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Calculation of Statically Indeterminate Systems based on the Theory of Plasticity.

Bemessung statisch unbestimmter Systeme nach der Plastizitätstheorie (Traglastverfahren).

Dimensionnement des systèmes hyperstatiques d'après la théorie de la plasticité.

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1) *Introduction.*

This paper has been written for the purpose of discussing the practical use of the theory of plastic equilibrium for the design of statically indeterminate structures composed of members stiff against bending, of defining the limits of its application, and of giving some examples of its use.

The usual method of designing steel structures proceeds from the assumptions that, under the influence of dead weight and live load, stresses develop which follow *Hooke*'s law, and that these stresses shall at no point exceed a certain limited proportion of the working strength of the material, which proportion is known as the permissible stress. The knowledge that in statically indeterminate structures the elastic limits can be exceeded locally without necessarily reducing the loadcarrying capacity and consequently the factor of safety of the structure, as the overloaded sections can be relieved by those less highly stressed, gave rise to another conception of safety in the design of such structures. The new definition of the safety factor should make it possible to take advantage of the properties of tenacity in the steel in the more economic design of statically indeterminate structures.

This definition, then, is as follows:

The factor of savety ν *is the ratio of the ultimate load to the useful load. By ultimate load is meant that limit of load up to which the load can be increased without causing in the structure inadmissible deformations due to frequent repetition of loading and release from load. If various cases of loading are possible, a factor of safety must exist even for any case of changing loading for any sequence of loading whatever.*

The dead weight of the structure is also included in the safe load.

In the method of designing steel structures up to now the factor of safety was expressed by the permissible stress through the ratio

$$\nu = \frac{\sigma_A}{\sigma_S} \qquad (1)$$

wherein σ_A is the working stress of the material, the upper limit of which, however, is considered as being fixed by the elastic limit in order that greater permanent deformation may be safely avoided.

The same value ν as expressed in equation (1) is also chosen as the safety factor for the design of statically indeterminate structures according to the new conception of safety. Employing then the same safety factor, the dimensioning of statically indeterminate structures according to the above conception would give exactly similar results to the dimensioning of statically determinate structures, which always proceeds on the basis of the proportionality between stress and strain.

The determination of dimensions of statically indeterminate structures based on this new conception of safety is called *design on the theory of plastic equilibrium*.

The theory of plastic equilibrium is based on two assumptions:

1) The material must show a stress-strain line in accordance with Fig. 1, with a completely elastic relationship up to the yield point σ_s, the material becoming plastic when this limit is reached. So that the method of plastic equilibrium demands a steel with a sufficiently high elongation[1].

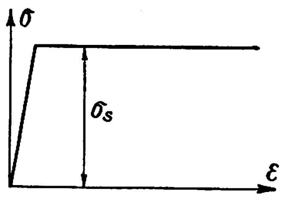

Fig. 1.

2) The material cannot be brought to fracture, in all those cases where the deformations are limited, in other words when a certain upper limit cannot be exceeded even for any number of loading repetitions, irrespective of the difference between the lower and upper limit of stressing. So the method of plastic equilibrium does not deal with the possibility of fatigue failure which can occur without appreciable deformation at the point of fracture. *Strictly speaking, therefore, the use of this method of design can only be permitted where the permanent strength does not come into account, that is to say where the number of loading repetitions during the life of the structure is limited, as is the case in the construction of roofs and floors.*

We must, however, compare the following facts with these observations: For many years now an enormous number of steel bridges have been undergoing in their riveted joints, local stresses in the form of what are called secondary

[1] In soft steels the total permanent deformation at the point of permanent hardening amounts to 10—20% of the measured length.

stresses, which in a considerable number of cases certainly reach the elastic limit. It is well known that in the case of these secondary stresses there are prominent cases of repeated loads producing limited deformation. Fatigue fractures are scarcely known in spite of the fact that in particular cases the number of load fluctuations runs into millions. On the other hand, however, laboratory tests have been carried out on drilled plates, which have shown clearly that in contrast with results obtained with bridges, fatigue strength depends largely on the magnitude of the stress differences and that the fatigue strength is mostly less than the yield point stress, although even with drilled plates there would occur, over the cross sections of the holes, local increases in stress in excess of the average, which are only looked upon as secondary stresses. The conditions have not yet been explained and until they have, precautions should be taken. That is why the line has been taken in the foregoing that dimensioning according to the plastic equilibrium method is limited to those cases where there is no question of fatigue strength. The exploitation of the properties of tenacity of the steel made possible by dimensioning according to this method is of especial significance from the economic point of view in those structures composed of a number of single members stiff against bending, and showing within a definite span or over a series of spans, a constant cross sectional area. If one part of the girder, in which the maximum stress has been reached, fails owing to the occurrence of permanent deformations, then a part of the girder which has up to that time not been completely utilised, will be loaded more heavily owing to the new distribution of the stresses. Nevertheless, in using this new method of dimensioning it is often expedient to carry out local strengthening of the girder.

The plastic equilibrium method, however, offers no further economic advantages in those instances where the cross section of the member is well adapted to the magnitude of the internal forces, as for example is the case in riveted or welded girders, the heights or flange thicknesses of which are designed to suit the bending moment. There is just as little advantage to be gained from the application of the plastic equilibrium method to the dimensioning of statically indeterminate lattice girders as the cross sections of the members of such structures are, as a rule, fitted fairly exactly to the forces in the members. But there is another very important reason why the dimensioning of statically indeterminate lattice structures according to this method is ill advised. The compression members of lattice structures show no tendency to plastic yielding on account of the tendency to buckle, but they fail suddenly, as in the case of tension members which are constructed from brittle material[2]. In this connection lattice work must not be regarded as consisting of members whose material is in every part in an equally elastic-plastic condition. Even in individual members the standard elastic properties as determined by the plastic equilibrium method may vary with changes in compression and tension. The bars may behave like tension bars in the elastic-plastic condition, but if the same beam is also subject to compression it may behave like brittle material.

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[2] This particularly applies to a member which fractures beyond the range of elastic deformations (after the elastic limit is passed).

2) The basic principles for calculation on the theory of equilibrium.

If the main stress at one extreme fibre is plotted to each cross section of a structure composed of members stiff against bending, the line, continuous or continuous in parts, which results is called a *stress line of the structure.* The separate ordinates of this line can naturally belong to different cases of loading of the structure. By selecting for each cross section that particular loading arrangement which creates for the extreme fibre the maximum tension or compression stress, we receive in this way two stress diagrams which are called max. σ and min. σ lines respectively. In asymmetric cross sections or in cases where there are longitudinal forces as well as bending moments, the stresses on the extreme fibres of the member are usually different. In such cases a line max. σ and a line min. σ belongs to each of the extreme fibres.

In each statically indeterminate structure as opposed to a statically determinate structure, stresses and reaction forces can occur, even if there are no external loads. Such a condition is called the *condition of self-stress.* The stresses appertaining to them are called *self-stresses* σ and the stress line, the *self-stress line.* In structures which are composed of straight members of invariable cross sections, this self-stress line is a composition of straight line.

The following general principle now becomes operative[3]:

According to the plastic equilibrium method, a statically indeterminate structure is capable of carrying loads when by appropriate selection of the statically indeterminate quantities it is possible to define a self-stress condition in such a way that at no point does the algebraic sum of the self-stress $\bar{\sigma}$ on the one hand, and the stress limit value max. σ or min. σ, which is calculated according to the usual theory in statically indeterminate structures, on the other, exceed the yield stress σ_s.

Thus, in general the following formula becomes operative:

$$| \max \sigma + \bar{\sigma} | \leqq \sigma_s \text{ and } | \min \sigma + \bar{\sigma} | \leqq \sigma_s \tag{2}$$

The object of *E. Melan*'s paper is the general proof of this principle. With the aid of the stress lines the above principle can be made easy to understand. In Fig. 2 the lines max. σ and min. σ are drawn for a girder on three supports with two equal spans l. The girder is only loaded with a movable load P. If it is now possible to plot a self-stress line, which has at no place a greater distance than σ_s from the lines max. σ and min. σ, then the structure is capable of being loaded with the movable load P. The load can infinitely often pass over the girder or change its position, without causing the permanent deformation of the girder to exceed a certain limit. If the ν-th part of the load P is taken as the safe load (the dead weight of the girder has been taken as nil for the sake of simplicity) then the girder has ν-fold safety.

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3 *H. Bleich.* The dimensioning of statically indeterminate structures taking the elastic-plastic behaviour of the constructional material into consideration. Der Bauingenieur, 1932, p. 261. On the condition that with a variable load other laws of dimensioning come into the question than when the same load changes between an upper and lower limit value, *G. v. Kazinczy* was, as far as I know, the first to give an example (three supporting girders). The further development of the plasticity member. Technika, 1931, Budapest.

If the conditions of the structure are observed, the self-stress line can be plotted within the limits which are given in the principles discussed above. Consequently the economic and constructive points of view can be considered when the choice of the plotted line of self-stress is made.

The self-stresses determined by the self-stress line can be regarded as *artificial pre-stressing* in the unloaded structure[4]. Then dimensioning according to the method of plastic equilibrium means nothing more than the superposition of such a system of artificial pre-stressing over the elastic stresses, produced by loading of a statically indeterminate system, that the maximum values of stresses at particular places of the structures are reduced accordingly.

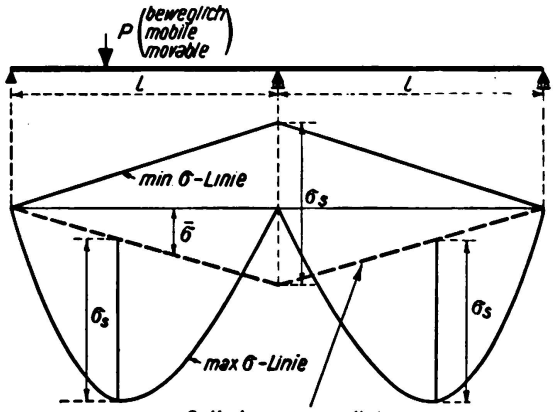

Fig. 2.

Such a condition of pre-stressing does actually occur if the structure is fully loaded and afterwards unloaded.

It is naturally not necessary in the practical application of the plastic equilibrium method to use the maximum loads and the max. σ or min. σ on the one hand and the yield limits σ_s on the other; it is more advantageous to determine the lines max. σ and min. σ corresponding to the actual loads and so to choose the self-stress line, that at no point does the interval between them and the external lines exceed the value of the permissible stress σ_{zul}. The factor of safety is then as required

$$\nu = \frac{\sigma_s}{\sigma_{zul}}$$

If the upper or lower extreme fibres have, in the more usual cases, different stresses in one cross section, then a special stress diagram (Fig. 2) holds for each extreme fibre. But the self-stress lines of both diagrams are no longer dependent upon each other, as they belong to the same condition of stressing.

The use of the stress lines in the graphic determination of the self-stress lines is naturally only possible when the cross sections of the members are already known, so that extreme fibre stresses can be calculated. But if the cross

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[4] In the example under consideration these initial stresses can be advantageously produced by raising the middle support to a certain extent.

sections require to be determined first, it is more advantageous to proceed from the lines max. M and min. M, which, reversed, are nothing but the stress lines multiplied by the modulus of section W. The problem can be carried through by suitable plotting of the line of self-stress moments M, so that the conditions

$$|\max M + \overline{M}| \leqq W_{s\,zul} \text{ and } |\min M + \overline{M}| \leqq W\sigma_{zul}$$

are fulfilled for each point of the structure. $W\sigma_{zul}$ is called the *permissible moment*[5]. The line of self-stress moments is always a straight line in the case of straight members. If normal stresses as well as moments are to be considered, then M and $\overline{M}$ represent the moments round the middle thirds of the section. It is also possible to proceed so that in the preliminary dimensioning the longitudinal forces N are considered in such a way that the value σ_{zul} is temporarily reduced by the estimated amount $\sigma = \frac{N}{F}$ (F = cross section of the member) and the permissible moment is expressed by the formula $W\left(\sigma_{zul} - \frac{N}{F}\right)$.

It is important to note that *singular overstressing* which can occur through a slight *subsidence of the supports* has no influence on the safety of the statically indeterminate structure, as in the most unfavourable cases it will lead to limited permanent deformations. On the other hand, temperature influences must be taken into consideration in designing as they are subject to unlimited repetition. This can easily be effected by adding the temperature stresses to the ultimate stress-lines for max. σ and min. σ.

In those cases where there is only a single kind of loading which fluctuates between a lower and upper limit, but where reversions of the load are excluded, the lines max. M and min. M coincide with the bending moment lines for this given case of loading. It is only in this case that the calculation no longer requires to be for a statically indeterminate system. The statically indeterminate quantities are chosen arbitrarily as long as the permissible moment $W\sigma_{zul}$ is not exceeded at any point.

The process of calculation according to the plastic equilibrium method can be briefly summarised as follows:

a) *Dimensioning.*

1. Calculation of the lines max. M and min. M, as well as the longitudinal forces N of the members in the statically indeterminate structure on the usual theory of statically indeterminate structures. This condition of stress is called *Condition I.*

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[5] The introduction of $W\sigma_{zul}$ as permissible moment means that even with bent girders the connection between stress and deflection according to Fig. 1 will be taken as simplified. The actual carrying capacity moment of an I-girder is, for example, as is well known, about 16 — 18% greater than $W\sigma_{zul}$. It is apparently incorrect to call on this last reserve in dimensioning statically indeterminate structures.

2. Determination of the condition of self-stress by means of determining the lines of self-stress moment $\overline{M}$ in such a way that the moment at no point exceeds $W\ \sigma_{zul}$.

   Calculation of the statically supernumerary quantities X_1, X_2, ... which belong to the condition of self-stress chosen, and which we shall designate as *Condition II.*

b) *Stress Calculation.*

1. Determination of the important extreme fibre stresses for Condition I.
2. Determination of the extreme fibre stresses for Condition II. These extreme fibre stresses must be less than σ_{zul}.
3. Forming the sum of the stress from I and II. This total shall not exceed σ_{zul}.

The total stress as determined according to 3 should be regarded as theoretical, since it represents only a sort of measure for the safety. The actual stress is given in each individual case by the stress as calculated according to 1, since, as a rule with the plastic equilibrium method of dimensioning. stresses due to safe load (dead weight + live load) also lie below the yield point stress. In such cases where the deflection due to safe load should be determined, these deflections should be calculated without consideration of the influences of self-stress conditions on to the stresses.

Deflections can therefore be determined in the usual way. Care must be exercised to ensure that no buckling of the flanges of girders in compression takes place.

3) *Application of the plastic equilibrium theory to the calculation of continuous girders.*

1st *Example.*

First of all we will deal with the simplest case, in which one kind of loading fluctuates between a lower and an upper limit, so that in the calculation it will

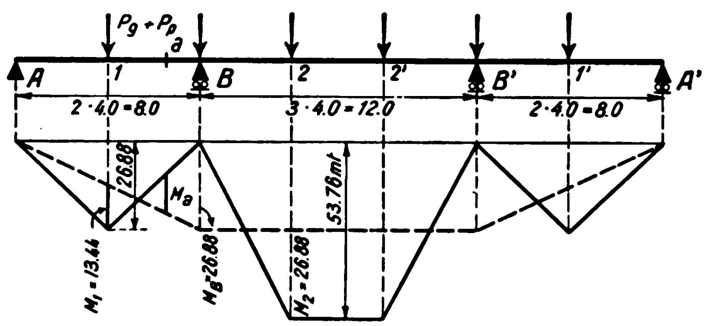

Fig. 3.

be possible to make use of the simplification mentioned on p. The roof joist, a continuous girder with a span $8 + 12 + 8$ m, shown in Fig. 3 a, will be taken as an example. The girder is loaded with a point load from the roof trusses at intervals of 4 m. The dead weight amounts to 180 kg/m², the live load (snow + wind) to 100·kg/m². The loading area for this joist is 12 m wide so that the following concentrated loads result:

from dead weight: $P_g = 12 \cdot 4 \cdot 0.18 = 8.64$ t.
from snow and wind: $P_p = 12 \cdot 4 \cdot 0.10 = 4.80$ t.

As all the loads in this case represent full loads there is but one kind of loading, whereby the loads fluctuate between $P_g = 4.80$ t and $P_g + P_p = 13.44$ t. The maximum values of the moments for statically determinate fundamental system composed of three single girders are shown in Fig. 3 b. Loading with the concentrated loads $P_g + P_p$. We now choose as statically indeterminate quantities the moments over the supports M_B and $M_{B'}$, in such a way that in the middle span the moment of support and the moment in mid-span are equal to each other. By this means the dotted line in Fig. 3 b is determined. The calculation is the following:

$$M_B = M_2 = \frac{53.76}{2} = 26.88 \text{ mt}; \quad M_1 = 13.44 \text{ mt}.$$

Based on $\sigma_{zul} = 1600$ kg/cm² we receive for the outer spans a joist I 34 and the middle span a joist I $42^1/_2$. The joint between I 34 and I $42^1/_2$ is placed in the outer spans; at point a in Fig. 3 b. The position of a must be plotted in such a way that $W_n \cdot \sigma_{zul} \geqq M_a$. W_n is the modulus of section of I 34 under consideration of rivet hole deductions for the position of the joint.

2^{nd} *Example.*

A floor beam passing over 4 spans, Fig. 4, is loaded in one thirds in the points of the span with point loads from dead weight loads $P_g = 4$ t and the live loads $P_p = 8$ t. The permissible stress is taken at $\sigma_{zul} = 1400$ kg/cm². The maximum and minimum values of moments which are obtained according to the usual theory of continuous beams are given in Fig. 4 a. In calculating the limit values of moments it was assumed that the individual spans of the beams were either completely loaded with the working load or were without load.

Dimensioning.

1^{st} *Solution, Fig. 4 b.* The lines of moment of self-stress $\overline{M}$ are plotted in such a way that $M_1 = -M_B = 20.15$ mt and $M_2 = -M_C = 15.74$ mt. Thus a rolled girder I 40 becomes necessary for the end spans and a girder I 38 for both the middle spans. With the moment of 17.70 tm, which a girder I 38 when $\sigma_{zul} = 1400$ kg/cm² is just capable of supporting, the theoretical situation of the welded joint next to support B is determined. The following values of statically supernumerary magnitudes are obtained for the condition of self-stress:

$$\overline{M}_B = \overline{M}_{B'} = +2.14 \text{ mt}, \qquad \overline{M}_C = +2.55 \text{ mt}.$$

2^{nd} *Solution, Fig. 4 c.* The line of moment is plotted in such a way that the absolute values

$$-M_B = M_2 = -M_C = 16.47 \text{ mt},$$

so that the maximum moment in the end span increases to = 21.34 mt. The end spans require a joist I $42^1/_2$ and for the middle spans a I 38 joist; the welded joint can, however, be placed immediately over the support B.

The condition of self-stress is denoted by the following values of the supernumeraries:

$$\overline{M}_B = \overline{M}_{B'} = + 5.81 \text{ mt}, \qquad \overline{M}_C = + 1.82 \text{ mt}.$$

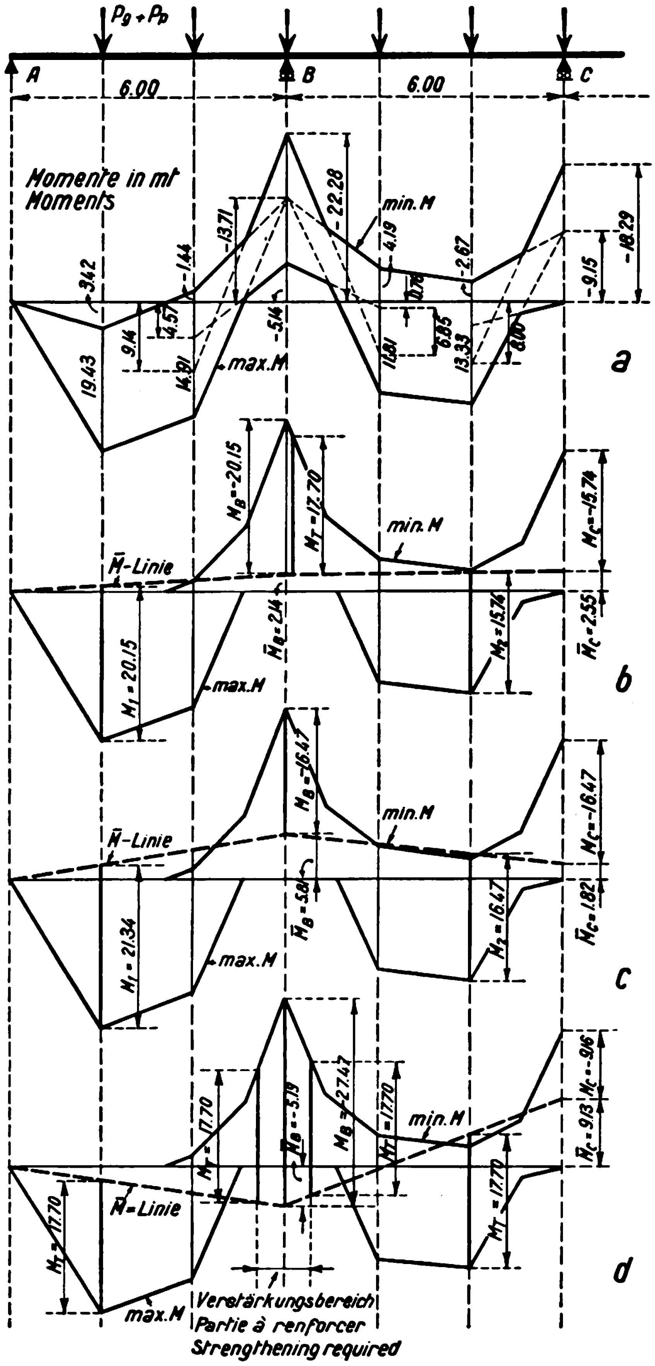

Fig. 4.

3rd *Solution, Fig. 4 d.* In the first and second bay at the points of largest positive moments are plotted the largest moments to which a joist I 38 is suited for $\sigma_{zul} = 1400$ kg/cm², in this case 17.70 mt.

These two points determine the self-stress bending moment line which produces a moment over the support B of 27.47 mt. This moment can be carried by strengthening the through girder I 38 with welded-on flange-plates of 160 · 12 in the region of the support.

The following values of the supernumerary quantities correspond to the condition of self-stress: —

$$\overline{M}_B = \overline{M}_{B'} = -5.19 \text{ mt}, \qquad \overline{M}_C = +9.13 \text{ mt}.$$

3rd *Example.*

The continuous beam for uniform distributed load plays a prominent part as floor girder in steel structures. Consequently in the following, simple rules for dimensioning such girders, which may have any desired number of equal spans, will be laid down. For this purpose a girder over three spans loaded with a permanent load g and a live load p, the latter always applied at the most unfavourable places, will be taken as an example. In Fig. 5 the lines

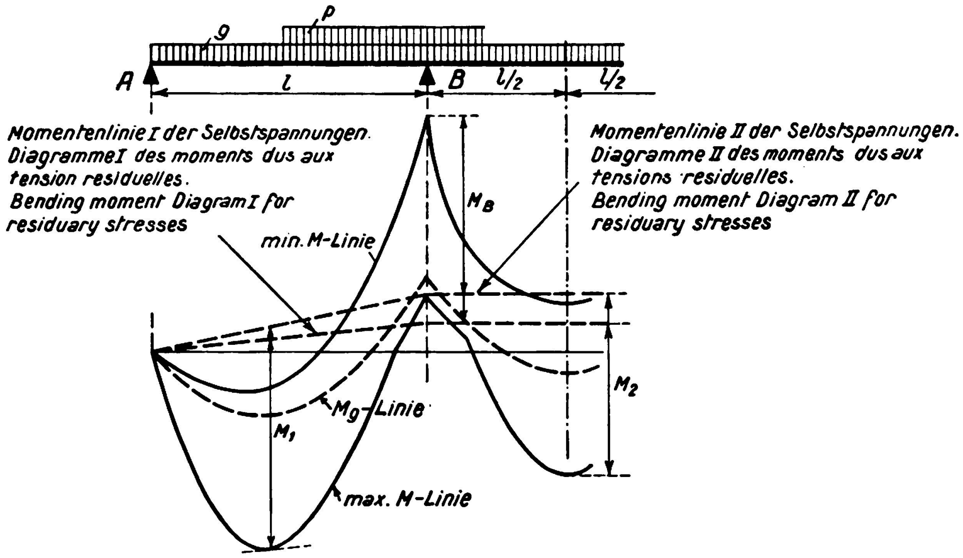

Fig. 5.

max. $M = M_g +$ max. M_p and min. $M = M_g +$ min. M_p are plotted beside the line M_g.

Two solutions of the problem will be considered: The line of bending moment for self-stresses I was plotted so that $M_1 = M_B$. The cross sections are determined in the end bays by the moment M_1, and in the middle span by M_2. It should be observed here that the stronger girder of the end bays must be projected into the middle span to a certain extent (by about 1/10) as M_B is greater than M_2, so that the joint of the girder is to the right of support B.

If the joint of the girder is to lie directly over support B, then the bending moment line for self-stresses should be plotted according to line 11 in Fig. 5. In this case $M_B = M_2$. The moment M_1 is greater than M_B, but the stronger girder of the end bay can now be jointed at B with the weaker girder of the middle span. The bending moment diagrams plotted in Fig. 5 were determined first under the assumption that the cross section of the girder was constant for all spans. So if exact calculations were desired, the bending moment diagrams would have to be re-calculated under consideration of the various cross sections. It becomes clear, however, that in calculating with the plastic equilibrium method the influence of difference in the cross section of the various spans is very slight, so that a single calculation nearly always suffices (assuming constant cross section). In order to understand the incidental correctness of this assertion it suffices to remember Ex. 1, which also represents a strict solution of the problem stipulated by the plastic equilibrium method. In that case the solution is entirely independent of the magnitudes of the moment of inertia of the cross sections in the individual spans.

Fig. 5 also shows that the maximum values of the moments M_g or M_r stipulated by the bending moment line for self-stresses occur either exactly at the same point or very close together. It is therefore possible to calculate the maximum values of the moments for dead weight and live load separately and then add them. The moments are then as follows:

for the end spans: $\max. M = c_1 \, gl^2 + d_1 \, pl^2$

for the middle span: $\max. M = c_m \, gl^2 + d_m \, pl^2.$

The coefficients c and d depend only, assuming equal spans, on the number of spans and can be calculated in advance. In Table I the coefficient for both solutions are grouped for the lines of self-stress I and II respectively. Since in the middle bays the maximum moments differ only slightly from one another, the maximum value of c and d, which holds good for all middle spans, was given for each case.

Table I. Moments for dimensioning continuous beams with equal spans.

End spans: max $M = c_1 \, gl^2 + d_1 \, pl^2$					Intermediate spans max $M = c_m \, gl^2 + d_m \, pl^2$				
(a) The stronger girder of the end spansprojects by about 1/10 into the second span					(b) The stronger girder of the end goes only as far as the first intermediary support				
No. of spans	End spans		Middle spans		No. of spans	End spans		Middle spans	
	c_1	d_1	c_m	d_m		c_1	d_1	c_m	d_m
2	0.0858	0.1048	—	—	2	0.0858	0.1048	—	—
3	0.0858	0.1061	0.0392	0.0858	3	0.0957	0.1109	0.0625	0.0957
4	0.0858	0.1061	0.0511	0.0942	4	0.0957	0.1104	0.0625	0.0971
>4	0.0858	0.1061	0.0625	0.0950	>4	0.0957	0.1098	0.0625	0.0972

4) *Application of the theory of plastic equilibrium to the calculation of frames.*

4th *Example.*

A frame with encastré ends, with a span of 16 m and uprights of 10 m, being a threefold statically indeterminate frame, shall be so designed that the uprights and brace could be made with rolled girders sections without local strengthening. The following loads are taken into account:

Permanent load p = 0.72 t/m,
Snow load s = 0.45 t/m, and
Wind pressure on the uprights w = 0.60 t/m.

The flow of the moments calculated in the usual way, for the three types of loading mentioned above, is shown in Figs. 6 b to 6 d. As there is a possibility of wind pressure from right and left, the lines max. M and min. M are symmetrical to the vertical axis of the frames. These lines are shown in Fig. 6 e.

We now determine a condition of self-stress which is given by the quantities $\overline{M}_A$, $\overline{M}_B$ and $\overline{H}$ and which fulfils the following conditions: It shall be $\overline{M}_A = \overline{M}_B$ and further that the encastré moment in A, the corner moment in C and the moment in F (in mid-span of brace) are each equal as regards numerical value.

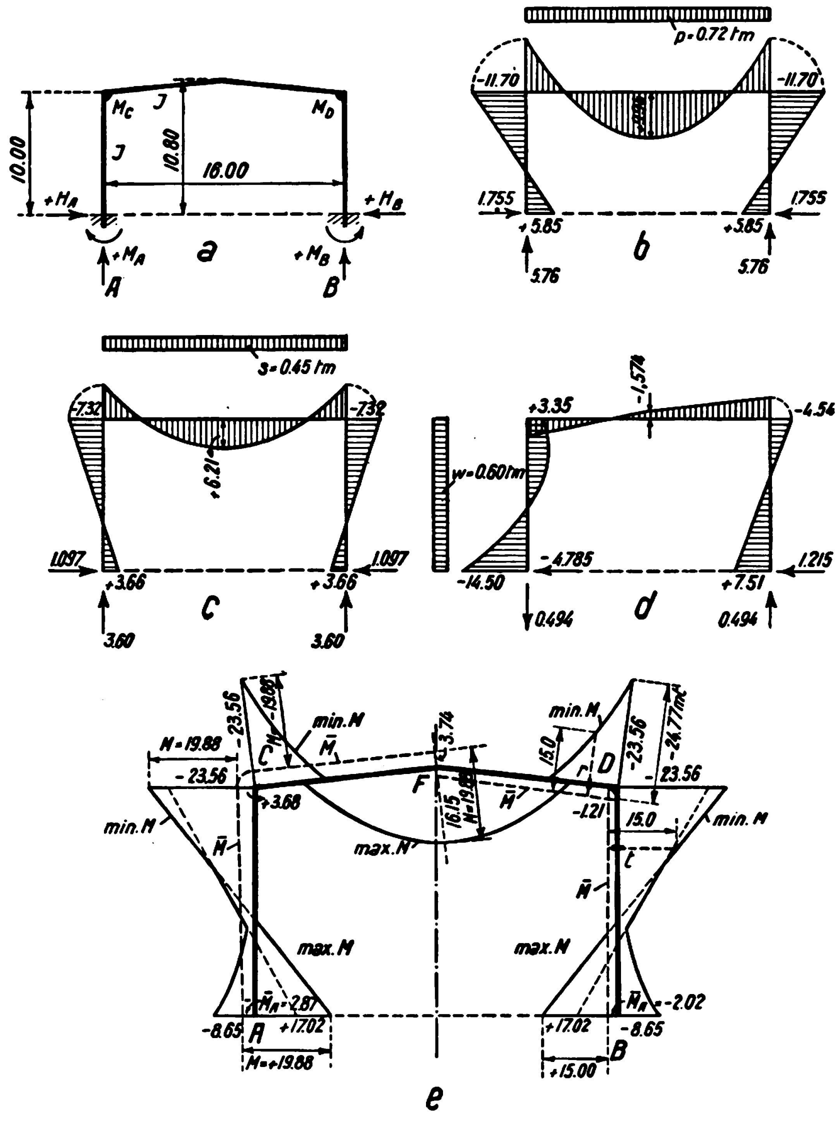

Fig. 6.

If the values for the limit moments as given in Fig. 6 e are considered, the following equations must exist:

at A $\quad 17.02 + \overline{M}_A = M,$

at C $\quad -23.56 + \overline{M}_A - 10\,\overline{H} = -M,$

at F $\quad 6.15 + \overline{M}_A - 10.8\,H = M.$

The unknowns $\overline{M}_A$, $\overline{H}$ and M can be determined from these three equations. The solution gives us

$$\overline{H} = -0.081 \text{ t}; \quad \overline{M}_A = \overline{M}_B = +2.87 \text{ mt}; \quad M = 19.88 \text{ t}.$$

The line of moments for self-stresses $\overline{M}$ is plotted in the left half of Fig. 6e with the values so found. The maximum moment, on which the dimensioning is based, amounts to M = 19.88 mt. If $\sigma_{zul} = 1600$ kg/cm² an I 40 girder will suffice[6]. One stipulation in this connection is that the security of the flanges under pressure has been provided for.

If, however, the possibility of local strengthening is considered, then it is possible to proceed in the following way:

As main section an I 36 girder is chosen whose permissible moment when $\sigma_{zul} = 1600$ kg/cm² amounts to $M_T = 1400 \cdot 1089$ kg/cm = 15.00 mt, with a reserve of 200 kg/cm² to cover stressing due to normal forces[7].

The condition of self-stress should be so chosen that the maximum moments at the point of fixing A, and in the middle of the brace F, do not exceed the value of 15.00 mt. Hence we have the conditional equations:

$$\text{for A:} \quad 17.02 + \overline{M}_A = 15.00$$
$$\text{for E:} \quad 16.15 + \overline{M}_A - 10.8\,\overline{H} = 15.00$$

From the solution of these two equations we see that $\overline{H} = -0.081$ t and $\overline{M}_A = -2.02$ mt. At points C and D a maximum moment $M_C = M_D = -23.56 - 2.02 + 0.81 = 24.77$ mt occurs.

The appertaining line of moment for self-stress condition is plotted in the right half of Fig. 6 e. With the permissible moment $M_r = 15.00$ the points r and t in Fig. 6 e, which define the limits of the range requiring to be strengthened by welding-on of flange-plates, are obtained, so that this range is rendered capable of withstanding a maximum moment of 24.77 mt.

We have used for the calculation of the second case the same lines max. M and min. M as in the first, although these lines should show a somewhat different course owing to local strengthening of the structure. The error thereby made is not so great, as it is shown again and again that the influence of conditions of rigidity on the moments which are decisive for dimensioning is comparatively small. A statically indeterminate structure designed after to the plastic equilibrium method behaves in accordance with the influence of rigidity of individual parts on the decisive quantities for dimensioning, similarly to a statically determinate structure. For the initial dimensioning it is therefore nearly always sufficient to take as basis of calculation a system with an approximate distribution of the moments of inertia only. For the actual stress calculation it is, however, recommended, in order that all conditions may be correctly conceived, that the correct lines max. M and min. M be used.

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[6] The normal forces in the uprights and brace must also be considered.

[7] The maximum normal force, for example, amounts in the upright to about 10 t.

Summary.

The paper deals with the practical application of the plastic equilibrium method to the dimensioning of statically indeterminate structures composed of members stiff against bending. Firstly the conception of the factor of safety in connection with the plastic equilibrium method is defined and in that connection the stipulations for its application, under consideration of the properties of the material, are discussed. It is concluded that the plastic equilibrium method should be limited above all to structures in which the fatigue strength of the material does not have to be considered. Its application to lattice structures is also inadvisable. Discussion of *H. Bleich's* principles based on the method and discussion of the method of calculation. Several examples are given of the practical application of the method, viz. in three different cases of continuous beams and one frame structure.