_{\alpha\mu} \partial_\nu \psi_2 + \partial_\nu \overline{\psi}_2 \gamma_{\mu\alpha} \partial_\alpha \psi_2 + \mathcal{L}^{(1)} \delta_{\mu\nu} \quad ,\end{aligned} \tag{22}$$

and

$$\begin{aligned}\mathcal{T}_{\mu\nu}^{(2)} &= \partial_\alpha \overline{\psi}_1 \gamma_{\alpha\mu} \partial_\nu \psi_2 + \partial_\nu \overline{\psi}_1 \gamma_{\mu\alpha} \partial_\alpha \psi_2 + \\ &+ \partial_\alpha \overline{\psi}_2 \gamma_{\alpha\mu} \partial_\nu \psi_1 + \partial_\nu \overline{\psi}_2 \gamma_{\mu\alpha} \partial_\alpha \psi_1 + \mathcal{L}^{(2)} \delta_{\mu\nu} \quad .\end{aligned} \tag{23}$$

As a result the first part of the Hamiltonian is[9]

$$\begin{aligned}\mathcal{H}^{(1)} &= \int \left[ -\partial_4 \overline{\psi}_2 \gamma_{44} \partial_4 \psi_2 + \partial_i \overline{\psi}_2 \gamma_{ij} \partial_j \psi_2 - \right. \\ &- \left. \partial_4 \overline{\psi}_1 \gamma_{44} \partial_4 \psi_1 + \partial_i \overline{\psi}_1 \gamma_{ij} \partial_j \psi_1 + m^2 \overline{\psi}_1 \psi_1 - m^2 \overline{\psi}_2 \psi_2 \right] d^3x \quad ,\end{aligned} \tag{24}$$

and the second part is

$$\begin{aligned}\mathcal{H}^{(2)} &= \int \left[ -\partial_4 \overline{\psi}_2 \gamma_{44} \partial_4 \psi_1 + \partial_i \overline{\psi}_2 \gamma_{ij} \partial_j \psi_1 - \right. \\ &- \left. \partial_4 \overline{\psi}_1 \gamma_{44} \partial_4 \psi_2 + \partial_i \overline{\psi}_1 \gamma_{ij} \partial_j \psi_2 \right] d^3x \quad .\end{aligned} \tag{25}$$

[8] I don't agree with the claim of the authors of ref. [1a,footnote 4] which states $v_1^\sigma(\vec{p})$ are not solutions of the equation (10). The origin of the possibility that the $u_i$- and $v_i$- bispinors in Eqs. (15,16) coincide each other (see Eqs. (13,14)) is the following: the Weinberg equations are of the second order in time derivatives. The detailed analysis will be presented in future publications.

In the meantime, I agree that it is more convenient to work with bispinors normalized to the mass, *e.g.*, $\pm m^{2j}$. In the following I keep the normalization of bispinors as in ref. [1].

[9] The Hamiltonian can also be obtained from the second order Lagrangian presented in [1b,Eq. (18)] by means of the procedure developed by M. V. Ostrogradsky [19] long ago (see also Weinberg's remark on the page B1325 of the first paper [11]). However, it would be difficult me to agree with the definition of momentum conjugate operators in the paper [1a]. The Ostrogradsky's procedure seems not to have been applied there to obtain momentum conjugate operators.

Using the plane-wave expansion

$$\psi_1(x) = \sum_\sigma \int \frac{d^3p}{(2\pi)^3} \frac{1}{m\sqrt{2E_p}} \left[u_1^\sigma(\vec p) a_\sigma(\vec p) e^{ipx} + v_1^\sigma(\vec p) b_\sigma^\dagger(\vec p) e^{-ipx}\right] \quad , \tag{26}$$

$$\psi_2(x) = \sum_\sigma \int \frac{d^3p}{(2\pi)^3} \frac{1}{m\sqrt{2E_p}} \left[u_2^\sigma(\vec p) c_\sigma(\vec p) e^{ipx} + v_2^\sigma(\vec p) d_\sigma^\dagger(\vec p) e^{-ipx}\right] \quad , \tag{27}$$

$E_p = \sqrt{\vec p^{\,2} + m^2}$, one can come to the quantized Hamiltonian

$$\mathcal{H}^{(1)} = \sum_\sigma \int \frac{d^3p}{(2\pi)^3} E_p \left[a_\sigma^\dagger(\vec p) a_\sigma(\vec p) + b_\sigma(\vec p) b_\sigma^\dagger(\vec p) + c_\sigma^\dagger(\vec p) c_\sigma(\vec p) + d_\sigma(\vec p) d_\sigma^\dagger(\vec p)\right] , \tag{28}$$

$$\mathcal{H}^{(2)} = \sum_\sigma \int \frac{d^3p}{(2\pi)^3} E_p \left[a_\sigma^\dagger(\vec p) c_\sigma(\vec p) + b_\sigma(\vec p) d_\sigma^\dagger(\vec p) + c_\sigma^\dagger(\vec p) a_\sigma(\vec p) + d_\sigma(\vec p) b_\sigma^\dagger(\vec p)\right] , \tag{29}$$

following the procedure of, *e.g.*, refs. [21, 22].

Setting up boson commutation relations as follows:

$$\left[a_\sigma(\vec p) + c_\sigma(\vec p), a_{\sigma'}^\dagger(\vec k) + c_{\sigma'}^\dagger(\vec k)\right]_- = (2\pi)^3 \delta_{\sigma\sigma'} \delta(\vec p - \vec k) \quad , \tag{30}$$

$$\left[b_\sigma(\vec p) + d_\sigma(\vec p), b_{\sigma'}^\dagger(\vec k) + d_{\sigma'}^\dagger(\vec k)\right]_- = (2\pi)^3 \delta_{\sigma\sigma'} \delta(\vec p - \vec k) \quad , \tag{31}$$

it is easy to see that the Hamiltonian is positive-definite and the translational invariance still keeps in the framework of this description (*cf.* with ref. [1]). Please pay attention here: there is no indefinite metric involved.

Analogously, from the definitions

$$\mathcal{J}_\mu = -i \sum_i \left\{ \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)} \phi_i - \overline{\phi}_i \frac{\partial \mathcal{L}}{\partial(\partial_\mu \overline{\phi}_i)} \right\} \quad , \tag{32}$$

$$Q = -i \int \mathcal{J}_4(x) d^3x \quad , \tag{33}$$

and

$$\mathcal{M}_{\mu\nu,\lambda} = x_\mu \mathcal{T}_{\lambda\nu} - x_\nu \mathcal{T}_{\lambda\mu} - i \sum_i \left\{ \frac{\partial \mathcal{L}}{\partial(\partial_\lambda \phi_i)} N_{\mu\nu}^{\phi_i} \phi_i + \overline{\phi}_i N_{\mu\nu}^{\overline{\phi}_i} \frac{\partial \mathcal{L}}{\partial(\partial_\lambda \overline{\phi}_i)} \right\} \quad , \tag{34}$$

$$M_{\mu\nu} = -i \int \mathcal{M}_{\mu\nu,4}(x) d^3x \quad , \tag{35}$$

I found the current operator

$$\mathcal{J}_\mu^{(1)} = i \left[\partial_\alpha \overline{\psi}_1 \gamma_{\alpha\mu} \psi_1 - \overline{\psi}_1 \gamma_{\mu\alpha} \partial_\alpha \psi_1 + \partial_\alpha \overline{\psi}_2 \gamma_{\alpha\mu} \psi_2 - \overline{\psi}_2 \gamma_{\mu\alpha} \partial_\alpha \psi_2\right] \quad , \tag{36}$$

and

$$\mathcal{J}_{\mu}^{(2)} = i\left[\partial_{\alpha}\overline{\psi}_2\gamma_{\alpha\mu}\psi_1 - \overline{\psi}_2\gamma_{\mu\alpha}\partial_{\alpha}\psi_1 + \right.$$
$$\left. + \quad \partial_{\alpha}\overline{\psi}_1\gamma_{\alpha\mu}\psi_2 - \overline{\psi}_1\gamma_{\mu\alpha}\partial_{\alpha}\psi_2\right] \quad . \qquad (37)$$

Using (34) the spin momentum tensor reads

$$S_{\mu\nu,\lambda}^{(1)} = i\left[\partial_{\alpha}\overline{\psi}_1\gamma_{\alpha\lambda}N_{\mu\nu}^{\psi_1}\psi_1 + \overline{\psi}_1 N_{\mu\nu}^{\overline{\psi}_1}\gamma_{\lambda\alpha}\partial_{\alpha}\psi_1 + \right.$$
$$\left. + \quad \partial_{\alpha}\overline{\psi}_2\gamma_{\alpha\lambda}N_{\mu\nu}^{\psi_2}\psi_2 + \overline{\psi}_2 N_{\mu\nu}^{\overline{\psi}_2}\gamma_{\lambda\alpha}\partial_{\alpha}\psi_2\right] \quad , \qquad (38)$$

$$S_{\mu\nu,\lambda}^{(2)} = i\left[\partial_{\alpha}\overline{\psi}_2\gamma_{\alpha\lambda}N_{\mu\nu}^{\psi_1}\psi_1 + \overline{\psi}_2 N_{\mu\nu}^{\overline{\psi}_2}\gamma_{\lambda\alpha}\partial_{\alpha}\psi_1 + \right.$$
$$\left. + \quad \partial_{\alpha}\overline{\psi}_1\gamma_{\alpha\lambda}N_{\mu\nu}^{\psi_2}\psi_2 + \overline{\psi}_1 N_{\mu\nu}^{\overline{\psi}_1}\gamma_{\lambda\alpha}\partial_{\alpha}\psi_2\right] \quad . \qquad (39)$$

If the Lorentz group generators (a $j = 1$ case) are defined from

$$\overline{\Lambda}\gamma_{\mu\nu}\Lambda a_{\mu\alpha}a_{\nu\beta} = \gamma_{\alpha\beta} \quad , \qquad (40)$$
$$\overline{\Lambda}\Lambda = 1 \quad , \qquad (41)$$
$$\overline{\Lambda} = \gamma_{44}\Lambda^{\dagger}\gamma_{44} \quad . \qquad (42)$$

then in order to keep the Lorentz covariance of the Weinberg equations and of the Lagrangian (9) one can use the following generators:

$$N_{\mu\nu}^{\psi_1,\psi_2(j=1)} = -N_{\mu\nu}^{\overline{\psi}_1,\overline{\psi}_2(j=1)} = \frac{1}{6}\gamma_{5,\mu\nu} \quad , \qquad (43)$$

see also ref. [16b, Eqs. (37,51,52)]. The matrix $\gamma_{5,\mu\nu} = i\left[\gamma_{\mu\lambda},\gamma_{\nu\lambda}\right]_-$ is defined to be Hermitian.

The quantized charge operator and the quantized spin operator follow immediately from (36,37) and (38):

$$Q^{(1)} = \sum_{\sigma}\int\frac{d^3p}{(2\pi)^3}\left[a_{\sigma}^{\dagger}(\vec{p})a_{\sigma}(\vec{p}) - b_{\sigma}(\vec{p})b_{\sigma}^{\dagger}(\vec{p}) + c_{\sigma}^{\dagger}(\vec{p})c_{\sigma}(\vec{p}) - d_{\sigma}(\vec{p})d_{\sigma}^{\dagger}(\vec{p})\right] , \qquad (44)$$

$$Q^{(2)} = \sum_{\sigma}\int\frac{d^3p}{(2\pi)^3}\left[a_{\sigma}^{\dagger}(\vec{p})c_{\sigma}(\vec{p}) - b_{\sigma}(\vec{p})d_{\sigma}^{\dagger}(\vec{p}) + c_{\sigma}^{\dagger}(\vec{p})a_{\sigma}(\vec{p}) - d_{\sigma}(\vec{p})b_{\sigma}^{\dagger}(\vec{p})\right] , \qquad (45)$$

$$(W^{(1)}\cdot n) = \sum_{\sigma\sigma'}\int\frac{d^3p}{(2\pi)^3}\frac{1}{m^2E_p}\overline{u}_1^{\sigma}(\vec{p})(E_p\gamma_{44} - i\gamma_{4i}p_i)\; I\otimes(\vec{J}\vec{n})\, u_1^{\sigma'}(\vec{p})\times$$
$$\times\left[a_{\sigma}^{\dagger}(\vec{p})a_{\sigma'}(\vec{p}) + c_{\sigma}^{\dagger}(\vec{p})c_{\sigma'}(\vec{p}) - b_{\sigma}(\vec{p})b_{\sigma'}^{\dagger}(\vec{p}) - d_{\sigma}(\vec{p})d_{\sigma'}^{\dagger}(\vec{p})\right] \qquad (46)$$

(provided that the frame is chosen in such a way that $\vec{n}\parallel\vec{p}$ is along the third axis). It is easy to verify the eigenvalues of the charge operator[10] are $\pm 1$, and of the Pauli-Lubanski spin operator are

$$\xi_{\sigma}^{*}(\vec{J}\vec{n})\xi_{\sigma'} = +1,\ 0\ -1 \qquad (47)$$

[10]In order to construct neutral particle operators one can use an analogy with a $j = 1/2$ case (to compare electron and neutrino field operators). At the present moment I would like again draw your attention to the fact that $u_i^{(j)}$ and $v_i^{(j)}$ coincide in the model studied here.

in a massive case and $\pm 1$ in a massless case (see the discussion on the massless limit of the Weinberg bispinors in ref. [2]).

As for the spin operator which follows from the Lagrangian (12) the situation is more difficult. If accept (40)-(42) we are not able to obtain the helicity operator (47) in the final expression. However, a cure is possible. We should take into account that the transformations

$$\overline{\Lambda}\gamma_{\mu\nu}\Lambda a_{\mu\alpha}a_{\nu\beta} = \mp\gamma_{\alpha\beta}\gamma_5 \tag{48}$$
$$\overline{\Lambda}\Lambda = \pm\gamma_5 \quad , \tag{49}$$

remain Weinberg's set of equations to be invariant. Equations are only interchanged each other. This is a cause of the possibility of combining the Lorentz and the dual (chiral) transformation for the Weinberg degenerate doublet. Thus, in order to keep the Lorentz and parity covariance of the Lagrangian of the form (12) one can impose

$$N_{\mu\nu}^{\psi_1,\psi_2(j=1)} = -N_{\mu\nu}^{\overline{\psi}_1,\overline{\psi}_2(j=1)} = \frac{1}{6}\gamma_{5,\mu\nu}\gamma_5 \quad . \tag{50}$$

After simple transformations I obtain

$$\begin{aligned}(W^{(2)}\cdot n) &= \sum_{\sigma\sigma'}\int\frac{d^3p}{(2\pi)^3}\xi^*_\sigma(\vec{J}\vec{n})\xi_{\sigma'}\times\\ &\times\left[c^\dagger_\sigma(\vec{p})a_{\sigma'}(\vec{p})+a^\dagger_\sigma(\vec{p})c_{\sigma'}(\vec{p})-b_\sigma(\vec{p})d^\dagger_{\sigma'}(\vec{p})-d_\sigma(\vec{p})b^\dagger_{\sigma'}(\vec{p})\right] \quad .\end{aligned} \tag{51}$$

I leave investigations of other possibilities for further publications.

Why "a queer reduction of degrees of freedom" did happen in the previous papers [5, 6, 9, 10]? The origin of this surprising fact follows from Hayashi's (1973) paper, ref. [9, p.498]: The requirement of *"that the physical realizable state satisfies a quantal version of the generalized Lorentz condition"*, formulas (18) of ref. [9],[11] permits one to eliminate upper (or down) part of the Weinberg bispinor and to remove transversal components of the remained part by means of the "gauge" transformation (7), what *"ensures the massless skew-symmetric field is longitudinal"*. The reader can convince himself of this obvious fact by looking at the explicit form of the Pauli-Lubanski operator, Eq. (46). Keeping all terms in field operators and in the Lagrangian (3), *cf.* with [5, 6], and *not* applying the generalized Lorentz condition, *cf.* with [9, 10], we are able to obtain transversal components, *i.e.*, a $j = 1$ particle in the massless limit of the Weinberg theory. The presented version does *not* contradict with the Weinberg theorem nor with the classical limit, Eqs. (21,22) of ref. [4]. Thanks to the mapping [3e,4] the conclusion is valid for both the Weinberg $2(2j+1)$ component "bispinor" and the antisymmetric (skew-symmetric) tensor field.

[11] Read: "a quantal version" of the Maxwell equations imposed on state vectors in the Fock space. See the papers of Ahluwalia *et al.*, *e.g.*, ref. [23, Table 2], for the discussion on the acausal physical dispersion of the equations (4.19) and (4.20) of ref. [11b], "which *are just Maxwell's free-space equations for left- and right- circularly polarized radiation."* See also the footnote # 1 in ref. [4] and ref. [3g]. Let me mention that this fact is probably connected with the indefinite metric problem.

Finally, for the sake of completeness let me re-write the Lagrangians presented above in the form:

$$\mathcal{L}_1 = -\partial_\mu \overline{\Psi} \Gamma_{\mu\nu} \partial_\nu \Psi - m^2 \overline{\Psi} \Psi \quad , \tag{52}$$

where

$$\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \quad , \quad \overline{\Psi} = ( \psi_1^\dagger \quad \psi_2^\dagger ) \cdot \begin{pmatrix} \gamma_{44} & 0 \\ 0 & -\gamma_{44} \end{pmatrix} \tag{53}$$

are the wave function of the degenerate doublet;

$$\Gamma_{\mu\nu} = \begin{pmatrix} \gamma_{\mu\nu} & 0 \\ 0 & -\gamma_{\mu\nu} \end{pmatrix} \quad . \tag{54}$$

The Lagrangian $\mathcal{L}_2$ could be written in a similar form if imply

$$\overline{\Psi} = ( \psi_1^\dagger \quad \psi_2^\dagger ) \cdot \begin{pmatrix} 0 & -\gamma_{44} \\ \gamma_{44} & 0 \end{pmatrix} \quad . \tag{55}$$

I take the liberty to name the field operator $\overline{\Psi}$ as the Weinberg-conjugated *dibispinor*.

My conclusions are: There exist the versions of both the Weinberg $2(2j+1)$ component theory and the antisymmetric tensor field formalism that answer for particles with transversal components. Thus, these versions do not contradict the Weinberg theorem and, in the case of $j = 1$ particle, the doublet field $(\psi_1, \quad \psi_2)$, or $(F_{\mu\nu}, \quad \tilde{F}_{\mu\nu})$, could be used in describing a photon. The origin of the contradictions met in the previous papers is the inadequate use of the generalized Lorentz condition which may be incompatible (in such a form) with the specific properties of antisymmetric tensor fields. The connection of the present model with the Bargmann-Wightman-Wigner-type quantum field theories deserves further elaboration.

As a matter of fact the present model develops Weinberg's and Ahluwalia's ideas of the Dirac-like description of bosons on an equal footing with fermions, *i.e.*, on the basis of the $(j, 0) \oplus (0, j)$ representation of the Lorentz group.

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