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Autor(en): **Straub, Erwin**

Objekttyp: **Article**

Zeitschrift: **Mitteilungen / Vereinigung Schweizerischer Versicherungsmathematiker = Bulletin / Association des Actuaires Suisses = Bulletin / Association of Swiss Actuaries**

Band (Jahr): **75 (1975)**

PDF erstellt am: **27.05.2024**

Persistenter Link: https://doi.org/10.5169/seals-967117

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Actuarial Remarks on the Insurance of Natural Hazards

By Erwin Straub, Zurich

1. Preliminaries

I confine myself to mentioning a few mathematical formulae. Proofs are omitted since all results are known from other actuarial areas. This note has served as mathematical skeleton for both my conference at the 1975 General Assembly of the Swiss Actuaries and the UNESCO-paper on "Insurance and the Economic Consequences of Earthquakes", jointly written with P. Perrenoud.

2. Basic Information

We divide the portfolio into earthquake zones or rather what we call *concentration areas* numbered from $j = 1$ to $j = J$.
By E_j we denote the total commitment *or exposure* of the company in area j.
Modified Mercalli *intensities* of earthquake shocks are then numbered from $i = 1$ to $i = I$.
The average number λ_{ij} of shocks of intensity i in area j is called *frequency*.
Similarly, we denote the average damage degree y_{ij} (expressed as a percentage of E_j) by *severity*.
Thus the minimum data forming the *frame* of the mathematical model described in the sequel consists of a set of *exposures*, *frequencies* and *severities*:

$$E_j, \lambda_{ij} \text{ and } y_{ij} \text{ for } i = 1 \text{ to } I \text{ and } j = 1 \text{ to } J.$$

3. Notations and Assumptions

$$\left.\begin{array}{l} K_{ij} = \text{number of shocks per year} \\ \left.\begin{array}{l} X_{ij} = \text{damage amount} \\ \eta_{ij} = X_{ij}/E_j = \text{damage degree} \end{array}\right\} \text{of a shock} \end{array}\right\} \begin{array}{l} \text{of intensity } i \text{ at concentration} \\ \text{area no. } j \end{array}$$

K_{ij} assumed to be Poisson with parameter λ_{ij}

η_{ij} assumed to be Beta with parameters $\alpha + \beta$ = constant

$$\text{and } \frac{\alpha}{\alpha+\beta} = E[\eta_{ij}] = y_{ij}.$$

Consequently $V_{ij}(x) = \text{Prob}\,[X_{ij} \leq x] = B\left(\frac{x}{E_j}\right)$

where $B(x)$ is the Beta-distribution of η_{ij}.

Further for the yearly total of claims Z_{ij}:

$$F_{ij}(x) = \text{Prob}\left[Z_{ij} = \sum_{k=1}^{K_{ij}} X_{ij,\,k} \leq x\right] = \sum_{n=0}^{\infty} \frac{\lambda_{ij}^{n}}{n!} e^{-\lambda_{ij}} V_{ij}^{(n)}(x)$$

where $X_{ij,\,k}$ stands for the *k-th* catastrophe claim.

The K_{ij}, $X_{ij,\,k}$ and thus also the Z_{ij} are assumed to be stochastically independent.

4. Total of Claims per Concentration Area and Overall

Considering

$Z_{\cdot j} = \sum_{i=1}^{I} Z_{ij}$ = total of claims in area j resulting from shocks of *all* intensity degrees

and

$Z_{\cdot\cdot} = \sum_{j=1}^{J} Z_{\cdot j} = \sum_{i,\,j} Z_{ij}$ = total of claims of the entire portfolio

leads to the following distributions

$$F_{\cdot j}(x) = \text{Prob}[Z_{\cdot j} \leq x] = \sum_{n=0}^{\infty} \frac{\lambda_{\cdot j}^{n}}{n!} e^{-\lambda_{\cdot j}} V_{\cdot j}^{(n)}(x)$$

$$\text{with } \lambda_{\cdot j} = \sum_{i=1}^{I} \lambda_{ij} \text{ and } V_{\cdot j}(x) = \sum_{i=1}^{I} \frac{\lambda_{ij}}{\lambda_{\cdot j}} V_{ij}(x)$$

and

$$F_{\cdot\cdot}(x) = \text{Prob}[Z_{\cdot\cdot} \leq x] = \sum_{n=0}^{\infty} \frac{\lambda_{\cdot\cdot}^{n}}{n!} e^{-\lambda_{\cdot\cdot}} V_{\cdot\cdot}^{(n)}(x)$$

$$\text{with } \lambda_{\cdot\cdot} = \sum_{j=1}^{J} \lambda_{\cdot j} = \sum_{i,\,j} \lambda_{ij}$$

$$\text{and } V_{\cdot\cdot}(x) = \sum_{j=1}^{N} \frac{\lambda_{\cdot j}}{\lambda_{\cdot\cdot}} V_{\cdot j}(x) = \sum_{i,\,j} \frac{\lambda_{ij}}{\lambda_{\cdot\cdot}} V_{ij}(x)$$

Expressing moments of $Z_{.j}$ and $Z_{..}$ in terms of the key figures exposure, frequency and severity yields

$$E[Z_{.j}] = \lambda_{.j}\, y_{.j}\, E_j \text{ and } \mathrm{Var}[Z_{.j}] = \lambda_{.j}\, c_j\, E_j^2$$

and

$$E[Z_{..}] = \sum_{j=1}^{J} \lambda_{.j}\, y_{.j}\, E_j, \mathrm{Var}[Z_{..}] = \sum_{j=1}^{J} \lambda_{.j}\, c_{.j}\, E_j^2$$

$$\text{where } y_{.j} = \sum_{i=1}^{I} \frac{\lambda_{ij}}{\lambda_{.j}} y_{ij}, \quad c_{.j} = \sum_{i=1}^{I} \frac{\lambda_{ij}}{\lambda_{.j}} c_{ij}$$

$$\text{with } y_{ij} = E[\eta_{ij}], c_{ij} = E[\eta_{ij}^2].$$

In the sequel we shall omit subscripts whenever possible, thus e. g. K may denote either K_{ij} or $K_{.j}$ or $K_{..}$

5. Maximum Probable Losses (MPL)

If X_m m-th individual catastrophe claim and K the number of claims, then

$$H(x) = \mathrm{Prob}[\max_{1 \leq m \leq K} X_m \leq x] = \sum_{n=0}^{\infty} \frac{\gamma^n}{n!} e^{-\gamma}\, V^n(x) = e^{-\gamma(1-V(x))}$$

which is the largest claim distribution having the nice property that

$$H_{.j}(x) = \prod_{i=1}^{I} H_{ij}(x) \text{ and } H_{..}(x) = \prod_{j=1}^{J} H_{.j}(x) = \prod_{i,j} H_{ij}(x).$$

With this we define MPLs as follows:
For a given ε (e.g. $\varepsilon = 1\%$) the maximum probable loss M is $M = x_\varepsilon$ where x_ε is the solution of $H(x) = 1 - \varepsilon$.

Note that $M_{.j} \geq \max_{1 \leq i \leq I} M_{ij}$

and $M_{..} \geq \max_{1 \leq j \leq J} M_{.j} \geq \max_{i,j} M_{ij}$.

6. Ruin Probabilities

Ruin probabilities are used as stability criterion when calculating premiums, maximum capacities, size of catastrophe fund and reinsurance corerage. As a first approximation we take the individual claims amount to be exponentially distributed, thus

$$\varphi(U,\pi) = \frac{\lambda\mu}{\pi} e^{-U\left(1-\frac{\lambda\mu}{\pi}\right)}$$

where

$\varphi(U,\pi)$ = ruin probability for an infinite planning horizon,
U = initial reserve (= catastrophe fund),
π = yearly premium income,
γ = mean of yearly number of claims,
μ = mean of individual catastrophe claim.

Therefore looking at the entire portfolio, we have to put

$$\lambda\mu = E[Z..] = \sum_{j=1}^{J} \lambda._j \, y._j \, E_j.$$

From this we can e.g. calculate maximum capacities E_j provided the rest of the parameters – the tolerable ruin probability in particular – are given.

A more accurate way is to replace the exponential by a Gamma distribution

$$G_n(x) = \frac{\gamma^n}{\Gamma(n)} \int_0^x z^{n-1} \, e^{-\gamma z} \, dz$$

for which

$$\frac{n}{\gamma} = E[Z..] \text{ and } \frac{n+n^2}{\gamma^2} = \frac{1}{\lambda} \operatorname{Var}[Z..].$$

This amounts to using a compound Poisson distribution having the same first *two* moments as the «right» one.

7. Excess of Loss Reinsurance

The basic considerations for the calculation of reinsurance premiums, catastrophe reserves and MPLs are the same as for direct insurance, however, the original variables have to be transformed according to the treaty conditions, i.e.:

$$X_{ij} \to X^*_{ij} = \begin{cases} 0 & \text{if} \quad X_{ij} \le R_j \\ X_{ij} - R_j & \text{if} \quad R_j < X_{ij} \le R_j + D_j \\ D_j & \text{if} \quad R_j + D_j \le X_{ij} \end{cases}$$

where

X_{ij} = original individual catastrophe claim,
X^*_{ij} = individual excess claim,
R_j = retention
D_j = cover amount
} for area no. j under an excess of loss treaty.

The key figures are also transformed of course, namely to

$$E^*_j = D_j$$

$$\lambda^*_{ij} = \lambda_{ij}\{1 - V_{ij}(R_j)\}$$

$$y^*_{ij} = \frac{1}{D_j\{1 - V_{ij}(R_j)\}} \left\{ \int_0^{D_j} V_{ij}(x + R_j) x dx + D_j \frac{1 - V(R_j + D_j)}{1 - V(R_j)} \right\}.$$

8. Final Remarks

The model can also be used for perils other than earthquake; for Australia, e.g. we replaced intensities by natural hazards putting $i = 1$ = bushfire, $i = 2$ = windstorm, $i = 3$ = flood and $i = 4$ = earthquake.
All calculations are performed on a Hewlett Packard desk computer in a few minutes. I would like, by the way, to thank Ron Grünig for his assistance in programming.