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Strickler formula, a Swiss contribution to hydraulics

A short note on the 100th anniversary of Strickler's birth

Daniel Vischer

Albert Strickler was born in Wädenswil near Zurich on July 25, 1887, and died in Küsnacht near Zurich on February 1, 1963. I have not known him personally; indeed I never met him. But in 1983 I paid a visit to his widow in Küsnacht to get a photo of him *(figure 1)* and to ask a few questions. As a souvenir of him she gave me two things: his logarithmic table and his slide rule, both much used and worn.

Fig. 1. Albert Strickler, 1887–1963.

And then it struck me: A logarithmic table is useful to deal with products and powers; a slide rule is based on logarithm as well and is therefore also a tool to deal with products and powers, but it is normally confined to exponents of 2, 3 and thus ½, ⅓ and ⅔. Therefore, Strickler's formula, which reads

$$v = k R^{2/3} S^{1/2} \qquad \text{(Eq 1)}$$

with

v = mean flow velocity,
k = roughness coefficient (Strickler coefficient),
R = hydraulic radius,
S = slope,

can easily be handled with a slide rule, or what is more, it was probably adapted to the capabilities of a slide rule: *a typical slide rule formula.*

Two or three years later Strickler's widow sent me her only copy of her late husband's booklet "Beiträge zur Frage der Geschwindigkeitsformel und der Rauhigkeitszahlen für Ströme, Kanäle und geschlossene Leitungen" (Contribution to the problem of the flow formula and of roughness coefficients for rivers, channels and conduits), published in 1923 [1].

So I feel indebted to her and in a way bound to write this short note. As an opportunity I gladly choose the coincidence of Strickler's 100th anniversary with the XXII Congress of the International Association of Hydraulic Research (IAHR), held in Lausanne, Switzerland (see also [2]).

Power type formulae versus other formulae

Strickler's booklet comprises 48 pages of text with an important appendix of 14 tables and 39 plates. It is a masterpiece of clear and concise treatise writing. The first chapter consists of an enumeration of the then known flow formulae for turbulent flow. Strickler reproduces 34 of them but states that he could easily add another set of 34 or more. But what is perhaps more important, he arranges these formulae in four classes:

a) power type formulae for open channels
b) other formulae for open channels
c) power type formulae for conduits, and
d) other formulae for conduits.

All the power type formulae can be written in the form

$$v = C R^a S^b \qquad \text{(Eq 2)}$$

with

C = roughness coefficient
a, b = exponents.

The authors of such formulae, quoted by Strickler, are given in the *tables I and II* together with the exponents they suggest.

In contrast to these formulae the other formulae contain more than only factors with exponents. But it is difficult to generalize them with a simple expression. Thus, we reproduce here as an example another Swiss contribution to the field, namely the formula of *Ganguillet* and *Kutter* for open channel flow. It was published in 1869, that is more than 50 years before Strickler's booklet was published, and reads:

$$v = \frac{23 + \frac{1}{n} + \frac{0.00155}{s}}{1 + (23 + \frac{0.00155}{S}) \frac{n}{R^{1/2}}} R^{1/2} \; S^{1/2} \qquad \text{(Eq 3)}$$

with n = degree of roughness.

This somewhat cumbersome expression was very popular in its time. According to [3] the demand for copies of the first

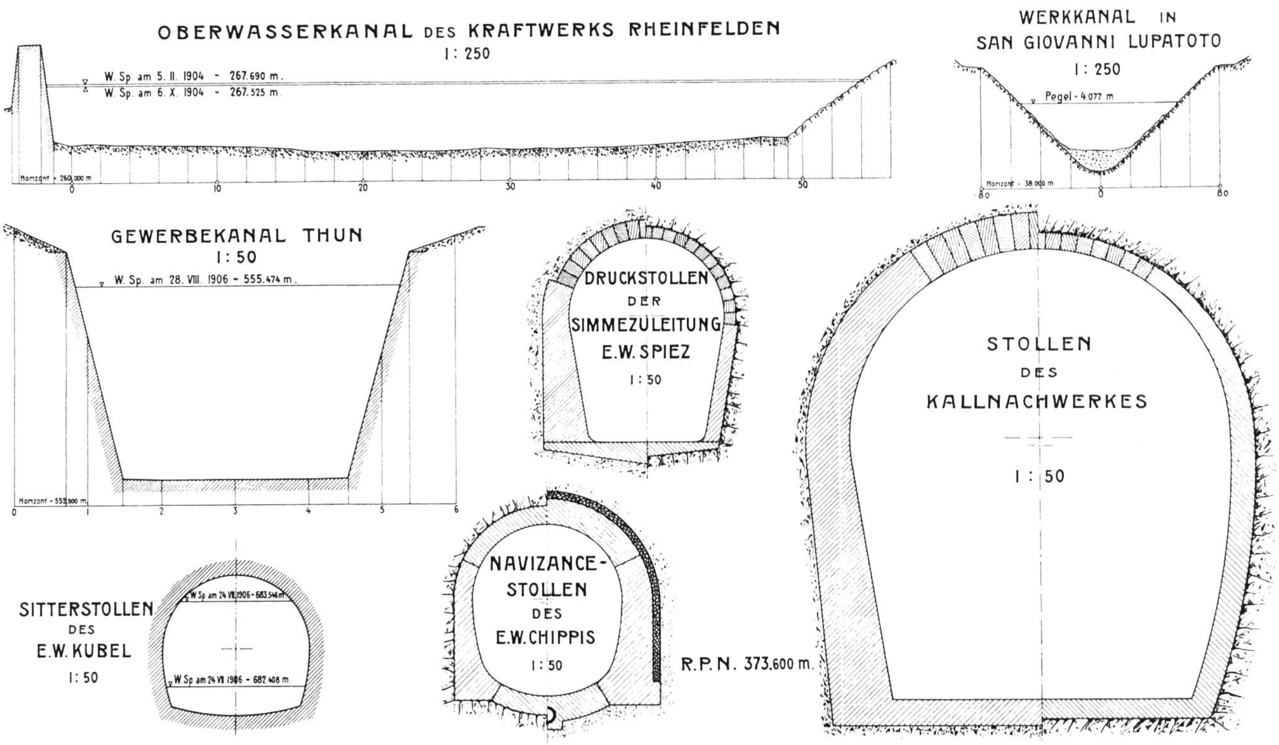


Figure 2. "Channels and tunnels". Plate 3 of Strickler's treatise [1]: Some of the free flow and pressure conduits investigated by the Swiss Federal Office of Water Resources.

paper of *E. Ganguillet* (1818–1899) and *W. R. Kutter* (1818–1888) "Versuch zur Aufstellung einer neuen allgemeinen Formel für die gleichförmige Bewegung des Wassers in Canälen und Flüssen" (an attempt to establish a new general formula for the uniform movement of the water in channels and rivers) was so great that the issue of the Austrian journal [4] in which it appeared was soon exhausted. But in 1877 the paper was enlarged and republished as a book, and this was translated into various other languages, e.g. in 1889 in English: "A general formula for the uniform flow of water in rivers and other channels" [4].

Strickler's power type formula

The second chapter in Strickler's booklet is called: The setup of a simple approximation formula for the mean velocity and the review of some existing formulae. Strickler used all the data available of other authors and some additional measurements of the Swiss Federal Office of Water Resources in the range of

0.037 to 7.14 m for R

0.04‰ to 2.5% for S

and of smooth concrete surface to boulders (size of a head) for resistance, to check every parameter. This range clearly shows that he aimed at a practical tool for hydraulic engineers. And it soon becomes apparent that he favoured the power-type formulae certainly – and as already indicated in the introduction – because of his concern for ease of computation.

Now from his treatise it is clear that Strickler was convinced that a flow formula for open channel flow must basically be identical with the formula for conduit flow. This means that the exponents in an appropriate flow formula should be equal for every type of turbulent flow, but the *tables I and II* show how these exponents vary from author to author, and often differ widely between open channel and conduit flow. The overall range is

$0.5 < a < 1$

$0.2 < b < 1$.

Therefore Strickler rearranged all data to find a compromise and finally arrived at formula 1, that is

$a = 2/3$ and $b = 1/2$.

His argument is set out in the treatise with many tables and plates, and in a straightforward and persuasive manner. As a byproduct he established the equally well known formula

$$k = \frac{21.1}{d^{1/6}} \qquad \text{(Eq 4)}$$

with d = size of gravel or boulders.

Table I: Exponents of the power type formula $v = k\,R^a S^b$ for open channels, streams and rivers.

Author	a	b	confined to
Chézy	1/2	1/2	
Lahmeyer	2/3	2/3	straight rivers
de Saint-Venant	11/21	11/21	
Humphrey and *Abbot*	1/2	1/4	
Gauckler	4/3	1	grades less than 7‰
	2/3	1/2	grades more than 7‰
Hagen	1/2	1/5	rivers
	1	1/5	small channels
	2/3	1/2	large channels
Manning	2/3	1/2	channels
Hermanek	1	1/2	0 < river depth < 1.5 m
	3/4	1/2	1.5 < river depth < 6 m
	0.6	1/2	river depth > 6 m
Forchheimer	0.7	0.5	
Beyerhaus	0.7	0.46	

Table II. Exponents of the power type formula $v = k\,R^a S^b$ for conduits.

Author	a	b	confined to
Eytelwein	1/2	1/2	
Woltman	4/7	4/7	
de Saint-Venant	7/12	7/12	
Lampe	0.69	0.55	
Fanning	1/2	1/2	
Christen	0.625	0.5	
Reynolds	0.765	0.59	large pipes
to	0.50	0.50	
Famant, Saph and *Schoder, Blasius*	0.71	0.57	
Tutton	0.66	0.51	cast iron pipes
etc.			

Strickler's generalized formula

Knowing that his power type formula 1 is only an approximation of the physics of turbulent flow, Strickler in the next chapter of his treatise developed a set of more sophisticated velocity and resistance formulae which are less known. For higher velocities in large channels and conduits e.g. he writes

$$v = [k^2 R^{4/3} S + (\frac{\pi \eta k^2}{\varrho g R^{2/3}})^2]^{1/2} - \frac{\pi \eta k^2}{\varrho g R^{2/3}} \qquad \text{(Eq 5)}$$

with η = viscosity of water
ρ = density of water
g = gravitation.

It is stated in the treatise that these formulae are not only valid for water but also for other fluids and even for gases. Further, various comparative computations demonstrate that the results of this formula do not differ much from those of the approximate formula 1 within the range of interest for hydraulic engineers, that is for turbulent water flows in rivers, streams, channels, tunnels and pipes, the latter with a diameter of more than 0.10 m.
The two last chapters of Strickler's book concern formulae for velocity distributions in various channels and conduits and guidelines for the computation of backwater curves.

Strickler versus Gauckler and/or Manning?

Strickler's formula is often called the *Gauckler* formula or the *Manning* formula and sometimes the *Gauckler-Strickler* formula, the *Manning-Strickler* formula or the *Gauckler-Manning* formula. This naming leads of course to questions of priority.

Now, Strickler never claimed any priority for his approximative formula 1. The very way he established it, excludes that. But his merit is to have put forward an expression, which covers almost the whole field of uniform turbulent flow in hydraulic engineering and is widely used.
As can be seen in table I, the Frenchman *Gauckler* (1826–1905) e.g. suggested 1868 two different power type formulae for open channels. But only the one for grades above 7‰ contains the exponents of $a = 2/3$ and $b = 1/2$. For conduits Gauckler published a completely different formula, namely

$$S = c \frac{(v^{1/2} + 0.25\ D\ v^{1/4})^4}{D^{4/3}} \qquad \text{(Eq 6)}$$

with D = pipe diameter.
About the same can be said for the German *Hagen* (1799–1884) who, according to table I, developed three different power type formulae for open channel flow. Only one, which

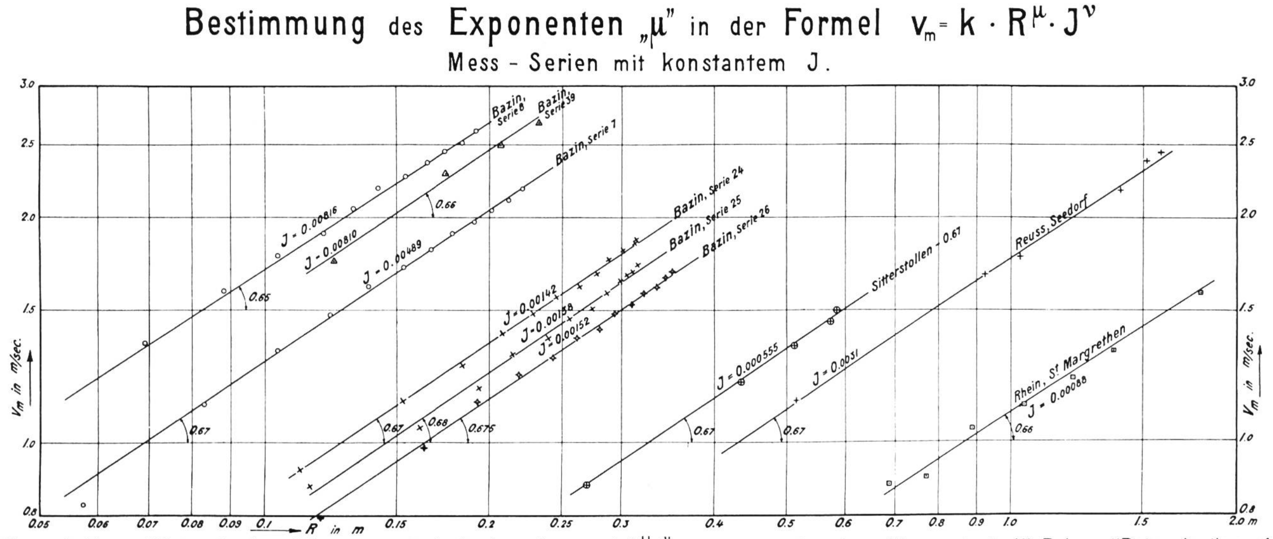
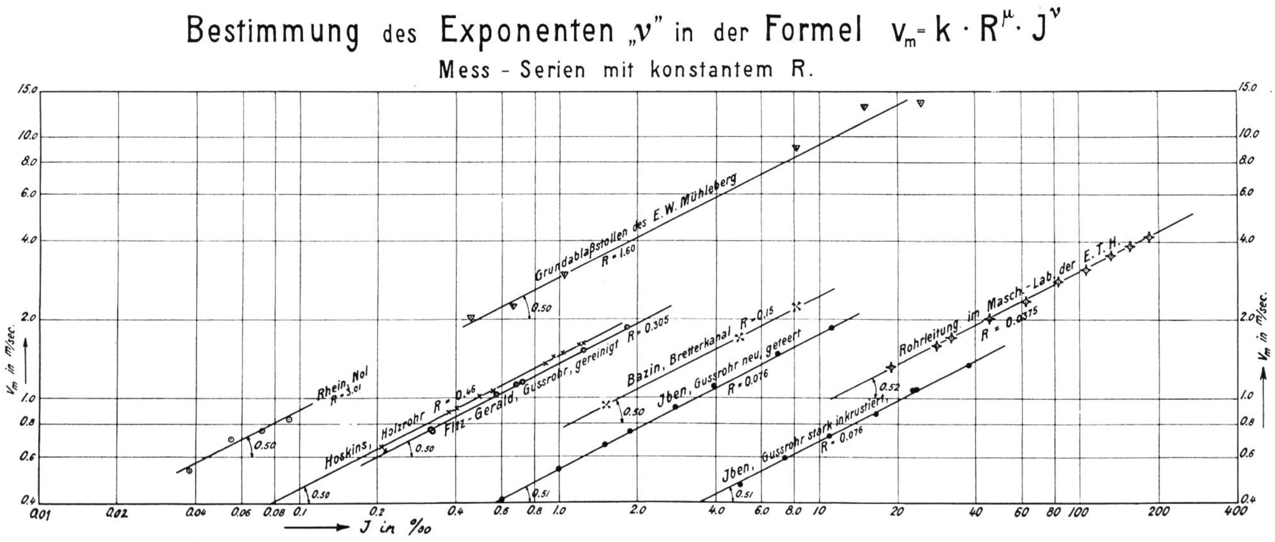


Figure 3. Above: "Determination of the exponent μ in the formula $v_m = k R^{\mu} J^{\nu}$, measurement series with constant J." Below: "Determination of the exponent ν in the formula $v_m = k R^{\mu} J^{\nu}$; measurement series with constant R." Plates 4 and 5 of Strickler's treatise [1] (corresponding notation in the present note: $v_m = v$, $J = S$, $\mu = a$, $\nu = b$).



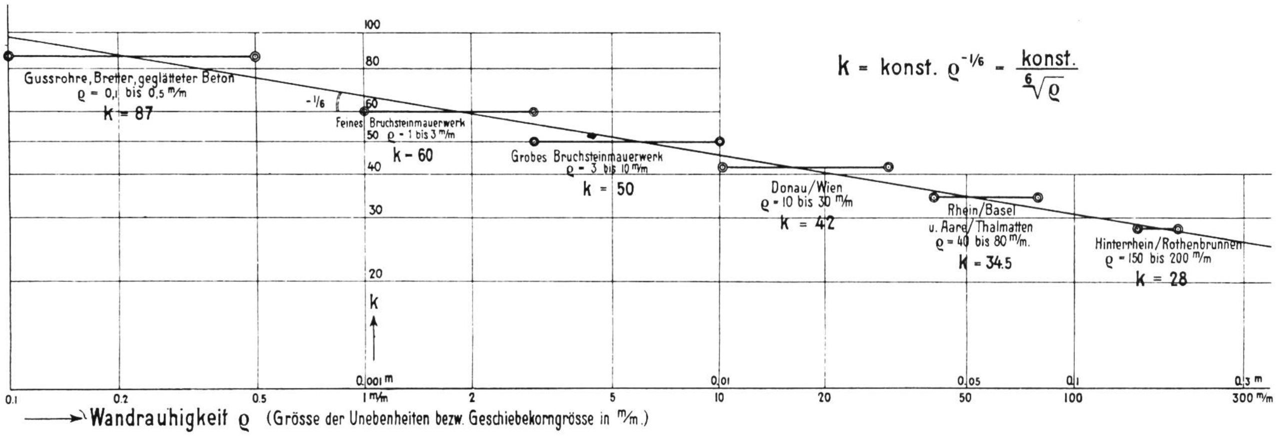


Figure 4. "Relationship between the k-values and the absolute wall roughness." Plate 17 of Strickler's treatise [1]. ρ is the wall roughness that is the "size of the wall uneveness or of the bed load grains" ($\rho = d$).

he derived from data from the Ganges River in 1881, shows the exponents $a = 2/3$ and $b = 1/2$.

The case of *Manning* (1816–1897), an Irish engineer, seems to be different. According to [3] and [5] he published the power type formula 1 in 1891 and stated that it is in better agreement with available data than any other then known formula both for open channels and conduits of nearly any size. He proved this in almost the same way as Strickler did 30 years later. But afterwards he discarded this formula because of theoretical objections as to its form and the tediousness of extracting a cubic root. He obviously did not have an appropriate slide rule. Instead of the power type formula he proposed the dimensionally homogenous relationship.

$$v = C\, g^{1/2}\, S^{1/2}\, [R^{1/2} + \frac{0.22}{m^{1/2}}\,(R - 0.15\, m)] \qquad \text{(Eq 7)}$$

with

C = a number "which varies with the nature of the surface"

m = height of a column of mercury which balances the atmospheric pressure.

A similar attempt of the American *Tutton* in 1896 resulted in the expression

$$v = \frac{1.54}{n}\, R^{2/3}\, S^{1/2} \qquad \text{(Eq 8)}$$

with n = Ganguillet and Kutter's degree of friction and probably led to the current writing of the so-called Manning formula

$$v = \frac{1}{n}\, R^{2/3}\, S^{1/2} \qquad \text{(Eq 9)}$$

with $n = 1/k$ = roughness coefficient, Manning coefficient (inverse to the Strickler coefficent)

Thus, it is of course questionable if formula 1 should really be called the Strickler formula. But it is also unsatisfactory to name it after Manning who rejected it. Should it be called the Tutton formula?

Certainly, Manning is at the origin of the wide spread use of the formula in the English speaking countries. But Strickler is responsible for its great popularity on the European continent, including Russia. It must not be overlooked that in 1923 the German language, the language of Strickler, was – in contrast to now – one of the main scientific languages and thus of great influence. Therefore I am in favour of naming the formula

$$v = k\, R^{2/3}\, S^{1/2} = \frac{1}{n}\, R^{2/3}\, S^{1/2} \qquad \text{(Eq 10)}$$

the *Manning-Strickler formula,* but I am not prepared to argue about it, because as is demonstrated in [5], the story of the formula is even more complicated.

The statement of Strickler's boss

When Strickler published his booklet in 1923, he was head of the Section of Hydro-Power Plants and Navigation in the already mentioned Federal Office of Water Resources in Bern. In fact, his booklet was a contribution of this Office edited by the Federal Department of the Interior. For this reason, his boss, the director *Carl Mutzner* (1885–1966), wrote the preface which is worth quoting at the end of this note:

"In the last decades the number of formulae for the computation of water velocity in rivers, channels and conduits has greatly increased. Therefore, the first objective of the here presented work was merely an investigation of the range of validity of the older and newer formulae, especially of those of a pure power type, which in very recent time have again received more attention from the engineers. It was by no means intended to increase the number of formulae.

But in the course of pursuing this objective, the necessity and the possibility arose to establish a new and universally valid formula, of which the pure power type $v = k R^a S^b$ is – if the exponents a and b are interpreted as constants – only an approximation, with a specific range of validity."

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