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GIRDERS, COMBINED WITH CONCRETE OR REINFORCED CONCRETE, SUBJECT TO BENDING

POUTRELLES MÉTALLIQUES COMBINÉES AVEC DU BÉTON OU DU BÉTON ARMÉ ET TRAVAILLANT A LA FLEXION

PROFILTRÄGER, KOMBINIERT MIT BETON ODER EISENBETON, AUF BIEGUNG BEANSPRUCHT

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Some years ago a series of tests of filler joist floors was made at the National Physical Laboratory, London, and the writer was recently engaged in analysing these tests and in seeking to fit the results to theory. The method adopted for calculating the strength of the combined section of concrete and steel joist was that due to Mohr.

In the N.P.L tests, separate tests were made of the concrete for elasticity, and these tests showed that the modular ratio varied between somewhat wide limits. The writer has been able to show that the calculated deflections of all the test specimens can be made to agree with the measured deflections, provided that a suitable value for the modular ratio is adopted for each test — such values lying within the range found in the test for elasticity.

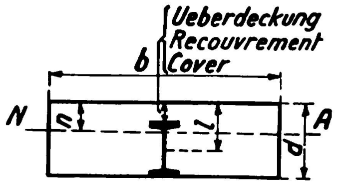

Figure 1.

A = area of section of joist.
l = depth to centre of gravity of joist.
d = overall depth to underside of joist.
b = width of slab acting with joist (centres of joists.)
m = the modular ratio.

Mohr's method of calculation.

For the purpose of obtaining the position of the neutral axis, the whole area of steel is considered as concentrated at its centre of gravity, and the value of n is then :

$$n = -\frac{m\,A}{b} + \sqrt{\frac{m^2\,A^2}{b^2} + \frac{2\,m\,A\,l}{b}}.$$

Equivalent Moment of Inertia and Section Modulus.

The value of n having been obtained, the values of the equivalent Moment of Inertia and Section Modulus are then, in steel units:

$$I = \frac{b\,n^3}{3\,m} + I_0 + A\,(l-n)^2 \qquad Z = \frac{I}{(d-n)}.$$

Where I_0 is the moment of inertia of the joist.

Variables in Calculations.

In any series of calculations on filler joist floors, there are three variables which affect the results :

1. Working stresses.
2. Percentage of steel, or centres of joists.
3. The modular ratio.

Economic Spacing.

For any given joists and depth of floor, there is a certain spacing which will develop the maximum stresses in the concrete and steel simultaneously, for a fixed value of the modular ratio. This may be termed the economic spacing.

If c and t are the maximum permissable stresses in concrete and steel, then the essential equations are :

$$\frac{c}{t} = \frac{n}{m(d-n)}$$

$$\frac{bn^2}{2} = \mathrm{A}m(l-n)$$

$$\text{or,} \qquad b = \frac{2\,\mathrm{A}\,m(l-n)}{n^2}$$

The First Report of the Steel Structures Research Committee, issued by the Department of Scientific and Industrial Research, recommends that the limit of stress in filler joists be 9 tons per square inch. Adopting this value, and limiting the stress in the concrete to 600 lbs. per square inch, the economic spacing for $6'' \times 3''$ joists and $4'' \times 1\frac{3}{4}''$ joists for two different values of the modular ratio have been worked out and are tabulated below :

Table 1. $m = 15$.

Joist Poutrelle	Economic spacing b in inches Écartement économique en pouces			
	cover 1″ recouvremt 1″	2″	3″	4″
$6'' \times 3''$	41.8	44.0	44.3	43.6
$4'' \times 1\frac{3}{4}''$	27.0	27.7	26.9	25.6

Table 2. $m = 24$.

Joist Poutrelle	Economic spacing b in inches Écartement économique en pouces		
	cover 1″ recouvrement 1″	2″	3″
$6'' \times 3''$	21.6	25.3	26.9
$4'' \times 1\frac{3}{4}''$	14.9	16.9	17.3

For any breadth less than those given in the above tables, the maximum permissable stress in the concrete will be reached before the steel is fully stressed. The value of the section modulus as calculated in steel units must therefore be reduced proportionately, or if

Z_2 = The section modulus in steel units reduced to ensure that the concrete stress is limited to 600 lbs. per square inch,

c' = The concrete stress when the steel is fully stressed,

$$\text{Then,} \qquad Z_2 = Z \times \frac{600}{c'} .$$

If now, the modulus be worked out for each of the sections given in table 2, first on the basis of m equal to 15, and then with m equal to 24, we get the values as set out in table 3. In the case of m equal to 15, the modulus is reduced to keep c equal to 600 lbs. per square inch.

Table 3.

Joist Poutrelle	Cover recouvrem'	Section Modulus per foot Module de la section par pied		Increase % for $m = 24$ augmentation en %
		$m = 15$	$m = 24$	
6″×3″	1″	3.49	4.43	26.9
»	2″	3.60	4.45	23.6
»	3″	4.15	4.98	20.0
4″×1 $\frac{3}{4}$″	1″	1.465	1.855	26.6
»	2″	1.80	2.15	19.4
»	3″	2.30	2.71	17.8

Some interesting points arise from this table. For instance, a floor 7″ thick with 6″ × 3″ joists at 21.6 inches centres calculated on basis of $m = 24$ is 6.75 % stronger than a floor 9″ thick with similar joists at 26.9 inches centres, calculated for $m = 15$.

From the point of view of load carrying, it would seem that nothing is to be gained by increasing the thickness of floors with 6″ × 3″ joists above 7″, since the extra dead weight more than absorbs the extra modulus.

Comparison of the Strength and Rigidity of Filler Joist Floors with reinforced concrete floors of the same effective depth.

For the purpose of this comparison, the breadth of filler joist floor in each case is taken such that the maximum stresses in both materials are developed at the same time.

The values worked to are :

c = 600 lbs. per square inch.
t = 20,160 lbs. per sq. inch.
m = 15.

For further comparison, the corresponding figures for reinforced concrete floors when $t = 16.000$ lbs. per square inch are given.

The values of the moment of inertia and section modulus for each section in steel units (inches^4 and inches^3 respectively) are also given. The moments of resistance are in 1000 inch pound units.

Table 4. — Tableau 4.

Filler Joist Floors.

Poutrelles en acier enrobé de béton.

Joist Poutrelles	d	b	Percentage of steel Pourcentage de l'acier	I	Z	Moment of resistance Moment de résistance
$6'' \times 3''$	7″	41.8″	1.15	42.3	8.74	176
$6'' \times 3''$	8″	44.0″	1.003	58.3	10.53	212
$6'' \times 3''$	9″	44.3″	0.886	78.6	12.65	255
$4'' \times 1\frac{3}{4}''$	5″	27.0″	1.96	9.0	2.59	52.2
$4'' \times 1\frac{3}{4}''$	6″	27.7″	1.59	14.4	3.45	69.5
$4'' \times 1\frac{3}{4}''$	7″	26.9″	.40	21.6	4.45	89.6

Reinforced concrete Floors.

Plancher en béton armé.

d	b	$t = 20{,}160 \left(I = \frac{bd^3}{351}\right)$			$t = 16{,}000 \left(I = \frac{bd^3}{263.15}\right)$		
		p.	I	M. R.	p.	I	M. R.
7″	41.8	0.458	40.9	170.5	0.675	54.5	195
8″	44.0	0.458	64.1	233.0	0.675	85.5	267
9″	44.3	0.458	92.0	297.0	0.675	123.0	342
5″	27.0	0.458	9.63	56.0	0.675	12.8	64
6″	27.7	0.458	16.25	82.7	0.675	21.7	94.8
7″	26.9	0.458	26.3	109.4	0.675	35.1	125

It is clear that the plain reinforced concrete floors have in most cases the advantage both in stiffness and in strength, while the area of steel per foot width is very much less. Reinforced concrete floors have the further advantage of being able to be made continuous.

Deflection of reinforced concrete beams and filler joist floors under uniform loading.

The deflection of a steel beam for which $E = 13{,}000$ tons per square inch, and whose depth is $\frac{1}{24}$ th part of the span, is $\frac{1}{325}$ th of the span when the flexural stress is 8 tons per square inch.

Reinforced concrete beams.

The equation connecting deflection and span is :

$\frac{\delta}{12L} = \frac{5.f.12L}{48.y.E}$, where L is the span in feet, δ is the deflection in inches.

If the deflection is to be limited to $\frac{\text{span}}{325}$ then

$$\frac{12\,L}{y} = \frac{48\,E}{325.f.5.}$$ from which we get :

a) Beams with less than the economic percentage of steel,

$$\frac{12.\,L}{(d\text{-}n)} = \frac{2240 \times 16{,}000}{41.67\,t} = \frac{53.76 \times 16{,}000}{t}.$$

b) Beams with more than the economic percentage of steel,

$$\frac{12.L}{n} = \frac{860{,}160}{m.c}.$$

These expressions hold good for any reinforced concrete beam since they depend merely on the stresses in the materials and on the modular ratio. They hold also for filler joist floors, and the ratio of deflection to span for a filler joist floor is therefore the same as that for a reinforced concrete floor or the same effective depth, modular ratio and maximum stresses.

Effect of varying the stress in the steel.

If for example we take two cases, in both of which $c = 600$ lbs, and $m = 15$, the effect of increasing the steel stress is shown by the value of the limiting span :

$$t = 16{,}000, \qquad \frac{12.L}{d} = 34.4$$

$$t = 20{,}160, \qquad \frac{12.L}{d} = 29.5$$

A variation in the value of the modular ratio has a similar effect.

Graphs for design.

If the values of the section modulus per foot width of floor for any given joist and depth of floor be plotted against the spacing or centres of joists, on logarithmic section paper, they lie on a straight line. This is true whether the value of the modulus is obtained by allowing the concrete to be overstressed, by reducing it to 600 lbs. per square inch (or other fixed value), or by taking suitable values of the modular ratio so as to obtain maximum stresses in both materials.

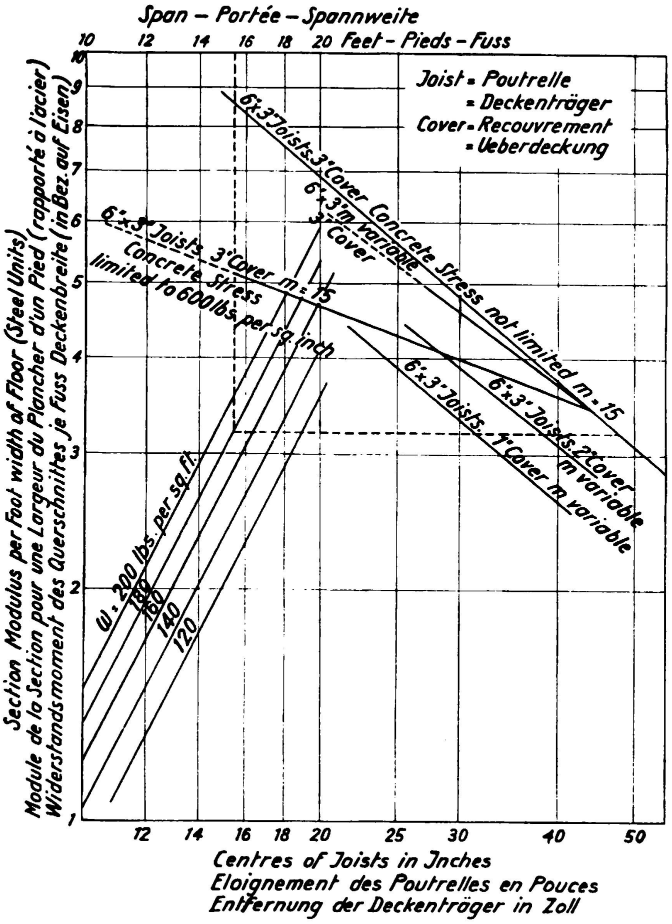

Fig. 2.

Diagram. — Diagramme.

Figure 2 gives the values of Z, the section modulus, per foot width of floor for $6'' \times 3''$ joists with $3''$ cover for $c = 600$ lbs per square inch, $t = 9$ tons per square inch, $m = 15$, also for m varying from 15 to 24; it gives the values for the same joists with $2''$ and $1''$ cover when m varies between the same limits.

Similar lines may be added for other joists. On the same graph are plotted the necessary values of the section modulus required for various weights per square foot on various spans. It becomes a simple matter to choose a section of floor suitable for any weight per square foot and span. The only drawback is that the weight of the floor is included in the total weight, but as there are so many variables it is not possible to eliminate all.

The dotted line on figure 2 shows the method. For a span of 15′ 6″ and total load of 180 lbs. per square foot, the modulus required per foot width is 3.2 and any suitable section can then be chosen.

TRADUCTION

L'auteur de cet article a été chargé dernièrement de l'analyse des essais entrepris, il y a quelques années, par les soins du National Physical Laboratory à Londres, qui concernent la question des poutrelles de planchers. Il s'agissait de mettre en rapport les résultats des essais avec ceux de la théorie. L'auteur base ses calculs sur la théorie de Mohr, c'est-à-dire sur la théorie du calcul des fatigues dans une section non homogène.

Des essais spéciaux ont été faits pour déterminer l'élasticité du béton. Quant aux modules d'élasticité ils ont donné une variation considérable. En choisissant un coefficient d'équivalence convenable, mais ne sortant pas des limites trouvées par les essais d'élasticité du béton, l'auteur obtient une coïncidence

Summary.

The author has undertaken to evaluate the results of the tests with concreted steel I beams, carried out a few years ago by the National Physical Laboratory, Teddington, near London. The tests simultaneously carried out to determine the modulus of elasticity of the concrete, showed that it varied greatly. The author bases his calculations on the method indicated by Mohr for calculating non-homogeneous cross-sections. For given values of the permissible stresses and of the modular ratio he tries to determine the most economically favourable distance apart of the concreted I beams, the concrete covering being chosen of different thicknesses.

From the graphic summary at the end, the dimensions of the necessary ceiling beams and the thickness of the ceiling may be determined for various loads and spans.

Résumé.

L'auteur a entrepris l'exploitation des résultats des essais faits il y a quelques années au « National Physical Laboratory » à Londres sur des poutrelles métalliques enrobées de béton. D'essais exécutés simultanément pour la détermination du coefficient d'élasticité du béton, il résulte que celui-ci est très variable. Dans ces calculs, l'auteur se base sur la méthode de Mohr pour la détermination des tensions dans une section hétérogène. En admettant diverses épaisseurs de la dalle de béton, il s'efforce, pour différentes valeurs des tensions admissibles et du coefficient d'équivalence, de trouver la distance la plus économique des poutrelles enrobées.

A la fin de son exposé, nous trouvons un graphique permettant de déterminer pour des charges et portées diverses, les dimensions des poutrelles nécessaires et l'épaisseur de la couche de béton.

Zusammenfassung.

Der Verfasser hat es unternommen, Versuchsergebnisse, die vor einigen Jahren durch das National Physical Laboratory in London mit einbetonierten eisernen I-Balken durchgeführt wurden, auszuwerten. Die gleichzeitig vorgenommenen Versuche zur Bestimmung des Elastizitätsmoduls des Betons ergaben eine starke Veränderlichkeit des Letzteren. Der Verfasser stützt seine rechnerischen Untersuchungen auf die von Mohr gezeigte Methode zur Berechnung inhomogener Querschnitte. Er sucht bei gegebenen Werten der zulässigen Spannungen und der Wertigkeit den wirtschaftlich günstigsten Trägerabstand der einbetonierten I-Träger zu bestimmen, wobei die Stärke der Betondeckschicht verschieden gross gewählt wird. Aus der graphischen Zusammenstellung am Schluss können für verschiedene Belastungen und Spannweiten die Abmessungen der erforderlichen Deckenträger und Deckenstärken ermittelt werden.