Zeitschrift: IABSE congress report = Rapport du congrès AIPC = IVBH Kongressbericht

Band: 2 (1936)

Artikel: Combined bending and shear beyond the range of purely elastic deformation

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DOI: https://doi.org/10.5169/seals-3241

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Combined Bending and Shear Beyond the Range of Purely Elastic Deformation.

Biegung mit Querkraft, außerhalb des Gebietes der rein elastischen Formänderung.

Flexion et effort tranchant en dehors de la zone de déformation purement élastique.

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Given the stress strain diagram for simple tension or compression it is possible, assuming that cross sections remain plane, to determine the distribution of stress over the cross section of a beam which is subjected to a shear force simultaneously with bending. An example of this will now be given.

Introduction.

It is known that the total deformation is capable of subdivision into two parts.

a) An elastic strain made up of components which satisfy the following equations of elasticity:

$$e_1 = \frac{1}{E} \cdot \left[\sigma_1 - \frac{1}{m}(\sigma_2 + \sigma_3)\right]; \quad e_2 = \text{etc., and}$$

b) A plastic strain made up of components which satisfy the equation of plasticity:

$$\delta_1 = \frac{1}{D} \cdot \left[\sigma_1 - \frac{1}{2}(\sigma_2 + \sigma_3)\right]; \quad \delta_2 = \text{etc.}$$

wherein E represents Young's modulus and D the modulus of plasticity. In the case of plastic deformations the coefficient of transverse strain m has the value 2.

Hitherto it has been customary to base all statical calculations on the assumption that the behaviour of the structure is purely elastic, but recently attempts have been made to take account also of the influence of plastic deformation on: — 1) The distribution of stress in a beam or over its cross section, and 2) The flow of forces in the structure as a whole (statically indeterminate quantities M, Q and N).

[1] *M. Roš* and *A. Eichinger:* Versuche zur Klärung der Frage der Bruchgefahr. Reports on Discussions, Swiss Federal Testing Station, Zürich. N°. 14 of Sept 1926; N°. 34 of Feb. 1929; N°. 87 of Apr. 1934.

Principles of the theory of plasticity.

It may be recalled[2] that in the case of simultaneous action of a normal and a shear stress the following equations apply:

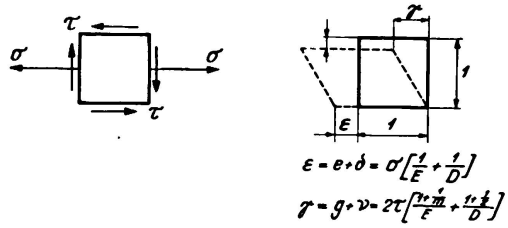

Fig. 1.
State of stress and strain in an element of a body.

for elastic deformation (strain of comparison)

$$e_g = \sqrt{e^2 \cdot \left(1 + \frac{1}{m}\right)^2 + \frac{3}{4} \cdot g^2} = \frac{\sigma_g}{E} \cdot \left(1 + \frac{1}{m}\right);$$

for plastic deformation

$$\delta_g = \sqrt{\delta^2 \cdot \left(1 + \frac{1}{2}\right)^2 + \frac{3}{4} \nu^2} = \frac{\sigma_g}{D} \cdot \left(1 + \frac{1}{2}\right)$$

wherein $\sigma_g = \sqrt{\sigma^2 + 3 \cdot \tau^2}$

denotes the stress of comparison. In such a case the elongation is expressed by

$$\text{elastic} \quad e = \frac{\sigma}{E}$$

$$\text{plastic} \quad \delta = \frac{\sigma}{D}$$

and the specific value of the slip is given by

$$\text{elastic} \quad g = \frac{\tau}{E} \cdot 2 \cdot \left(1 + \frac{1}{m}\right)$$

$$\text{plastic} \quad \nu = \frac{\tau}{D} \cdot 2 \cdot \left(1 + \frac{1}{2}\right).$$

The total deformation is equal to the sum of elastic and plastic deformation, namely,

deformation (strain of comparison) . $\varepsilon_g = e_g + \delta_g$
elongation . . . . $\varepsilon = e + \delta$ and
slip . . . . $\gamma = g + \nu$

If, then, the stress-strain diagram for a particular material subject to ordinary tension or compression is known, it becomes possible by the above formulae to arrive at the fundamental relationship between the stress of comparison and

[2] Discussion by *M. Roš* and *A. Eichinger:* The buckling of rectangular slabs compressed in excess of the elastic limit symmetrically along both axes. Final Report of the First Congress, I.A.B.S.E.

the deformation (strain of comparison) depending on this alone (changes in volume being always of an elastic character), see Fig. 2.

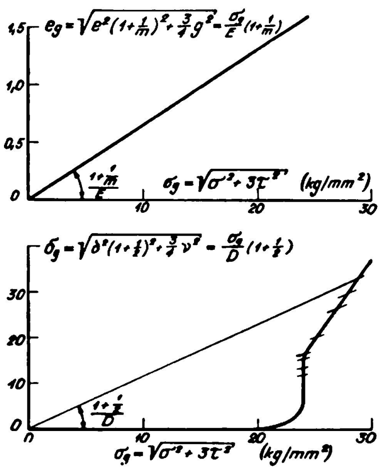

Fig. 2.
Relation of elastic and plastic strain of comparison to the stress of comparison.

Distribution of stress over the cross section of the beam.

Before attempting to estimate the influence of the statically indeterminate quantities M, Q and N upon the flow of forces within the structure it is necessary to determine the effect of plastic deformation on the distribution of stresses over the cross sectional area.

Assuming that the cross section remains plane throughout (or in more accurate language, assuming that the elongation follows a linear law) the elongation undergone by a fibre at any given distance y from the neutral axis is expressed by

$$\varepsilon = \varepsilon_r \cdot \frac{y}{h/2}$$

where ε_r denotes the elongation of the extreme fibre (Fig. 3). Since, moreover, the shear stress and therefore the amount of slip that occurs at the extreme fibre must be zero, it becomes possible to determine the extreme fibre stress σ_r by reference to Fig. 2.

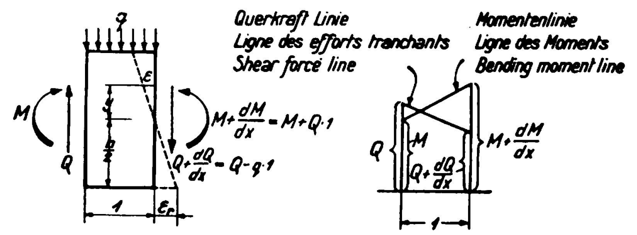

Fig. 3.
Element of a beam of unit length 1 under the influence of M, Q and q.

The distribution of normal stress over the cross sectional area is usally assumed to occur from $\sigma = 0$ to $\sigma = \sigma_r$ by analogy with a portion of the $\sigma - \varepsilon$ diagram. This assumption, however, is only justified if $\tau = 0$ throughout. If this condition is not satisfied the distribution of the normal stresses over the cross section may be very different, for the total elongation ε would be produced by a smaller normal stress σ according to the magnitude of the shear stress τ acting on the same element of material.

For the present, instead of attempting to determine the distribution of σ and τ for a given bending moment and shear force, let us be content with the σ distribution as in Fig. 4. On this assumption τ can be determined for every point in the cross section, since

$$\varepsilon = \varepsilon_r \cdot \frac{y}{h/2} = \sigma \cdot \left[\frac{1}{E} + \frac{1}{D}\right]$$

whence it follows that:

$$\frac{1}{D} = \frac{\varepsilon}{\sigma} - \frac{1}{E}.$$

In other words it is possible to determine the modulus of plasticity D in respect of every value of y and by drawing a line at an angle of $\frac{1+\frac{1}{2}}{D}$ through the origin of the coordinates as far as the σ_g—δ_g curve in Fig. 2 we can find the stress of comparison σ_g and hence determine the required shear stress from the formula

$$\tau = \sqrt{\frac{\sigma_g{}^2 - \sigma^2}{3}}.$$

Since the relation

$$\frac{\partial \tau}{\partial y} = \frac{\partial \sigma}{\partial x}$$

applies in the case of sectional elements of constant width the distribution of stresses σ' and τ' in a neighbouring section is also known, as in Fig. 4. It follows from this that the distribution of normal stresses σ over a cross section does not

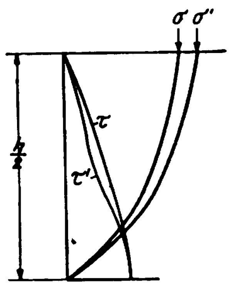

Fig. 4.
Distribution of stress in the upper half of cross section.

here depend entirely on the moment M which exists in this section, as is true for elastic deformation, but is influenced also by the shear force Q and by the distributed load, $q = \frac{dQ}{dx}$.

Strictly speaking, these considerations are valid only on a single occasion of stressing in excess of the limit of purely elastic behaviour. They are, therefore, of little practical importance. Whereas the first time that the elastic limit is exceeded a large amount of plastic deformation takes place at once, the material may fracture through fatigue even without any visible sign of permanent deformation.

It should also be noticed that in spite of the implied change in the upper limit of load or stress in the most heavily stressed member, which is attributable to plastic deformation, no change occurs in the amplitude of the range of loading (B — A) even at the critical points considered (where B represents the upper and A the lower limits of load). Since, however, in most forms of construction the fatigue strength depends mainly on the amplitude of the alternating stresses, and only slightly on the magnitude of the basic stress $\frac{A+B}{2}$, the advantage actually gained is much smaller than might have been expected from the reduction in the upper limit of stress as indicated in Fig. 5. For these

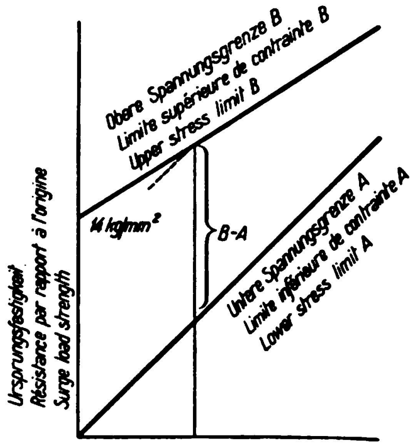

Fig. 5.
Tensile fatigue tests for butt welds. (Swiss Federal Testing Station, Zürich).

reasons the plastic deformation should be kept in reserve against accidental overloading in all cases where fatigue effects are likely to be present, the only exception being where it is due to buckling. Calculations for estimating the factor of safety against fatigue should continue to be based on the principles of elasticity, and exceptions to this rule should be admitted only where they are supported by endurance and fatigue tests as distinct from short period tests.

$\frac{y}{h/2}$	σ	ε	$\frac{\varepsilon}{\sigma}$	$\frac{1}{D}\cdot 10^8$	σ_g	τ	$\frac{\partial\tau}{\partial y}$	σ'	ε'	$\frac{\varepsilon'}{\sigma'}$	$\frac{1}{D}\cdot 10^8$	σ'_g	τ'
	kg/mm²	‰	·10³	mm²/kg	kg/mm²	kg/mm²	kg/mm³	kg/mm²	‰	10³	mm²/kg	kg/mm²	kg/mm²
1.0	25.0	14.58	0.584	0.534	25.0	0	0.40	29.0	23.5	0.810	0.760	29.0	0
0.8	23.2	11.66	0.503	0.453	24.2	4.0	0.35	26.7	18.8	0.705	0.655	27.1	2.7
0.6	20.8	8.75	0.421	0.371	24.0	6.9	0.29	23.7	14.1	0.595	0.545	25.1	4.8
0.4	17.0	5.84	0.343	0.293	24.0	9.8	0.28	19.8	9.4	0.475	0.425	24.1	7.9
0.2	10.5	2.92	0.278	0.228	24.0	12.5	0.20	12.5	4.7	0.376	0.326	24.0	11.8
0	0	0	—	—	—	13.8	0	0	0	—	—	—	13.8

Note: The section σ'—τ' is at a distance $\frac{h}{10}$ from the section σ—τ. See Fig. 4.