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ELASTICITY, PLASTICITY AND SHRINKAGE

ÉLASTICITÉ, PLASTICITÉ ET RETRAIT

ELASTIZITÄT, PLASTIZITÄT UND SCHWINDEN

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The design of reinforced concrete in practically all regulations and text books is based on the following assumptions :

1) that concrete is an elastic material.

2) that it has a definite modular ratio as compared with steel, which is constant during the life of the structure.

If these assumptions were reasonably or approximately correct, then a beam of reinforced concrete when loaded would show a finite elastic deflection which would remain sensibly constant however long the load were applied, the deflection returning to its original value when the load is ultimately removed.

In the case of steel beams stressed inside the elastic limit, this occurs in accordance with the ordinary elastic theory. If experiments show that it does not do so for concrete even approximately, then it follows that the application of the ordinary elastic theory to reinforced concrete will lead to very considerable errors in the computation of stresses. It does not necessarily follow that reinforced concrete if designed in accordance with customary methods will no longer be properly regarded as a safe material, and it is not by any means the author's view that reinforced concrete so designed is not safe. It must however be admitted that a system of calculations which indicates that a structure is stressed to a certain stress must be considered most unsatisfactory if it can be shown that actually the stress may be several times as great, even if it can subsequently be show that the persistence of a higher stress does not render a structure unsafe. A more accurate knowledge of these matters is in the author's view extremely desirable as only more intimate knowledge can ultimately lead to more economical design, and we wish to know as accurately as possible what are the real stresses and what stresses may be in various circumstances regarded as safe.

The following is intended to be an attempt to help in proceeding a little way along this line in the hope that other engineers may carry the matter still further.

In a paper submitted to the Institution of Civil Engineers on the 15th November 1927 the author described some research work which throws some light on the questions involved.

Four beams of reinforced concrete 15 ft. span were at the age of 28 days suspended between two supports 15 ft. apart and loaded in such a manner as to stress the concrete and the steel to the following values :

Table I.

	Concrete Stress. Compression.	Steel Stress. Tension.
Beam N° 1	676	13750
» 2	946	19250
» 3	1216	24750
» 4	1487	30250

these stresses being based on the ordinary straight line theory, neglecting tension of concrete and taking Young's modulus for concrete as 4,000,000 lbs. per square inch (which was approximately the experimental value obtained with the concrete in question, the modular ratio thus being taken at 7 1/2).

On the basis of this theory the deflection of the beams which were 5″ deep, 2″ wide, reinforced with .06 square inches of steel 4 1/2″ from the compressed edge, is as follows :

Table II.

	Deflection. Inches.
Beam N° 1	.474
» 2	.622
» 3	.770
» 4	.918

It was however quite impossible to find any evidence of cracks however minute on the tension side of the concrete, and it was therefore thought of interest to calculate what the stresses would be if the concrete were not considered as having failed on the tension side.

The stresses on this basis work out as follows :

Table III.

Beams	Concrete stress in compression. lbs. per sq. in.	Concrete stress in tension. lbs. per sq. in.	Steel stress. Tension. lbs. per sq. in.
N° 1	389	367	2180
» 2	545	514	3060
» 3	700	661	3920
» 4	856	808	4800

The deflection of the beam under this condition of stress with the concrete considered not cracked on the tension side is calculated as follows :

Table IV.

	Deflection. Inches.
Beam N° 1	.128
» 2	.167
» 3	.207
» 4	.247

The observed deflection of the four beams when first loaded was .3″, .35″, .4″, .5″ respectively.

The comparison between the calculated and the observed deflections is therefore as follows :

Table V.

Beam.	Calculated deflection, ignoring tension of concrete.	Calculated deflection, concrete considered as taking tension in accordance with Table III.	Observed Deflection.
N° 1	.474	.128	.30
» 2	.622	.167	.35
» 3	.770	·207	.40
» 4	·918	·247	.50

It will be seen that the observed deflection lies intermediate between the values calculated in the two manners previously described, and the observed deflection is therefore consistent with the concrete being stressed in tension, but to a value considerably lower than that given in Table III. This might be quite consistent if the stress-strain diagram for concrete curves considerably when the ultimate tensile stress is approached.

For the concrete in question a breaking tensile stress of 230 lbs. per square inch might not unreasonably be expected, and if such a stress is considered as acting below the neutral axis and the rest of the tension carried on the steel, a value for the deflection is arrived at not differing greatly from the observed deflection.

It is interesting to observe that the actual deflection lies almost exactly half-way between the deflection calculated by the two methods.

Now it is clear that if concrete were an elastic material like steel the observed deflection would remain constant and the deflection-time curve would be a straight line parallel to the axis of time.

The deflections were accordingly noted every week to ascertain whether this were so. The results of these measurements are recorded in Fig. 1, which shows how the deflection increased with time. It will be seen that this increase is not a small one.

Roughly speaking, the deflection in ten weeks is twice the deflection when the load is first applied at four weeks, and increased to nearly three times the initial deflection at the end of a year.

The readings were actually continued for five years with the results as shown in Fig. 1, the diagram containing a break in the horizontal scale as the variation in the latter period was much less marked. The diagram indicates the effect of removing and replacing the applied load other than the dead load, and it will be seen that when the load is removed the elastic deflection is

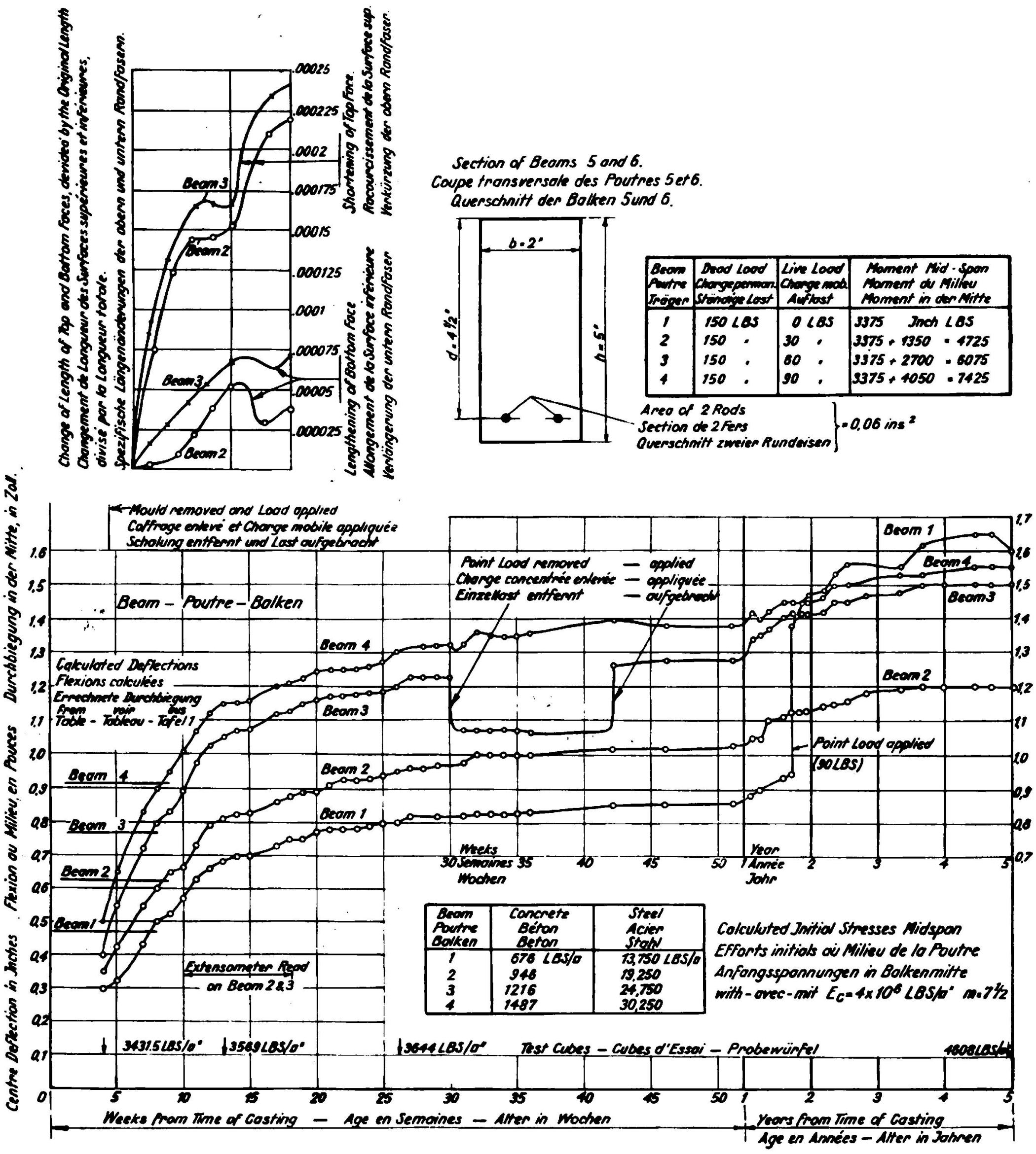

Fig. 1.

Deflection of Concrete Beams, reinforced on Tension Side only.
Flexion des poutres en béton, armées seulement dans la zone travaillant à la traction.
Durchbiegung der nur auf der Zugseite bewehrten Eisenbetonbalken.

almost the same as it was originally, but there is a large permanent set which remains.

At first it was considered possible that the increase in the deflection was due to the gradual breaking down of the concrete on the tension side, and that this might cease when the deflections reached those calculated on the basis of the concrete having cracked in tension, but as the figure shows, this was not the case and there were no discontinuities in the curve either at this or at any other point.

Another very interesting point is that beam N° 1, which was for the first year stressed to the lowest stresses was then loaded to produce stresses similar to those in beam N° 4, and ultimately gave a deflection not very dissimilar from that in beam N° 4 which had carried these heavier stresses for the whole period.

Another point of great interest is that though at the end of five years three of the beams were deflected by over 1.5″ on a span of 15 ft., which is of

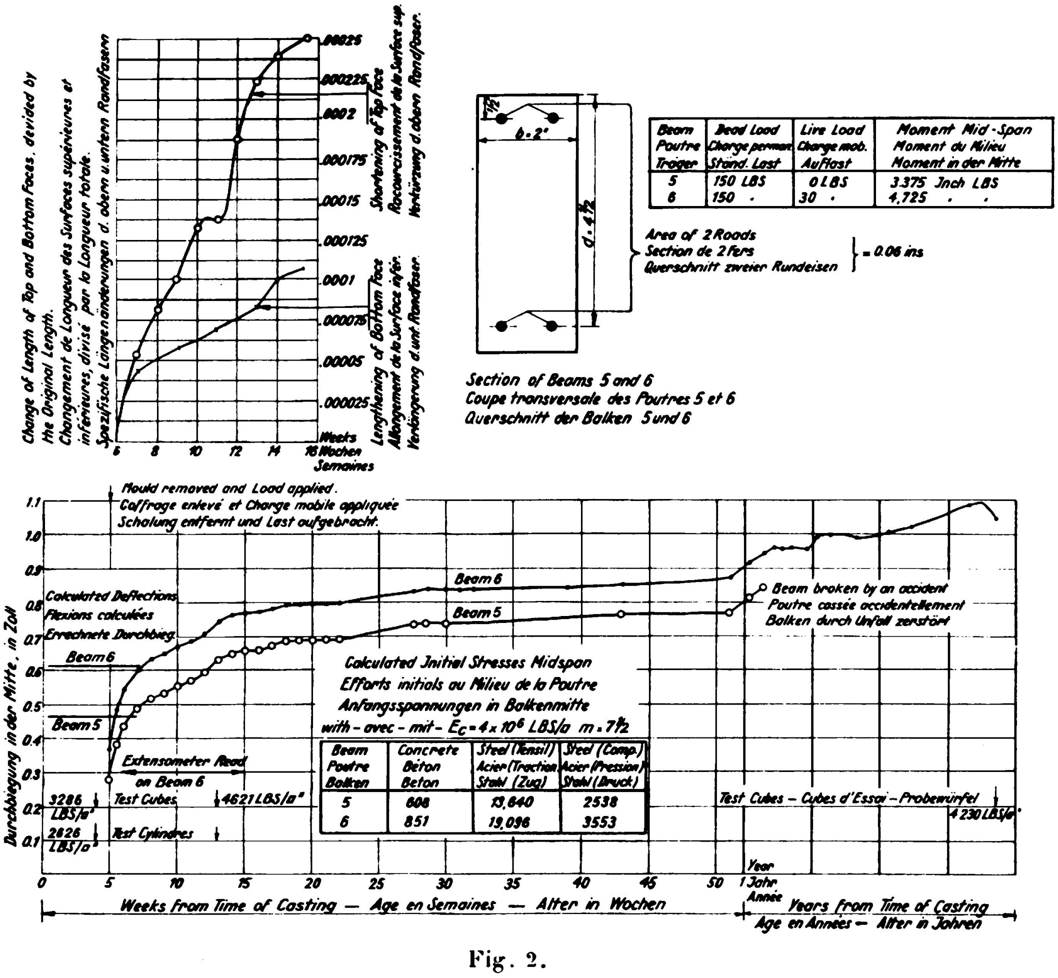

Fig. 2.

Deflection of Concrete Beams, reinforced equally on Top and Bottom.

Flexion des poutres en béton, avec armature travaillant à la traction et à la compression.

Durchbiegung der beidseitig bewehrten Eisenbetonbalken.

course a very heavy deflection, yet a most careful search with a powerful magnifying glass failed to indicate the slightest indication of any crack anywhere, and the absence of any discontinuity in the deflection curve is additional evidence that in fact no crack occurred.

Fig. 2 shows a similar effect with two beams, similar except that they were reinforced in compression with the same amount of steel as in the tension side. The ultimate deflection is considerably reduced, but the general increase of deflection with time remains similar.

All the beams had extensometers clamped to their top and bottom faces in such a manner as to enable the extension and shortening to be measured to one millionth of an inch by optical means, the extensometers being designed by Professor Lamb and kindly lent by Mr. A. Macklow-Smith.

It was found that the deflection was proportional to the difference in the extensometer readings. The total shortening of the top flange consists of three items, as follows :

1) Strain due to the elastic stress.
2) Shortening due to shrinkage.
3) Plastic yield due to stress under the action of time.

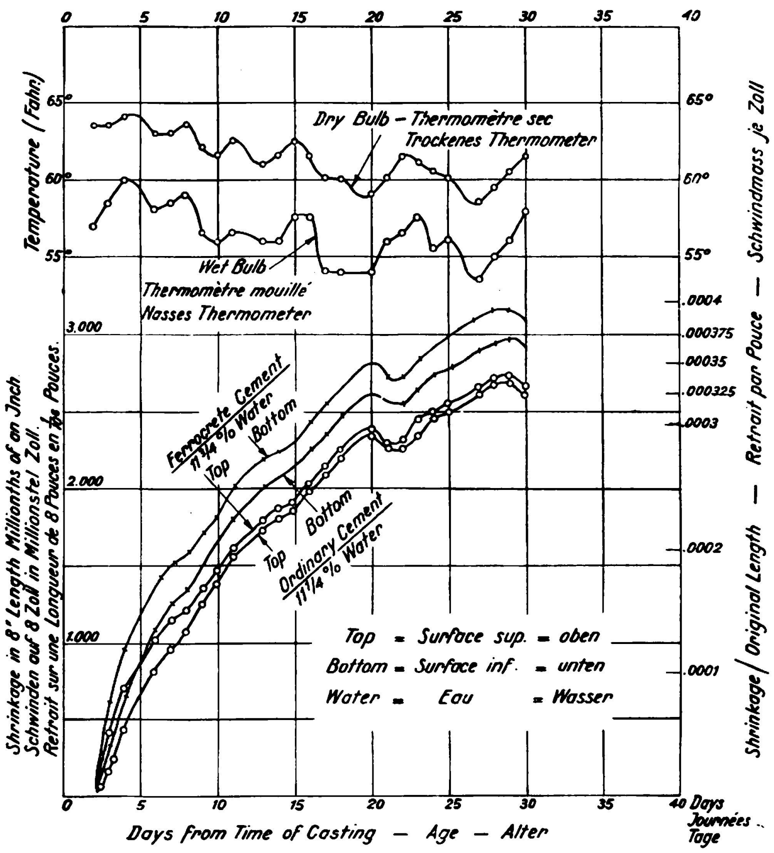

Fig. 3.
Shortening of Tops of Beams 1 to 4 from all Causes.
Raccourcissement des surfaces supérieures des poutres 1 à 4.
Verkürzung der oberen Randfaser der Balken 1 bis 4.

The shortening due to shrinkage was measured on similar beams unstressed, but in all other respects similar.

The shortening due to elastic compression can be calculated from the known value of Young's modulus which was measured on specimens made of identical concrete and the difference between these and the total shortening observed represents the residual plastic yield.

These quantities are calculated in this manner and shown on Fig. 4 in which the shortening of the top flange is plotted against the concrete stress, from which it will be seen that not only is the elastic strain proportional to the concrete stress, but the subsequent plastic yield is also proportional to the concrete stress. Superimposed on both these is the shrinkage which is independent of stress.

It is further observed that the total shortening is many times greater than that due only to the elastic strain which has been the only item normally taken into account in most current regulations.

The results of Figs 1 and 2 appear to the author to indicate as clearly as possible that the assumption of concrete acting as an elastic material is bound to lead to extremely erroneous calculations of stress based on this assumption, because the deflection diagram does not even approximately conform to a

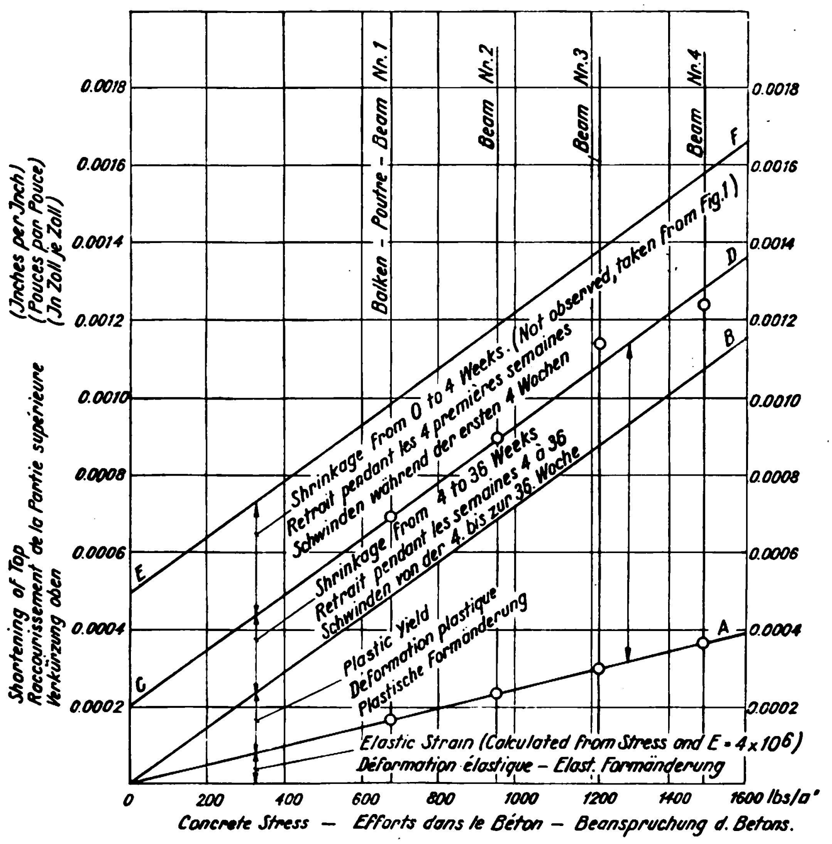

Fig. 4.

Shrinkage of Specimens 18" long, 5" deep, 2" wide. Cement = 1 part, Sand = 2 parts (by weight). Specimens not under stress.

Retrait de quelques éprouvettes 46 cm × 12, 5 cm × 5 cm. Ciment : Sable = 1 : 2 (Rapport des poids). Éprouvettes non chargées.

Schwindproben, gemessen an einigen Betonkörpern von 46 cm × 12,5 cm × 5 cm. Zement : Sand = 1 : 2 (Gewichtsteile). Die Probekörper waren unbelastet.

straight line parallel to the horizontal axis as it should do if this theory were even approximately true.

All these beams were kept in a laboratory for the whole period of their test in a temperature varying between 58° and 61° F. so as to eliminate errors due to change of temperature as much as possible.

Tests of shrinkage were made on various kinds of cement and are shown in Fig. 3, the humidity of the air and its temperature being simultaneously recorded, and it will be seen that the fluctuations in the shrinkage diagram are accounted for by variations in the atmospheric conditions, upward variations in the one diagram corresponding to downward variations in the other, particularly in the case of a considerable variation which occurred 22 days after casting.

As a result of a great many observations the author finds that the average shrinkage of ordinary concrete may be taken somewhat as follows :

Average Shrinkage of Concrete.

Shrinkage. Inches per inch.	Age in Weeks.
0	0
.0002	2
.0003	5
.0004	12
.0005	50

Compression Steel in Concrete.

It is now of interest and importance to consider the effect of the phenomena previously observed and studied in relation to the stresses in structures, and in particular in relation to the stress in steel in the compression area of a concrete beam or a concrete column.

It is clear that as the observed shortening is many times greater than the calculated shortening on the ordinary elastic theory (see Fig. 4), the stress in the compression steel must also be many times greater than the stress calculated in accordance with the ordinary elastic theory, and detailed study and further observations have confirmed that this is indeed the case.

Without going into the matter in great detail here as it has already been explained in great detail in the paper previously referred to, it may be stated that the shortening of the compression steel could be calculated from the observations taken, with the result that in beam N° 6 (reinforced in compression) it can be shown that the compressive stress on first loading was 3,553 lbs. per square inch, which was increased during a year by shrinkage and plastic yield by 11,700, giving a final stress of 15,253 lbs. per square inch. It is indeed obvious that as the concrete continues to shorten, the steel is forced to shorten with it, and is therefore subject to increased stress until the yield point is reached.

This increased stress in the steel causes it to carry a greater proportion of the total compression, thus relieving the concrete of stress, which accounts for the reduced deflection of the beams with compression reinforcement as compared with those without it. For this reason alone compression reinforcement therefore serves a very useful function.

It is also noteworthy that the high compressive stress of 15,253 lbs. per square inch is carried without the slightest signs of distress, though the beams were not provided with binding or stirrups of any kind.

The measured yield point of the steel was 28,627 lbs. per square inch, so that the steel had not approached this stress by a very large margin.

Very careful consideration was given to the question as to whether the high stresses in the steel had relieved themselves by slipping between the steel and the concrete. It was however conclusively shown that this was not the case, the adhesion stress being very low (of the order of 17 lbs. per square inch average and possibly double this value maximum).

It has been shown in Fig. 4 that the plastic yield was proportional to the stress. Therefore the ratio of eventual strain to original strain is a constant, k, independent of the stress and increasing with time, to which the author gives the name « factor of plasticity. » Hence the initial strain (elastic strain = stress/E_c) due to a stress gradually increases to plastic strain ($k \times$ stress/E_c), the value of k depending on the length of time the load is applied and the age of the concrete. Fig. 4 shows that between 4 and 36 weeks k is approximately 2.9 (that is, the final strain is 2.9 times the elastic strain).

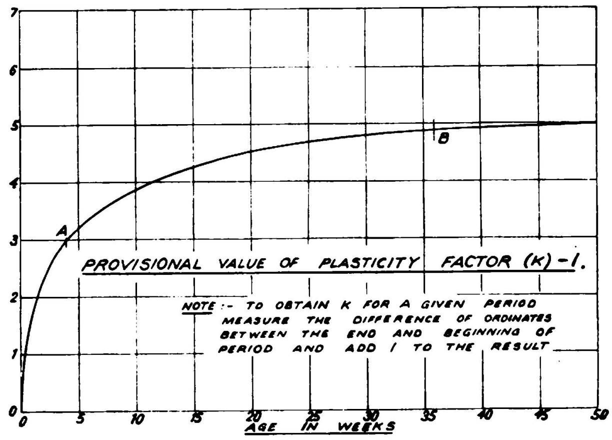

Fig. 5.

Valeur provisoire du facteur de plasticité (k) — 1.

Vorläufiger Wert des Plastizitätsfaktors (k) — 1.

(Note : Pour obtenir k correspondant à une certaine période, il faut mesurer la différence des ordonnées entre le début et la fin de cette période et y ajouter 1.

Bemerkung : Um das zu einem bestimmten Zeitraum gehörende k zu erhalten, ist die Ordinatendifferenz zwischen dem Anfangs- und Endpunkt zu messen und um den Wert 1 zu vermehren.)

It is shown later that the effect on stress calculations is that mk should be substituted for m in the ordinary formulae used in reinforced concrete design. More research is necessary before the value of k can be definitely determined for all ages and conditions. In the meantime, the author offers Fig. 5 as a provisional first approximation, based on the results of the tests described. This curve is intended to be used differentially, for example, the value of (k—1) between 4 weeks and 36 weeks is given by $4.9 - 3 = 1.9$, whence k (in the same period) $= 1.9 + 1 = 2.9$, which accords with the tests (Fig. 4).

The only information available for drawing this curve is that the difference between the points A and B (4 weeks and 36 weeks respectively) is 1.9, the curve is to have the general form of the curves in Figs. 1 and 2, and the ordinates of A and B are to be raised to such values as will allow the smooth curve to go through the origin.

Effect of Shrinkage and Plasticity on Stresses in Actual Structures.

In the original paper it is shown how the stresses in actual structures may be calculated on another basis taking shrinkage and plasticity into account, and without wearying the readers by a repetition of these calculations it is perhaps of interest to give their results. For this purpose a reinforced column 10″ square with 1 % of longitudinal steel is considered which at the age of four weeks is loaded with 60,000 lbs.

Taking Young's modulus as 4,000,000 lbs. per square inch and the modular ratio $m = 7\frac{1}{2}$ it can easily be shown that the stresses as ordinarily calculated would be 564 lbs per square inch in the concrete and 4230 lbs per square inch in the steel. Taking shrinkage and plastic yield into account it is however shown that the stresses would be as follows:

1) At four weeks, old, just previous to loading, 67 1/2 lbs. per square inch in tension in the concrete, 6,750 lbs. per square inch compression in the steel.

2) At four weeks old, just after loading these become, concrete in compression 496 lbs., per square inch, steel in compression 10,980 lbs. per square inch.

3) 52 weeks after casting, concrete in compression 389 lbs. per square inch, steel in compression 21,500 lbs. per square inch.

The difference between four weeks and 52 weeks with the load applied in both cases, shows how the shortening of the concrete has put additional compressive stress on the steel, and to this extent has relieved the concrete of part of its stress, and comparing the stresses at the age of 52 weeks with those as ordinarily calculated it will be seen that the concrete stress is only about two-thirds of that normally calculated, while the steel stress is about five times as great.

Since this research was first carried out, a most extensive series of tests has been made at the Building Research Station at Watford on compression members loaded by specially designed springs, and these have in general afforded the most valuable confirmation of the results set forth in this paper.

The author was led to carry out this research by observing in practice that reinforced concrete beams allowed to stand under load for long periods did in fact increase progressively in their deflection, in some cases sufficiently so to produce objectionable cracks in partitions resting upon them.

It is of course a matter of the greatest importance to consider whether the effects observed and discussed in this paper indicate that reinforced concrete, especially with compression reinforcement, is a less safe material than would have appeared to be the case previous to an understanding of the effects of shrinkage and plasticity on the distribution of stresses.

In the author's opinion the answer to this is sufficiently given by the experience of reinforced concrete on a large scale in practice where it is found that notwithstanding the high stresses which must be reached in compression steel after a long period of time a combination of the two materials, steel and concrete, remains perfectly sound and safe, and the author considers that it has been adequately established both by experiment and by practical experience that the stresses previously referred to may be safely carried without in any way producing weakness or a reduced factor of safety in reinforced concrete structures. What it appears to come to is in fact the following.

We start off with two materials, concrete and steel, which share the total load (considering for the moment a column fairly concentrically loaded). Owing to the progressive shortening of the concrete the steel eventually carries a much bigger proportion of the total load than was previously thought to be the case, until the steel stress may nearly reach, and may even in some cases actually reach the yield point of the steel. It is clear that the margin of safety on the steel has then been reduced, but by the higher stressing of the steel the stress in the concrete has been reduced and the margin of safety on the concrete has been correspondingly increased.

It does not by any means follow that there has been any change in the factor of safety of the composite structure. It must be remembered that if the steel stress ultimately reaches the yield point, and further loading of the column continues, the steel stress then remains constant and any increase in load has to be carried solely on the concrete, and ultimate failure does not arise until the steel stress is carried at its yield point and the concrete is loaded to its ultimate stress.

In the author's view therefore it is not to be lightly assumed that the high stresses which this experimental work have shown to exist are any indication of any unsafety in the composite material which we know as reinforced concrete, and he considers that there is ample proof, both laboratory and of actual practice, to the contrary.

It is of interest to note that while a more modern assumption of Young's modulus for concrete would put it at about 4.000,000 lbs. per square inch rather than 2,000,000 lbs. per square inch which was more currently accepted some twenty years ago, giving a value for the modular ratio m as 7 1/2 rather than the generally accepted value of previous years of 15, the effect of plastic yield is to increase the shortening with time in such a manner as to produce somewhat similar results as if Young's modulus had been lowered and the modular ratio increased, and the actual stresses are not dissimilar at the end of a year to those which would have been calculated on the basis of taking m nearer to 22 1/2 than to 7 1/2 if the plasticity factor in taken as 3, the effect of shrinkage being however separately allowed for, as it is not a function of the stress.

It must not however be assumed that because reinforced concrete structures are no less safe as a result of the stresses found to exist by this research, that an accurate knowledge of shrinkage and plastic yield are unimportant. On the contrary, they unable us to foresee with much greater accuracy what will happen in many practical cases where the old incomplete theory would certainly have given us wrong results. For example, it undoubtedly means that the deflection on concrete beams which are kept fully loaded will be two or three times as great at the end of a year or so as compared with their original deflection when first loaded. There are many places where the deflection is so small that this is a matter of no concern at all. There are other cases where the effects of this deflection have been most objectionable.

The author's experience has included large girders of great spans carrying partitions in which this very deflection was first noticed to produce objectionable cracking in the partitions and their finishings a year or more after the structure was completed, though the loads and stresses were in all cases low and well below those considered safe in accepted practice.

There are cases of beams carrying long lengths of line shafting where a gradual increasing deflection of this kind would be extremely damaging, and on the other hand enormous areas of warehouse flooring where such deflections would be unimportant, and an accurate knowledge of the phenomena enables an engineer to distinguish between the cases where it would be objectionable and the cases where it would be unimportant and to assume with accuracy the deflection in any given case at any particular period of time.

The author's thanks are particularly due to Mr. R. H. H. Stanger, A. M. Inst. C. E., in whose laboratory the tests described were made under his personal supervision, and who very kindly provided the facilities for the research which has been described, and who, with the author, actually made the specimens and the observations from which the subsequent results have been calculated.

TRADUCTION

par L. Descroix, Ing., Paris.

Les projets de construction en béton armé sont basés, dans la plupart des règlements comme des manuels, sur les hypothèses suivantes :

1° Le béton est une matière élastique ;

2° Le rapport de son module d'élasticité à celui de l'acier est défini et d'ailleurs constant pendant toute l'existence de la construction.

Si ces hypothèses étaient correctes dans une limite raisonnable, approximativement une poutre en béton armé, supportant une charge donnée, accuserait une flexion élastique déterminée qui resterait sensiblement constante quelle que soit la durée d'application de la charge, et reprendrait sa valeur initiale quand cette charge cesserait d'agir.

C'est bien là ce qui se produit conformément à la théorie de l'élasticité dans le cas de poutres en acier travaillant au-dessous de la limite d'élasticité.

Si l'expérience prouve qu'il n'en est pas ainsi, même approximativement pour le béton, il s'ensuit que l'application de la théorie ordinaire de l'élasticité au béton armé peut conduire à des erreurs importantes dans l'évaluation des efforts. Il ne s'ensuit pas nécessairement que les constructions en béton armé, calculées d'après les méthodes courantes, ne puissent être considérées comme présentant une sécurité suffisante, et il n'entre nullement dans les intentions de l'auteur de regarder le béton armé, ainsi calculé, comme peu sûr.

Il faut pourtant bien admettre, qu'une méthode de calcul qui indique qu'une construction est soumise à certains efforts doit être considérée comme peu satisfaisante, si l'expérience prouve que ces efforts pourraient en réalité être

Summary.

The shrinkage of concrete and its plastic behaviour under the influence of long-continued internal stresses change in course of time the distribution of the internal forces between the concrete and the reinforcement.

The author investigates the development and changes of the internal stresses in reinforced concrete beams by means of measurements of elongation and bending, and for beams with and without compression reinforcement.

These changes in stress reach extraordinarily high values, without however being capable of affecting the safety of structures dimensioned according to the methods of calculation hitherto employed; for when the point is reached where deformation is no longer proportional to the load, phenomena of flow, i.e. plasticity, appear in the reinforcement, thereby setting a limit to the translation of forces between concrete and reinforcement treated here.

Knowledge of the influence of shrinkage and plasticity on the internal stresses, and of the deformations thereby caused in the structure, enables the designer to foresee many undesirable secondary effects and to take suitable steps to render them harmless.

Résumé.

Le retrait du béton ainsi que sa plasticité, sous l'influence de tensions internes prolongées, altèrent à la longue la distribution des efforts entre le béton et son armature.

L'auteur a suivi le développement et le déplacement des tensions internes dans des poutres en béton armé au moyen de mesures d'allongement et de flexion, en opérant sur des poutres avec et sans armature de compression.

Ces modifications atteignent des valeurs extrêmement élevées sans toutefois pouvoir compromettre la sécurité des ouvrages établis d'après les méthodes de calcul jusqu'ici usitées. En effet, lorsque la limite d'allongement proportionnel de l'acier se trouve dépassée, il se produit dans l'armature des allongements ou phénomènes de déformation plastique qui opposent une limite aux déplacements d'efforts étudiés ici.

La connaissance de l'influence du retrait et de la plasticité sur les tensions internes et le travail qui en résulte dans les constructions permet à l'ingénieur constructeur de prévoir bien des phénomènes parasites indésirables et de les rendre inoffensifs grâce à des mesures appropriées.

Zusammenfassung.

Das Schwinden des Betons und sein plastisches Verhalten unter dem Einfluss lang andauernder innerer Spannungen verändern im Laufe der Zeit die Kraftverteilung zwischen Beton und Bewehrung.

Der Verfasser verfolgte die Entwicklung und Verschiebung der inneren Spannungen in Eisenbetonbalken mittels Dehnungs- und Durchbiegungsmessungen, und zwar sowohl für Balken mit und ohne Druckarmierung.

Diese Verschiebungen erreichen ausserordentlich hohe Werte, ohne jedoch