The relations M st (P) and M F (P) in girders continuous over three spans carrying a load P in the central span

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Objekttyp: **Article**

Zeitschrift: **IABSE congress report = Rapport du congrès AIPC = IVBH Kongressbericht**

Band (Jahr): **2 (1936)**

PDF erstellt am: **11.07.2024**

Persistenter Link: https://doi.org/10.5169/seals-3245

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The Relations M_{st} (P) and M_F (P) in Girders Continuous over Three Spans Carrying a Load P in the Central Span

(see Preliminary Publication, pages 121—126).[1])

Die Beziehungen M_{st} (P) und M_F (P) beim durchlaufenden Balken mit drei Öffnungen, belastet durch P im Mittelfeld (siehe Vorbericht Seite 126—128).[1])

Les expressions M_{st} (P) et M_F (P) dans la poutre continue à trois ouvertures, soumise à une charge P agissant dans la travée médiane (voir la Publication Préliminaire, pages 121—126).[1])

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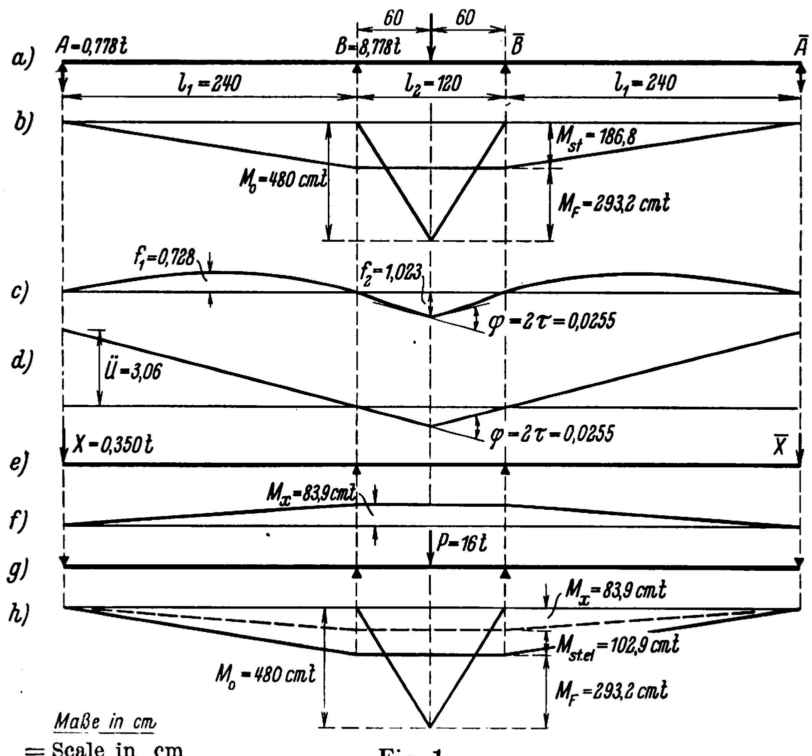
If the girder shown in Fig. 1 be subjected to a load increasing from 0 to P the first result is a bending moment diagram calculable from purely elastic



= Scale in cm

Fig. 1.

Behaviour of girder under a load P = 16 tonnes.

[1] See also the journal „Der Stahlbau", 1936, No. 20, pp. 153 foll.

principles and characterised by M_o, M_{st}, M_F. When $P = P_s$ ($= \sim 11$ tonnes) the yield stress σ_s is attained at the extreme fibre in the middle of the span.

As soon as $P > P_s$ (as, for instance, when P = 16 tonnes), M_F cannot increase appreciably above $M_s = W \cdot \sigma_s$. With the aid of *Mohr's* theorem an expression for the angle φ of the line of bending at the middle of the girder can be derived by considering the line of moments (Fig. 1b) from A to B and from B to the point where the line of bending has a kink. In the present case

$$EI\varphi = 6600\, P - 280\, M_F.$$

If the load is removed the shape of the beam in Fig. 1d shows at the ends an amount ü, and before the load is re-imposed ü has to be eliminated by the two

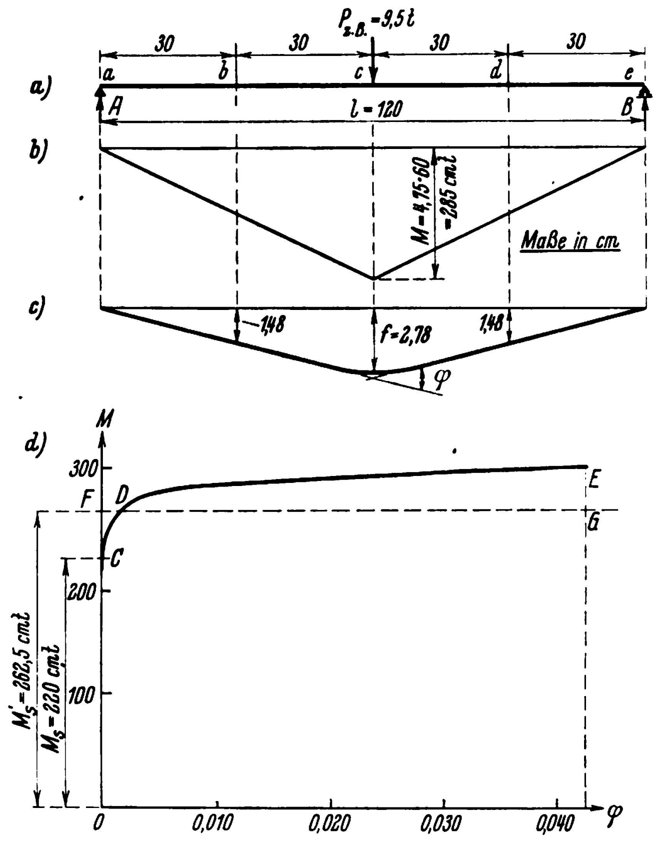


Fig. 2.
Relations of φ (M) in the simple beam.

forces X and $\overline{X}$ corresponding to the moments at the supports M_x. On reloading with P = 16 tonnes, M_x is augmented by the purely elastic support-moment M_{stel} (line of moments corresponding to Fig. 1h).

How great is M_F and, therefore, the support moment $M_{st} = M_o - M_F$ and the values of φ, ü and X?

In a simple beam, for instance in the beam of comparison $l = 120$ cm $= l_2$ the relation φ (M) may be obtained by purely experimental means as shown in Fig. 2, where M is the moment at the middle of the beam. In order to indicate the actual behaviour of the continuous beam this relationship may be transferred to the continuous beam, that is to say $M_{(\varphi)} = M_F\, (\varphi)$. On the other hand we may write the relationship stated above:

$$EI\varphi = 6600\, P - 280\, M_F$$

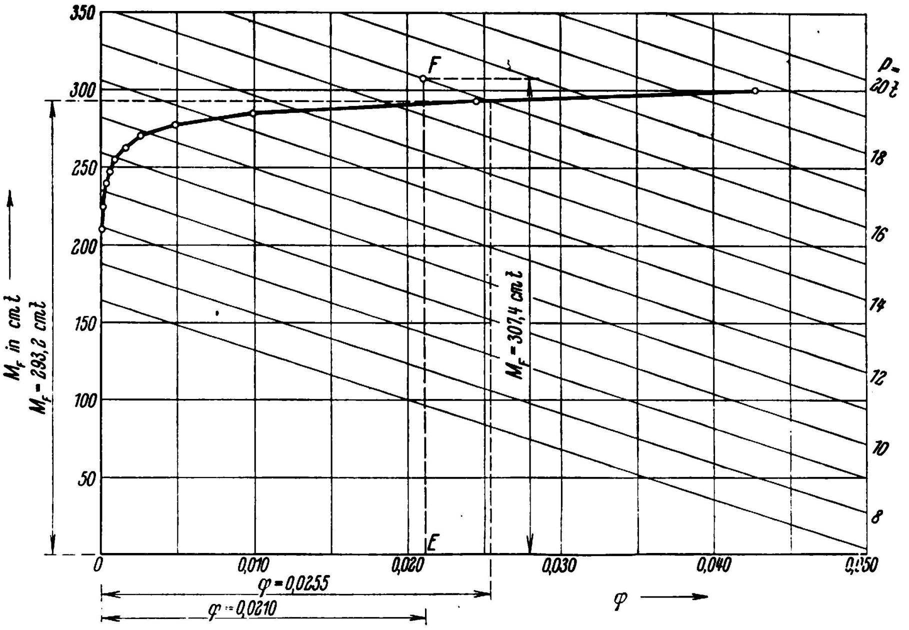


Fig. 3.

Determination of M_F for $l_1 = 240$ cm and $l_2 = 120$ cm, with the aid of $EI\varphi = 6600\ P - 280\ M_F$ (particularly with P = 16 tonnes).

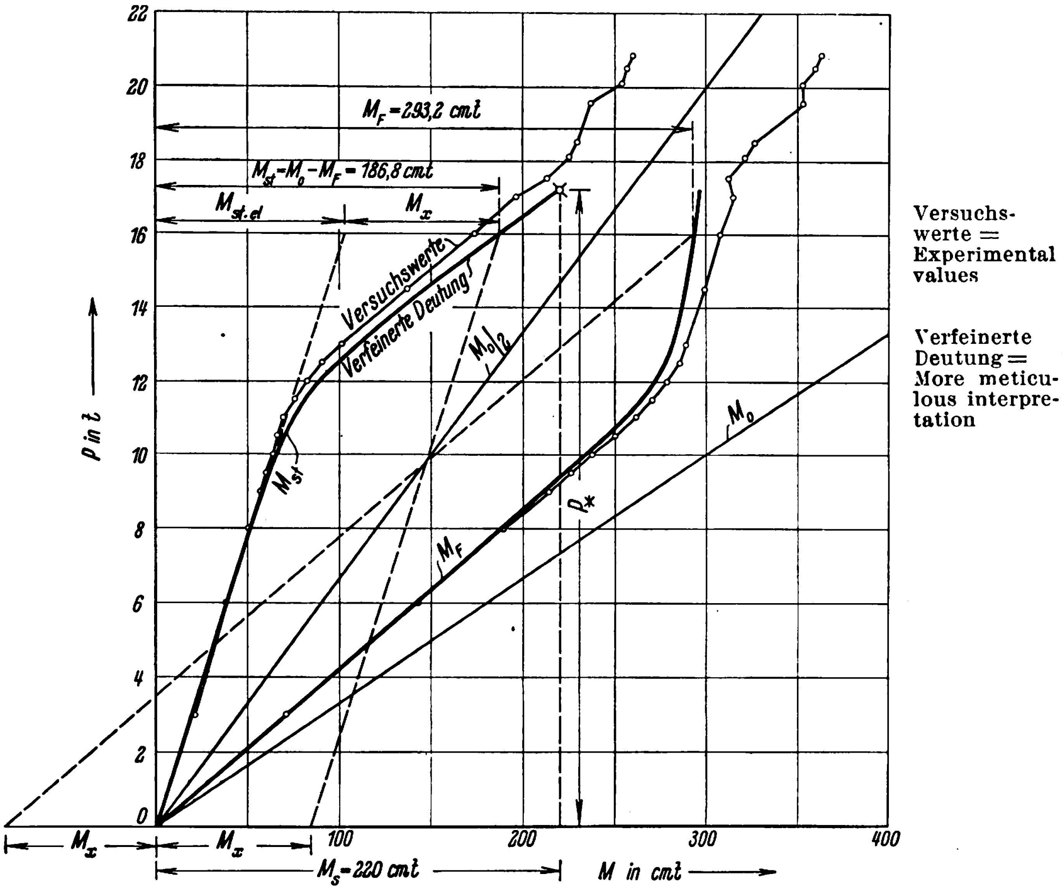


Fig. 4.

Relations M_{st} (P) and M_F (P) for $l_1 = 240$ cm.

Fig. 3 shows both the relationships. Where $P = 16$ tonnes we obtain $M_F = 293.2$ cm tonnes and $\varphi = 0.0255$ (the more precise hypothesis of interpretation).

If this determination is effected for the other loads also, we obtain the picture shown in Fig. 4. M_{st} and M_F at first increase according to linear law, then from $P'_s = 11.12$ tonnes onwards by curves. With $P = 16$ tonnes the values of M_x and $M_{st\,el}$ established above (Fig. 1h) are entered in the diagram. It may also be seen from this diagram how a reimposition of load takes place after removal of load, and that the values in the more exact hypothesis of interpretation agree well with the experimental values indicated by the lighter lines.

In Fig. 3 for $P = 16$ tonnes is given the experimental value $M_F = 307.4$ cm tonnes as the ordinate EF. The ordinates of the curve φ (M_F) are, therefore, greater than those of the curve φ (M) which appertain to the beam of comparison with $l = 120$ cm. This is due to the fact that reference should properly be made for shorter experimental beams of comparison with a span equal to the distance between the points where the moment in the central opening is zero; a fact which is confirmed by experiments carried out subsequently to the Congress with $l = 950$ cm corresponding to P_s and $l = 730$ cm corresponding to P_T.

The foregoing provides a basis for a more exact solution than has hitherto been available of the problem of the actual carrying capacity of continuous girders of structural steel, and closes a gap to which *J. Fritsche* has drawn attention in the journal Der Stahlbau, Vol. 9 (1936), page 67. In future, therefore, as is done in the „Traglastverfahren“ [carrying capacity method] it will not be necessary to rely upon the overprimitive assumption of equalisation of moments.