The mass ratio in spectroscopic binaries

THE MASS RATIO IN SPECTROSCOPIC BINARIES

BY

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It is a well known fact that the majority of spectroscopic binary only exhibit one spectrum. All information concerning the mass of the binaries is contained in the mass-function

$$f(\mathfrak{M}) = \frac{\mathfrak{M}_2^3 \cdot \sin^3 i}{(\mathfrak{M}_1 + \mathfrak{M}_2)^2} \tag{1}$$

which can be calculated from observable orbital parameters

$$f(\mathfrak{M}) = 1.038 \times 10^{-7} (1 - e^2)^{3/2} K_1^3 P \tag{2}$$

where K_1 = semiamplitude, in km/sec

P = period, in days

$\mathfrak{M}_1, \mathfrak{M}_2$ = masses of the components, in solar masses

$\mathfrak{M} = \mathfrak{M}_1 + \mathfrak{M}_2$ = sum of the masses

i = angle of inclination

e = excentricity

Several attempts have been made to extract from the knowledge of $f(\mathfrak{M})$ the distribution of the mass ratios and its average. The reader is referred specially to the papers by BEER (1956) and HYNEK (1951).

Because of the recent publication of the " Sixth Catalogue of the Orbital Elements of Spectroscopic Binary Systems " (BATTEN (1967)) it seemed appropriate to use the large material collected there to rediscuss the average mass ratio and its interpretation.

MATERIAL

Since the aim is to work on the statistics of the material, one must be careful to avoid selection effects. Such a bias exists surely if one uses all the material of Batten's catalogue without regard to its magnitude limit. This is well illustrated by the fact that out of 16 systems brighter than $4^{m}0$, 5 are eclipsing binaries (EB), while out of 11 systems fainter than 10^m, all are EB. This introduces a heavy bias toward large sin i values if all SB's are used. It was therefore decided to include only systems brighter than 6^m5, in order to diminish as much as possible the selection effect. EB's were included if at maximum light they were brighter than 6^m5. This reduces Batten's material to 448 systems. For these systems it was attempted to ascertain the spectral type of the primary, using the MK classifications quoted by Batten, supplemented from the types given by JASCHEK et al. (1964), ABT and BIDELMAN (1969), HILTNER et al. (1969) and HARLAN (1969). For 41 systems no MK classification could be found. For 22 of them the classification " d " and " g " provided by HOFFLEIT (1964) was used and the remaining 19 stars were left out. This seemed permissible because it represented only 4% of the material.

Next all systems whose primaries were not dwarfs or giants were discarded, because it turned out that in luminosity classes IV, II and I there were too few systems for a meaningful statistics. This leaves a final number of 254 systems upon whose discussion the remainder of the paper is based.

For each of these systems the mass function was taken from Batten, and in the case of SB's showing two spectra, $f(\mathfrak{M})$ was calculated with the formula (2). Five systems were ommitted from the statistics, in which the mass ratio was larger than one, namely the objects N. 52, 428, 481, 607 and 643 in Batten's catalogue and also the system β Lyrae. Table 1 gives the values of the average mass function for each group, as also some quantities which will be used later. Groups with less than 5 SB's were ommitted from further discussion.

TABLE 1

Numerical data for statistics

Group	N_1	N_2	$\langle \mathfrak{M}_1 \rangle$	$\langle f_1 \rangle$	$\langle f_2 \rangle$	$\langle \mu_2 \rangle$	$\langle f_1^{1/3} \rangle$
B2-B3 V	12	8	11	0.1584	0.4970	0.63	0.456
B4-B6	5	7	7	.2032	.7398	.78	.573
B7-B9	5	6	4.5	.0933	.3607	.72	.479
B9.5-A1	8	10	3.5	.0493	.3072	.86	.331
A2-A3	9	10	2.7	.0408	.4849	.77	.318
A4-F4	12	8	1.9	.0202	.2500	.89	.409
F5-F8	17	8	1.3	.0476	.2556	.91	.314
F9-G2	11	2	1.1	.0415			.247
B7-A3 III	6	3	3.5	.2042			.539
A4-F8	7	3	1.5	.0367			.276
F9-G5	8	1	2.5	.1669			.526
G6-K0	23	2	3.5	.0716			.369
K1-M7	18	—	5.0	.1100			.425

The meaning of the different columns is the following

N_1 :	number of systems having one spectrum visible
N_2 :	number of systems having two spectra visible
$\langle \mathfrak{M}_1 \rangle$:	average mass of the primary, in solar masses
$\langle f_1 \rangle$:	average mass function for systems exhibiting one spectrum
$\langle f_2 \rangle$:	average mass function for systems exhibiting two spectra
$\langle \mu_2 \rangle$:	average mass ratio of systems exhibiting two spectra
$\langle f_1{}^{1/3} \rangle$:	average of $f_1{}^{1/3}$

The usual procedures to handle the mass function will be discussed next

a) Hynek's procedure. This author writes

$$\langle f_1 (\mathfrak{M}) \rangle = \langle \mathfrak{M}_1 \rangle \cdot \langle \sin^3 i \rangle \langle \frac{\mu^3}{(1+\mu)^2} \rangle \qquad (1)$$

which is strictly true if masses, inclinations and mass ratios are mutually independent. Although they are independent in principle, the observations one obtains are certainly biased in the sense that observationally stars with large inclination are favored because of the larger probability of discovery. To take this into account, Hynek uses for $\langle \sin^3 i \rangle$ the value

TABLE 2

Average data for statistics

Group	(1)	(5)	(6)
B2-B4 V	0.0211	0.160	0.020
B5-B6	.0428	.234	.064
B7-B9	.0305	.227	.069
B9,5-A1	.0207	.172	.043
A2-A3	.0221	.179	.019
A4-F4	.0156	.267	.022
F5-F8	.0538	.225	.053
F9-G2	.0552	.187	—
	.033 ± 5	0.21 ± 1	0.41 ± 8
B7-A3 III	.0855	.276	
A4-F8	.0360	.190	
F9-G5	.0980	.301	

TABLE 2 *(continued)*

Group	(1)	(5)	(6)
G6-K0	.0300	.190	
K1-M7	.0324	.194	
	.056 ± 14	.23 ± 2	
Final grand average			
	.045 ± 12	.22 ± 1	

The meaning of the different headings in the following:

— (1) gives $\langle \frac{\mu^3}{(1+\mu)^2} \rangle$

— (5) gives $\langle \frac{\mu}{1+\mu} \rangle$

— (6) gives $\langle \frac{\mu_1^3}{(1+\mu_1)^3} \rangle$

the errors quoted are $\left[\frac{\sum (\bar{x}-x)^2}{n\,(n-1)} \right]^{\frac{1}{2}}$

TABLE 3

Assumptions concerning $f(\mu)$

Form	Integr. limits for		$\langle \mu^3/(1+\mu)^2 \rangle$	$\langle \frac{\mu}{1+\mu} \rangle$	$\langle \mu \rangle$
a) $f(\mu) = c$	0	1	0.079	0.31	0.50
b) $f(\mu) = c$	0.2	1	.100	.36	.60
c) $f(\mu) = \alpha\mu$	0	1	.220	.39	.67
d) $f(\mu) = 1-\alpha\mu$	0	1	.020	.23	.33
e) $f(\mu) = \mu^{-4/3}$	0.07	1	.030	.19	.34
f) $f(\mu) = \mu^{-7/3}$	0.15	1	.025	.22	.30
g) $f(\mu) = \mu^{-7/3}$	0.20	1	.036	.27	.38

$$\langle \sin^3 i \rangle = \frac{\int_0^{\pi/2} \sin^5 i.di}{\int_0^{\pi/2} \sin^2 i.di} = 0.68 \qquad (2)$$

in which the distribution function of i is sin $i.di$, to which one adds also a detection probability of the form sin i. Hynek's procedure is probably the most direct approach to the problem. Its difficulty lies in the use of $\langle f(\mathfrak{M}) \rangle$ which has a wide range of values, its mean being therefore not very stable. A second difficulty is that Hynek uses afterwards

$$\langle \frac{\mu^3}{(1+\mu)^2} \rangle = \frac{\langle \mu \rangle^3}{(1+\langle\mu\rangle)^2}$$

which is mathematically incorrect. We will later show how this can be handled properly.

b) One can also write

$$f_1^{1/3}(\mathfrak{M}) = (\mathfrak{M}_1 + \mathfrak{M}_2)^{-1/3} \sin i \frac{\mathfrak{M}_2}{\mathfrak{M}_1 + \mathfrak{M}_2} \qquad (3)$$

By introducing

$$f^{1/3}(\mathfrak{M}) \cdot (\mathfrak{M}_1 + \mathfrak{M}_2)^{-1/3} = y$$

and

$$\frac{\mathfrak{M}_2}{\mathfrak{M}_1 + \mathfrak{M}_2} = x$$

$$y = \sin i. \, x$$

If one knows the distribution functions of y, x and i it can be shown that

$$\langle y \rangle = \langle \sin i \rangle \cdot \langle x \rangle$$

By repeating what was said above about the distribution of " sin i ", i.e. including the detection probability factor

$$\langle \sin i \rangle = \frac{8}{3\pi} \qquad (4)$$

and therefore

$$\langle f_1^{1/3} \cdot (\mathfrak{M}_1 + \mathfrak{M}_2)^{1/3} \rangle = \frac{8}{3\pi} \cdot \langle \frac{\mu}{1+\mu} \rangle \qquad (5)$$

The advantage of the procedure is that since the range of $f_1{}^{1/3}$ is much smaller than that of f_1, its average is more stable. For $\mathfrak{M}_1 + \mathfrak{M}_2 = \mathfrak{M}_1 \,.\, (1+\mu)$ one uses an approximate value.

c) Beer (1927) derived a formula which in our notation is

$$\left[\frac{\langle f_1 \rangle}{\langle f_2 \rangle}\right] = \left[\langle \frac{\mu_1^3}{(1+\mu_1)^3} \rangle\right] \times \left[\langle \frac{\mu_2^3}{(1+\mu_2)^3} \rangle\right]^{-1} \qquad (6)$$

where he has introduced $\mathfrak{M}_1 (1+\mu) = \mathfrak{M} = \mathfrak{M}_1 + \mathfrak{M}_2$
f_2 and μ_2 are the mass function and mass ratio for binaries with two spectra visible. By using

$$\left[\langle \frac{\mu^3}{(1+\mu)^3} \rangle\right]^{1/3} = \frac{\mu}{1+\mu}$$

he then simplifies further his formula, but this is again incorrect. It should also be noticed that formula (6) is not entirely correct because really $\mathfrak{M}$ is not the same for the average of spectroscopic binaries with one and with two spectra visible.

The correct formula is

$$\left[\frac{\langle f_1 \rangle}{\langle f_2 \rangle}\right] = \left[\langle \frac{\mu_1^3}{(1+\mu_1)^3} \rangle\right]\left[\langle \frac{\mu_2^3}{(1+\mu_2)^3} \rangle\right]^{-1} \times \frac{\langle 1+\mu_1 \rangle}{\langle 1+\mu_2 \rangle} \tag{7}$$

Since in general $1 + \mu_1$ is smaller than $1 + \mu_2$, the values of (6) will be too large. As an average correction factor one can take $\frac{1.30}{1.80} = 0.72$.

It should be noticed here that what one obtains from this procedure is $\frac{\mu_1^3}{(1+\mu)^3}$ which when corrected for the above factor gives $\frac{\mu_1^3}{(1+\mu_1)^2}$ which is the same as formula (1).

Values of the different quantities needed for the use of the three formula (1), (5) and (6) are given in table 1, and from these one can calculate the values quoted in table 2.

The next step is to derive $\langle \mu \rangle$ from the data of table 2. As remarked before, one can only calculate $\langle \mu \rangle$ if $f(\mu)$ is known. But conversely it is also true that knowing $\langle \frac{\mu^3}{(1+\mu)^2} \rangle$ and $\langle \frac{\mu}{1+\mu} \rangle$ some restrictions on $f(\mu)$ result.

The easiest way to see this is to postulate different functions, derive analytically the averages and compare with the results of table 2. The results of this procedure are given in table 3. A few comments may be in place regarding the selection of trial functions. In first place the simplest possible functions were considered-rectangular, triangular and power laws. It became however soon evident that great importance is to be attached to the lower integration limit, because all averages are very sensitive to the particular value chosen.

Of all the possible distribution laws, the only one which has some theoretical background is the power law with exponent $-$ 7/3. This is the " original mass function " of SALPETER (1955); WARNER (1961) and others have shown its importance for binary statistics. So for instance this power law permits to explain the observed distribution function of the Δm for visual binaries. If this law is relevant for visual binaries, it should be applicable also to spectroscopic binaries, and therefore preferential attention has been paid to it.

DISCUSSION

The data assembled in table 2 show very clearly several facts. First, that the scatter in the results from (1) is larger than in those from (5). This is evidently due to the use of $f_1^{1/3}$ which is more stable than f_1. In second place procedure (1) gives higher values for giants than for dwarfs. Now it must be kept in mind that the masses (which enter in a direct way in formula (1)) for giants are not very well known and some part of the difference can be due to that. There is also the question of possible mass-exchange between the components, which if it exists, would tend to give larger values for evolved systems than for the unevolved one. It seems however unsafe to base much speculation on the small difference and one simple grand average value for all systems will be adopted. The final adopted values are 0.045, 0.22 and 0.041. The near coincidence of the latter value with the first illustrates the fact already mentioned that Beer's procedure gives the same quantity as Hynek's. The small difference is very probably due to the fact that the use of an average correction factor in Beer's formula, as given in (7), does not produce a full correction. Therefore conclusions based upon these results should be given less weight, and the $\langle \frac{\mu^3}{(1+\mu)^2} \rangle$ value has been ommitted from further consideration.

The next step is to interpret the averages found, namely

$$\langle \frac{\mu}{1+\mu} \rangle = 0.22$$

$$\langle \frac{\mu^3}{(1+\mu)^2} \rangle = 0.045$$

For this purpose one can use the trial functions assembled in table 3. It is evident that the first two functions (cases a, b, c) do not satisfy the numerical values adopted.

The next three trial functions are better; i.e. they lie in the right direction. If one adopts the power law $\mu^{-7/3}$ as the most convenient theoretically, one can calculate which lower integration limit reproduces the observed averages. The answer can be found through numerical integration and gives the results assembled in table 4.

TABLE 4

Lower limits to mass ratios

Observed Average and its variance		Lower integr. limit	$\langle \mu \rangle$
$\langle \frac{\mu}{1+\mu} \rangle$	$= 0.22 \pm 1$	0.15 ± 1	0.30
$\langle \frac{\mu^3}{(1+\mu)^2} \rangle$	$= 0.033 \pm 5$ (dwarfs)	$.19 \pm 2$	0.36
	$= 0.056 \pm 14$ (giants)	$.27 \pm 5$	0.46
	$= 0.045 \pm 12$ (average)	$.24 \pm 5$	0.43

Although the results do not agree too closely with each other, it is clear that they all point to lower limits for the mass ratios of the order of 0.2 ± 0.05. If an average mass-luminosity relation of the type

$$\log \mathfrak{M} = \alpha M_v + \beta, \quad \text{with } \alpha = 0.115 \qquad (8)$$

is used, this mass ratio corresponds to a limiting average magnitude difference between the primary and the secondary of about 6^m, if both are on the main sequence.

By knowing the $f(\mathfrak{M})$ law one can also calculate the average mass ratio $\langle \mu \rangle$. As can be seen from table 3, there are no large changes in this quantity for all three laws considered, (d, e, f) which is a very convenient circumstance. The values tabulated in the last lines of table 3 lie all between 0.30 and 0.38. On the other hand if one accepts the $\mu^{-7/3}$ law, one can derive, from the knowledge of the lower integration limit, the value of $\langle \mu \rangle$; the results lie between 0.30 and 0.46. It seems reasonable to adopt

$$\langle \mu \rangle = 0.35$$

as a compromise between the two ranges. This value corresponds to an average magnitude difference of 4^m between the primary and the secondary components, if both are on the main sequence.

If one compares this value with others given in the literature, one obtains the results of table 5, where all Δm values are obtained with formula (8).

There are no data in the literature concerning the lower mass (or luminosity) limit for the companion. But there should be such a limit because in order that a binary may exist, with its primary being a main sequence star, the secondary must have contracted at least inside its Roche equipotential surface. If for the benefit of simplification we accept that both stars are on the main sequence (which should not change greatly the results) then GIANNONE and GIANNUZZI (1969) have shown that the maximum possible magnitude difference between the components must be smaller than about 6^m, because otherwise the primary would evolve off the main sequence, before the secondary arrives at it.

TABLE 5

⟨μ⟩ and Δm

Author	Year	⟨μ⟩	Δm
Beer	1927	0.46	3^m
Hynek	1951	0.19	6^m3
Kurzemniece	1954	0.44	3^m1
Beer	1956	0.44	3^m1
This paper	1971	0.35	4^m

This is in line with the lower limit we found earlier. — Thanks are due to Prof. P. Bouvier for revising the manuscript.

SUMMARY

The mass ratio of spectroscopic binaries showing a single spectrum is derived anew, correcting methods formerly used. An average mass ratio of 0.35 is found, which corresponds to an average magnitude difference of about 4^m for main sequence binaries. A lower limit for the mass ratios is calculated and found to be 0.2, in agreement with theory.

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