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Shell Construction in Reinforced Concrete.

Die Flächentragwerke des Eisenbetonbaues.

Les surfaces portantes dans la construction en béton armé.

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Twelve years have passed since the introduction — by Messrs. Dyckerhoff & Widmann AG., in conjunction with Messrs. Zeiss, Jena — of the shell method of reinforced concrete construction, in which the distribution of loading is effected solely by elongation forces. During these twelve years this type of construction has advanced by leaps and bounds; its development was only possible after the theoretical side of these tri-dimensional structures had been greatly elaborated through extensive experiments and in a surprisingly short space of time. The result was that this theory has opened up new fields of work in monolithic reinforced concrete construction as applied to wide-spanned hall-possibilities which by far exceed those offered by the theory of crosswise reinforced and mushroom slabs. By employing shells and saw-type-roof shell structures span widths can be obtained that were formerly regarded as impossible in massive constructions. It must, however, be remembered that little more than a decade has gone by since shell construction was discovered. In this short period hundreds of thousands of square metres of large halls with spans up to 100 metres wide have been erected.

The present paper may be divided into two sections, the first of which gives a review of the development of the theory since the last Congress and the constructional progress made as demonstrated by some practical examples. The second section deals with the problem of continuous cylindrical shells or pipes.

1. Development of the Theory of Shell Construction since the Congress in 1932.

As regards the various forms of shells mentioned in this paper, we would refer to *W. Petry*'s work for the Paris Congress of 1932 (Theme II, 4). Vol. I of the 'Publications', which appeared in the same year, contains an article by *U. Finsterwalder*[1] on the problem offered by the Zeiss-Dywidag system of cylindrical barrel shell, a combination of cylindrical shell and frame supporting its edges on either side. This forms a uniform system in space which can be described as a T-beam in space in which the shell represents the flange of the beam. In contrast to the commonly known T-beam systems — in which, when the webs are widely spaced, the flange only takes up a limited amount of the compressive forces — in the case of these T-beams in space the whole shell acts as compression flange. This results from the fact that in ordinary T-beams as shown in Fig. 1a), the participation of the flange for the transmission of compressive forces N_x forcibly must be established by shear stresses N_{xy} acting between the beam and the flange. The active width is therefore a function of the length of beam. However, the compressive stresses are not evenly distributed over the whole width of the flange, since the strips of slab situated farther away from the beams fail to co-operate on account of deformations due to shear.

The action of the T-beams in space, as shown in Fig. 1b), is essentially different; for, as is clear from Eq. 2 (Section II), compressive forces N_x arise in the shells — even when no account is taken of the compressive forces $N_{x\varphi}$ acting between shell and lateral beam — which are governed by the dead weight of the shell and consequently the whole width of the shell is operative in taking up the compressive forces. This occurs to a greater extent the higher the cross-sectional curve of the shell is raised above the funicular polygon. For this reason shell structures whose cross-sectional curve follows the form of flat elliptical segments possess an essentially better girder action from cross-panel to cross-panel than circular cylindrical shells. Furthermore, the bending moments

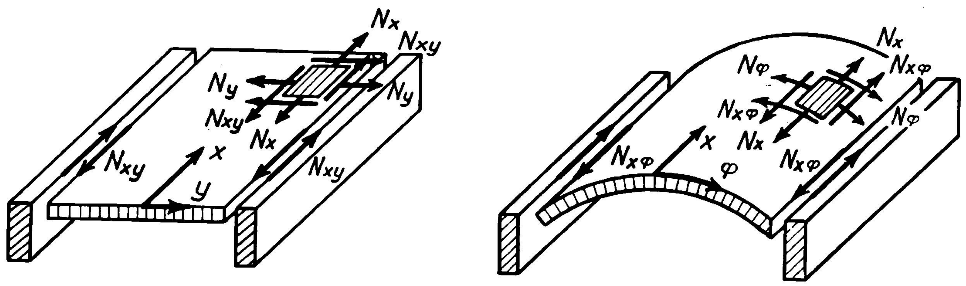

Fig. 1 a. Fig. 1 b.

which arise in these steeply arched systems are much smaller because the compressive forces N_x, necessary here for the transference of the external bending moments, are for by far the greater part produced by the dead weight of the shell itself and not by shear forces $N_{x\varphi}$. The magnitude of the bending moments which occur in the direction of arch are dependent upon that portion of the compressive forces N_x which must of necessity be produced under strain by the shear forces $N_{x\varphi}$. For these reasons it becomes apparent at once that essentially smaller bending moments are caused in shell systems of steeply arched cross-sectional curve, than in circular cylindrical shells. I shall refer to this point again at a later stage.

Between shell and lateral frame there are four statically indeterminate forces acting, namely: — 1) the arch action N_φ, the transverse force Q_φ, the bending moment M_φ and the shear force $N_{x\varphi}$. Thus for both edges together we have eight statically indeterminate quantities, and consequently the basis of the shell problem must be a differential equation of the eighth order or a system of three differential equations corresponding thereto, for in accordance with the eight statically indeterminate quantities we need eight constants for the closing of the two joints between shell and lateral frame. In arriving at his solution *U. Finsterwalder* started with the assumption that when the cross panels are located at relatively large intervals, the bending moments M_x prevent the shell from transmitting loads on to the cross panels; he therefore assumed the moment M_x and the pertaining transverse force Q_x, together with the (twist moment) $M_{x\varphi}$ as being zero. In consequence of this approximation it was possible to work out the problem in the form of a differential equation of the eighth order, introducing a stress function in which the internal forces of the shell could be represented in the same manner as for Airy's stress function for panels, as derivatives of this stress function.

When the cross panels are situated at relatively small intervals in comparison with the radius of curvature of the circular cylindrical shell, *U. Finsterwalder*'s assumption $M_x = Q_x = M_{x\varphi} = 0$ are no longer permissible. It is for this reason that the author has been endeavouring to find a strictly accurate solution for circular cylindrical shells in these cases, which are of importance in the construction of halls with wide arch spans. As the shells of these wide-spanned arches must be strengthened with ribs to ensure safety against buckling, I have also extended my investigations to cover anisotropic shells.[2] Here three linear simultaneous differential equations with constant coefficients are obtained. Part-solutions of these differential equations can be arrived at by following *H. Reissner*[3] and representing the dead weight by circular functions in the form of double trigonometric series. The investigations now show that there are three possibilities for transferring loads in a closed pipe: — 1) transference of loads to the gross panels system by means of elongation forces (membrane theory); 2) transference of loads to the cross panel systems by means of bending moments M_x in the shell (slab action); and 3) equalization of the loads of the higher harmonics by means of bending moments in the direction of the curve. This equalization in the direction of curve is only possible because there is no real vertical load resultant to correspond to the higher harmonics in relation to the whole cross section of the shell. The actual loading is transferred to the cross panel systems by actions 1) and 2). In order to fulfil the support conditions required for the two lateral beams in the case of the Zeiss-Dywidag barrelsystem, the above-mentioned part-solution must be supplemented by a solution of the homogeneous system of differential equations. The latter system is fulfilled in the same manner, introducing the exponential term $e^{m\varphi} \cos \lambda x$, as for the problem solved by *K. Miesel*[3] in 1930, which we shall discuss below. Thus the three differential equations resolve into three ordinary homogeneous equations leading to one equation of the eigth order, from whose solution we obtain the wave length and attenuation of the double oscillations issuing from the two edges of the shell. This equation of the eight order has been solved for about one hundred different cases. The values for wave length and attenuation of the oscillations obtained from it were worked up into diagrams from which the values can be read without any calculation whatever. With the assistance of the above-mentioned basic term not only the eight support conditions along the lateral edges, but also the support conditions for the cross panel systems can be satisfied.

As has already been mentioned, the problem offered by the support conditions of closed circular cylindrical pipes at the cross panels was solved as early as 1930 by *K. Miesel* for any desired variation of the support conditions. Here

Miesel also took the elasticity of the stiffening panels into account — a problem which plays an important part in submarine construction. *Finsterwalder,* too, investigated this problem in his work cited under 1) and found an approximative solution for it, again in the form of a stress function. Here, however, in contrast to the corresponding solution for Zeiss-Dywidag shells, not the quantities M_x, Q_x, $M_{\varphi x}$ but the values M_φ, Q_φ, $M_{x\varphi}$ have been neglected. With much less calculation work and for reasonably small values of the harmonics, this approximative solution coincides very well indeed with *Miesel*'s strict solution. And in our practical constructional problems there are no very high values of the harmonics involved.

The more rigid the shell is constructed as regards bending in the direction of arch, the nearer does the law of stress distribution of the N_x forces approach *Navier*'s straight line law in the case of Zeiss-Dywidag barrels, because then the work of deformation of the bending moments in the direction of arch are insignificant compared with those of the elongation forces. The thinner the shell is, however, the greater is its tendency to reduce the bending moments for correspondingly unfavourable distribution of the elongation forces. If, nevertheless, a more favourable distribution of the N_x forces is to be attained, these thin shells must be combined with correspondingly high lateral beams.

At the commencement of my expositions I pointed out that in greatly raised cross-sectional curves, as for example in flat elliptical segments, smaller bending moments are produced and a more favourable girder action obtained. The larger the cylindrical shells are made, the more necessary does it become to replace them by shells of steep curvature. Thus, for the huge halls constructed for the German Air Ministry, practically only shells with elliptical cross-sectional curves were employed, calculated according to a suggestion made by *U. Finsterwalder.* on the theory of circular cylindrical shells, in such a manner that the elliptical segment was approximated by a three-centre arch. This naturally involves very intricate calculation, as there are now four edges to cope with and the oscillations starting from these have a mutual influence on one another. It is therefore urgently necessary for these cross-sectional curves to be resolved in a strictly accurate manner. One of my assistants has succeeded in doing this, and the solution will shortly be published in the form of a dissertation.

Shell systems are frequently constructed as continuous systems over several spans. As these shell systems are extremely high in relation to their span, the moments at the support are in parts substantially influenced by deformations due to shear. This fact has already been pointed out by *W. Flügge*[4]. As is also well known, the influences of these shear deformations are deliberately neglected as being insignificant in the case of slender beams. For shell systems, however, this omission is not always permissible. In Section II of this paper I have given detailed proof of the influence of these shear deformations on the support moments, and with the assistance of *Flügge*'s three-moment equations developed a process by which the support moments for isotropic and anisotropic shell systems can be obtained for any desired span and loads, both in the direction of arch and also lengthwise.

As the width of spans in shell systems increases, so does the problem of buckling grow in importance. In this connection distinction must be made between two cases, namely: — a) Buckling of the shell in direction of arch, and b) buckling in the direction of the generatrices. The first problem was treated as early as 1914 by *R. von Mises*[5] and the second even earlier by *Lorenz*[6] and *Timoshenko*[7]. For shell systems of large arch and beam spans, however, both these problems appear in combination, so that too favourable results would be obtained if the two cases of buckling were calculated separately. This combined case of buckling, so important in the construction of cylindrical shells, was solved by *W. Flügge*[8] in 1932 and worked out in a very detailed manner and in a very practicable form. Here the influence of the combined buckling becomes apparent in an unfavourable manner. *Flügge*'s investigations also extend to the case of the anisotropic circular cylindrical shell, which is bound to be involved when it is a question of large spans. By means of a transition process *W. Flügge* shows that his equations can also be applied to the special case of buckling in slabs.

As it is assumed when deducing buckling conditions that the deformations of the shell are small in proportion to the thickness of the latter, but that, on the other hand, it is extremely difficult to adhere to this condition in practical constructional work, since very considerable deformations are set up with large span widths, it becomes necessary to reckon with very much higher factors of safety against buckling in shells than in simple arches. These safety factors can easily be attained by reinforcing the shell with ribs. These have further the advantage of being able to reduce the deformations considerably and of simultaneously taking up the bending moments of the shell as well.

During the last few years cylindrical shells have been coming steadily to the fore in practically every country. Such shells have been constructed with girder spans of up up to 60 m and arch spans of 45 m, i. e. with ground plan areas of as many as 2700 square metres. For the reasons mentioned above, elliptical cross-sectional curves have been employed for shells of large arch spans and large shell spans. On the other hand, a number of halls have been constructed with arch spans up to 100 m wide and relatively small intervals between the cross panels. Fig. 2 shows an Airplane Hangar of this type seen from the outside, with a very large arch span; Fig. 3 is an interior view of an Aircraft Hangar of large arch and girder span, for the reproduction of which I am indebted to the courtesy of the German Air Ministry. Figs. 4 and 5 illustrate the use of shells in industrial structures, Fig. 4 being an interior view of Bamberg Postal Car Hall and Fig. 5 depicting the application of cylindrical shells in the form of shed-type roofs for a plate mill in Buenos Aires.

2) *Shed-Type Roofs.*

In shed-type roofs the bent cross-sectional curve of the shells is replaced by a polygon and the shell thus replaced by a system of planes. The problem is naturally quite the same as for the cylindrical shells. Instead of differential equations we have equations of difference of the same order. Now in addition to the bending moments emanating from the shell action appear others from the slab action, since the individual slabs must first transfer their loads by bending moments to the edges of the shed-roof system, the loads being then

Fig. 2.

transferred by the action of the shell, i. e. of the shed-roof, from here to the stiffening panels by means of elongation forces. The bending moments emanating from the shell action were first considered in this problem by *E. Gruber*[9] and *G. Grüning*[10]. Both authors thereby neglected the influence of the rigidity against torsion of the lateral beams. In this respect the abovementioned works were complemented by *R. Ohlig*[11], who also took the rigidity against torsion of the lateral trusses into account in the same manner as has always been customary for shell systems. Plane systems are less economical than

Fig. 3.

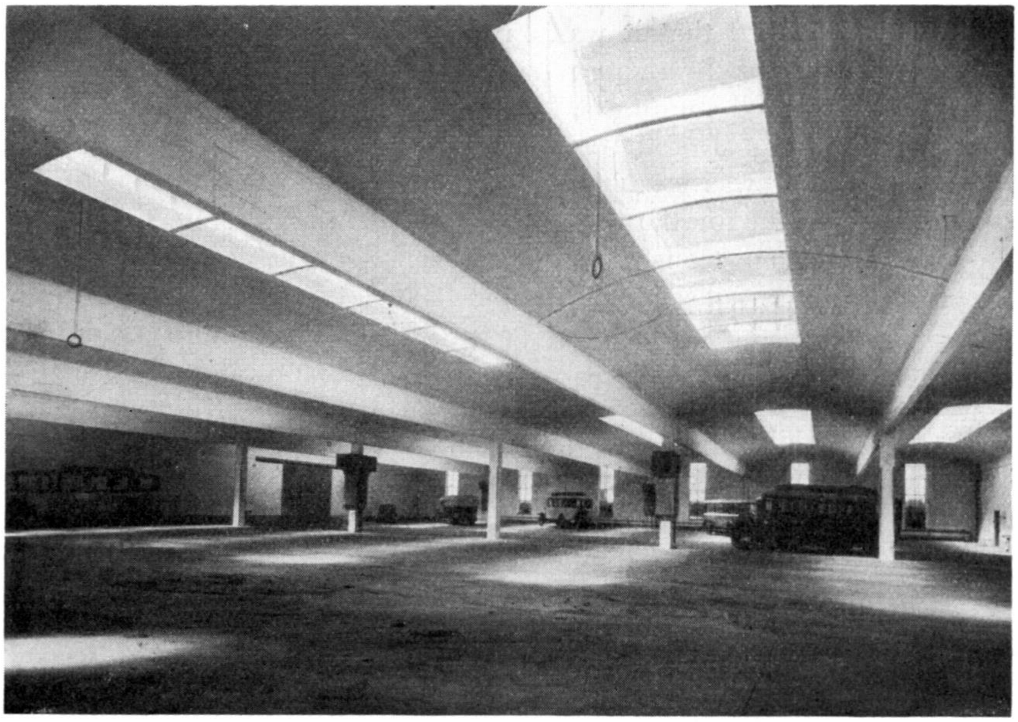

Fig. 4.
Postal motor coach garage, Bamberg

shells in consequence of their greater bending moments, so that as yet no very large structures of this type have been carried out. Of course the reason for this is also to be sought in the fact that the patents for shells and shed-roof types are the property of the one concern — Messers. Dyckerhoff & Widmann AG.

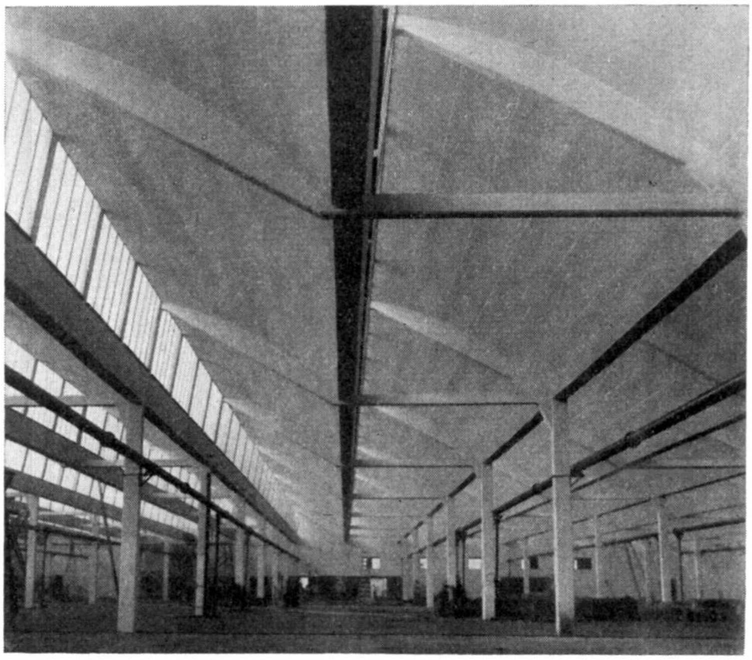

Fig. 5.

3) Polygon Domes composed of Cylindrical Shells.

It will be remembered that it was on this system that the monolithic domes of the great Market Hall in Leipzig, at present the largest in the world with a span of 76 m, and the dome of the Market Hall at Basle, 60 m span, were constructed. Their vaulting is of the so-called monastery type. Although the theory of monastery vaulting was established and published a considerable time ago,[12] the same cannot be said for the theory of cross vaulting. Architecturally beautiful domes, perfect from an acoustical point of view, can be constructed with cross vaulting. Fig. 6 shows a hectagonal dome of this type. Apart from their good acoustical qualities, these domes can be provided with beautiful and effective lighting systems by means of large windows let into the calottes through which the light is reflected right into the middle of the room from the cylindrical shells. I elaborated the theory of these domes in 1930 in a paper written for the competition organized by the Academy of Architecture. In this paper I showed that it is possible to keep the stiffening ridges free from bending moments. As the space allowed me in the present article is not sufficient for the purpose, this theory will shortly be published in a technical periodical.

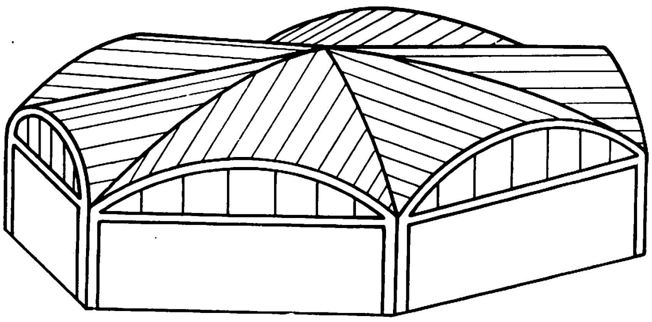

Fig. 6.

4) Shells with Double Curvature.

The membrane and the bending theory of rotary shells continuously supported on their springing has long ago been established. The following types are important forms which have since developed: — a) Rotary shells supported at a few points only, their girder action being superimposed on the dome action so that the shell can transmit its loads to pillars situated a considerable distance away. b) Rotary and translation shells with rectangular or polygon-shaped horizontal projections. c) Apse domes.

The theory of these various forms of shells with double curvature was elaborated by the author in 1930 for the competition already mentioned. The Academy had intended to issue these works in book form, but was obliged to withhold publication owing to lack of funds. I therefore abbreviated the works for publication in the 'Bauingenieur'[13]. As regards rotary shells on single columns it should be mentioned that their girder action produces the astonishing result, coinciding with the well-known slab action, that the height of girder and with it the lever arms of the internal forces are proportional to the distances between the girders in transmitting loads to the pillars. The stresses arising from the girder action are therefore independent of the girder spans.

From this it follows that with these shells, just as with polygon-shaped domes, very large girder spans can be attained. Here, however, the shells do not remain free from bending. A work by *A. Havers*[14], who employs a spheric function to treat and solve the problem of the disturbance of support conditions on a latitude circle of a spherical shell for an arbitrarily selected harmonic, has made it possible also to calculate bending moments arising in shells, knowledge which is absolutely indispensable when carrying out large constructions. It would be extremely useful to carry through the complete calculation of an example, for although this naturally takes a great deal of trouble, it would definitely show what spans can be attained with these shells and whether they posses economic advantages over the shells classified under b), in which the load is transmitted almost entirely by means of elongation forces and shell-thickness therefore depends solely on the safety against buckling. This because the permissible stresses cannot be fully utilised in these forms of shell, even when the spans are maximum. The calculation of the rotary shells with rectangular or polygon-shaped horizontal projection can be effected in a very simple manner by means of the procedure, already quoted, of the differential equations of the membranal condition of stress.

Fig. 7 shows a very flat shell of this type with a rectangular plan; it was carried out for one of the buildings of the Danzig Institute of Technology. The shell has a span of 12 m, but only a rise of 0.77 m. The ratio l/f between rise and span amounts to 15.6, being therefore smaller than for that of the flattest bridge. This illustration shows quite clearly that a shell system of this type is nothing else but a T-beam in space, but one distinguished from the ordinary T-beam in that the whole shell acts as a compressive slab. Fig. 8 depicts the employment of these shells with double curvature and rectangular plan for a clinker factory in Beocin, and also the application of the apse domes mentioned under c). As I elucidated in my article published in the 'Bauingenieur'[13] there is a state of membranal stress in these semi-spherical domes if the shell is stiffened by rings at the springing. As this type of dome can be constructed as an independent structural member and, in combination with cylindrical shells, can be used for structures which are more or less oval in plan, they prove to be a very important new structural element in the construction of large halls or airplane hangars. In the latter form of structure they are therefore frequently employed as terminal features, with spans up to 40 m. The hangar illustrated in Fig. 3, which is constructed of one longitudinal shell, is terminated at both ends by apse domes. Lastly, Fig. 9 shows another semi-apse dome of this type constructed for the Music Pavilion at Schwalbach Spa.

5) *The principle of calculating the statical balancing of masses, applied to affined shells.*

The types of shell discussed in the previous section of this paper can be calculated with the assistance of the differential equations of the state of membranal stress because the spherical shell in itself is easy to estimate mathematically. The principle of the statical balancing of masses now enables us to calculate in an extremely simple manner the forms of shell affined to them as well. I elaborated this principle in 1928 and elucidated it in the 'Handbuch für

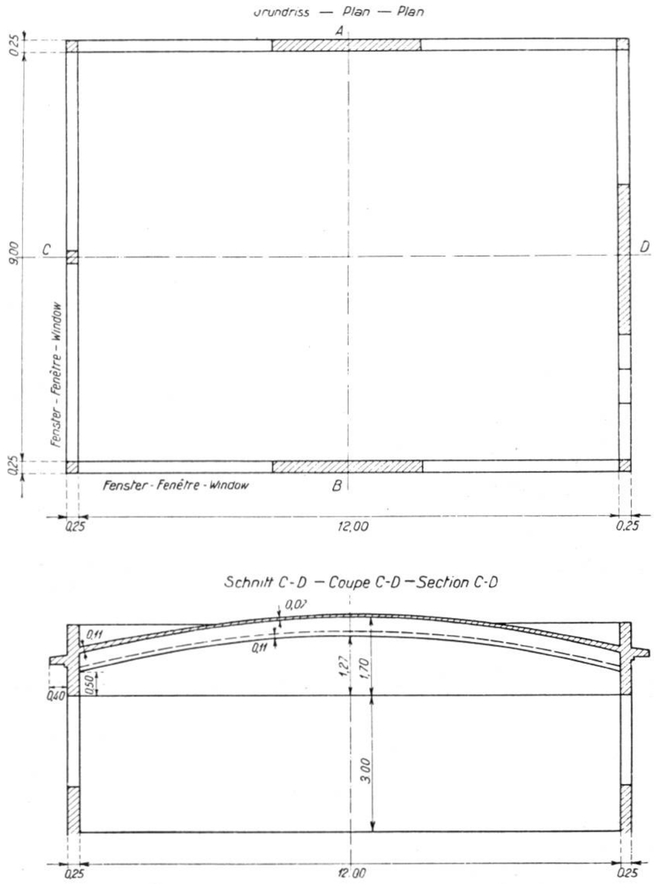

Fig. 7.

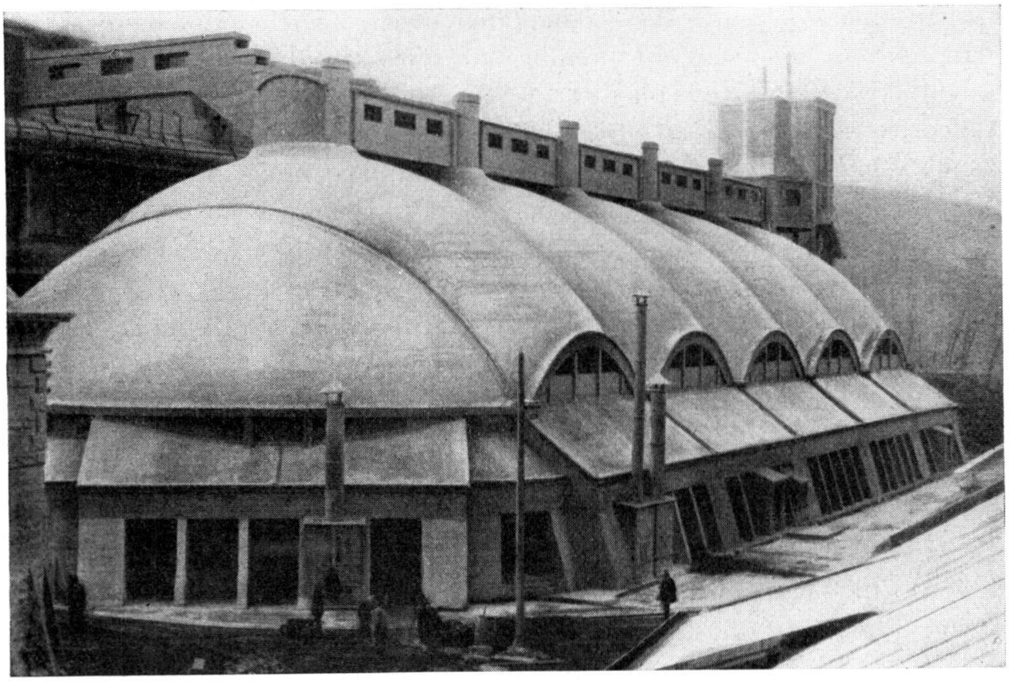

Fig. 8.

Clinker factory, Beocin

Eisenbetonbau'[15] for specific cases. Then, in 1930, I employed the differential equations for shells of any form to give a general outline of the problem: this work was written for the competition already mentioned and was subsequently published in the 'Bauingenieur'.[16] On this basis a shell with elliptical plan, for instance, can be calculated on the basis system of a rotary shell. The numerous other problems that can be solved in this manner are set forth in the above-mentioned article. It may, however, be briefly stated that affined shells can also be estimated in a simple manner.

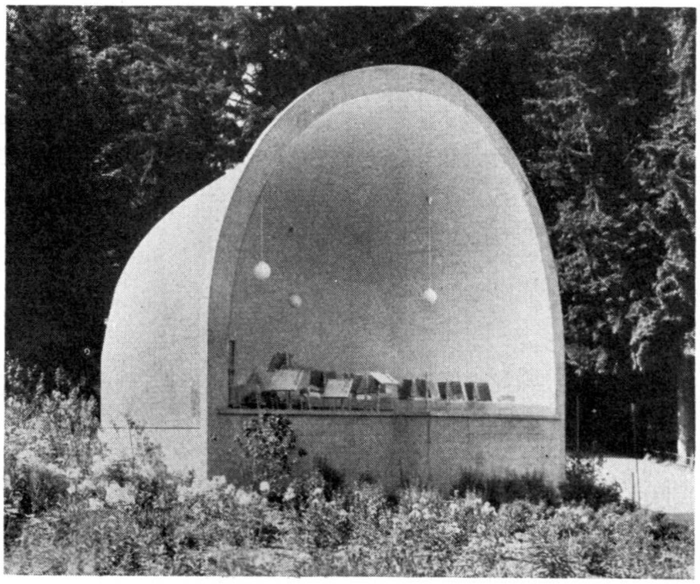

Fig. 9.

Music pavillon Spa Schwalbach

6) Shells of any curvature.

No solutions can be obtained by means of the differential equations of the membranal state of stress for shells with double curvature shaped to any type of surface, because the three partial differential equations thereby evolved cannot be integrated. We are obliged to find another means of approach and solve these equations by difference calculation.

An extremely clear and easily applied method for the solution of problems of this type was elucidated by *Pucher*[17] in 1931. This simple solution is rendered possible by showing that the three differential equations can be combined to form a single one; this is done by introducing a stress function which completely describes the stress conditions. The internal forces of the membranal state of stress can hereby be deduced in a similar manner as from Airy's stress function. As the only assumption that can be made as regards the form of surface is that of continuity, all the forms of shells used in practical construction can be calculated if the circumferential conditions are known and are compatible with the conditions of membranal stress. The method of differences should always be applied if, as mentioned above, a solution is possible with the differential equations. More recent French works follow in principle the line developed by *Pucher*. It is on this theory that the increasing employment of shell construction in France, in the form of non-developable straight-line surfaces, is based. The specific case of translation surfaces has been solved by *Flügge* in the same manner by means of equations of difference.

In conclusion I should like to mention an interesting construction illustrated in Fig. 10. It is the dome in the 'Haus des Deutschen Sports', which was built for the Olympic Games. The Project was prepared by Mr. *Marchi*, architect, and designed by Mr. *U. Finsterwalder*. The skylight is eccentrically placed to afford good ligthting for the platform. The dome in itself, however, exerts no dome action. because the separate shell sectors, which are stiffened by sturdy ribs, project beyond the springing without mutual support.

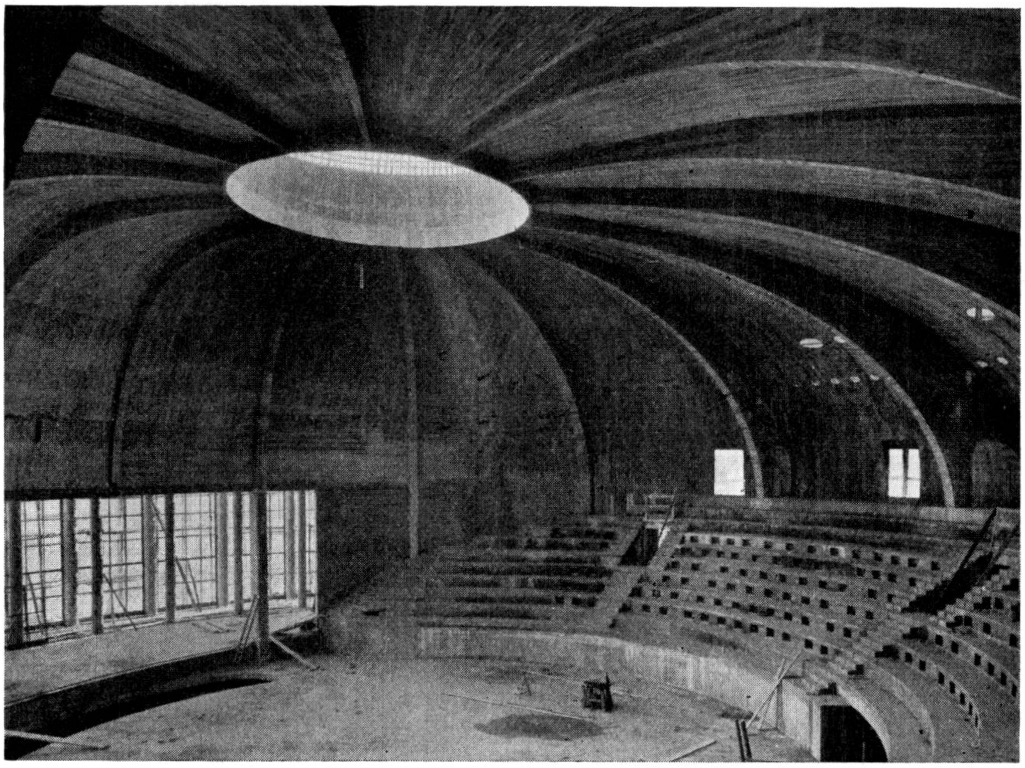

Fig. 10.

„Haus des deutschen Sports". (House of German Sport) Berlin-Reichssportfeld.

References.

[1] *U. Finsterwalder:* Die Theorie der zylindrischen Schalengewölbe (theory of cylindrical shell vaulting), System Zeiß-Dywidag, Intern. Asscn. for Bridge and Structural Engineering, Publ. Vol. 1, 1932, and Diss. München 1930, desgl. Ing. Arch. Bd. IV. (1933).

[2] *Fr. Dischinger:* Die strenge Theorie der Kreiszylinderschale in ihrer Anwendung auf die Zeiß-Dywidag-Schalen (The strict theory of circular shells in its application to Zeiß-Dywidag shells), Beton und Eisen 1935, H. 16—18.

[3] *K. Miesel:* Über die Festigkeit von Kreiszylinderschalen bei nicht achsensymmetrischer Belastung (On the strength of circular cylindrical shells with loading which is not axially symmetrical), Ing. Arch. Bd. 1 (1930).

[4] *W. Flügge:* Statik und Dynamik der Schalen (Statics and dynamics of shells), Jul Springer, Berlin 1934.

[5] *R. v. Mises:* Z. VDI 58, 1914 p. 750 ff.

[6] *R. Lorenz:* Z. VDI 52, 1908 p. 1766 ff.

[7] *S. Timoshenko:* Z. Math. Phys. 58 (1910) p. 378 ff.

For [6] and [7] see also *K. v. Sanden* and *F. Tölke:* Stabilitätsprobleme dünner Kreiszylinderschalen (Problems of stability for thin circular cylindrical shells) Ing. Arch. Bd. 3 1932.

[8] *W. Flügge:* Die Stabilität der Kreiszylinderschale (The stability of circular cylindrical shells) Ing. Arch. Bd. 3 1932.

[9] *E. Gruber:* Berechnung prismatischer Scheibenwerke (Calculation of prismatic slab systems) I.A.B.St.E., Pub. 1 (1932), p. 225, and Pub. 2 (1934) p. 206.

[10] *G. Grüning:* Die Nebenspannungen der prismatischen Scheibenwerke (Secondary stresses in prismatic slab systems) Ing. Arch. Bd. 3 (1932).

[11] *R. Ohlig:* Beitrag zur Theorie der prismatischen Faltwerke (Contribution to the theory of prismatic shed-roof systems) Ing. Arch. Bd. 6 (1935) und: Die Nebenspannungen der Randträger prismatischer Faltwerke (Secondary stresses in the lateral beams of prismatic shed-roof systems) Diss. Darmstadt (1934).

Re: the membrane theory [9] and [11] see further *H. Craemer:* Allgemeine Theorie der Faltwerke (General theory of shed-roof systems) Beton und Eisen 1930, S. 276, and *G. Ehlers:* Die Spannungsermittlung in Flächentragwerken (The calculation of stresses in shell constructions) Beton und Eisen 1930, p. 28.

[12] *Fr. Dischinger:* Theorie der Vieleckskuppeln (Theory of polygon-shaped domes) Diss. Dresden 1929 und Beton und Eisen 1929, p. 100.

[13] *Fr. Dischinger:* Die Rotationsschalen mit unsymmetrischer Form und Belastung (Rotary shells unsymmetrical in form and loading) Bauing. 1935, Heft 35—38.

[14] *A. Havers:* Asymtotische Biegetheorie der unbelasteten Kugelschale (Asymtotic bending theory of unloaded spherical shells) Ing. Arch. 6 1935.

[15] *Fr. Dischinger:* Der Spannungszustand in affinen Schalen und Raumfachwerken (The state of stress in affined shells and lattice constructions in space) Bauing. 1936, p. 128, and further.

[16] *Fr. Dischinger:* Hdbuch für E. B., 3. Auflage Bd. 12 1928, und *W. Flügge* [4].

[17] *A. Pucher,* Beitrag zur Theorie tragender Flächen (Contribution to the theory of load-carrying surfaces) Diss. Graz 1931 und unter dem Titel: Über den Spannungszustand in doppelt gekrümmten Flächen (On the state of stress in surfaces with double curvature) in abgekürzter Form, Beton und Eisen 1934 Heft 19.

Summary.

The theory of slender continuous beams purposely neglects the influences of deformation due to shear, since they are very small. These influences, however, cannot be neglected in the case of continuous stiffened tubes or cylindrical Zeiss-Dywidag shells. In the treatise following, a general procedure for determining these influences is given and the influences themselves are shown by examples. It is also shown that continuity conditions entirely disappear for boundary cases of small spans in relation to the tube diameter.