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Formulae for the Stability of Eccentrically Loaded Steel Columns.

Formelmäßige Lösung des Stabilitätsproblemes exzentrisch gedrückter Stahlstäbe.

Les formules de la stabilité des barres excentriquement comprimées.

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The classical problem of stability of a straight column subjected to a concentric compressive load was satisfactorily solved by the researches of *Euler*, *Engesser* and *Kármán*.[1] This type of loading represents an ideal which hardly never occurs in practice, because even the smallest deviations from the assumptions underlying it — as, for instance, the unavoidable and vanishingly small eccentricities of imposition of the load, or curvatures of the column — cause additional bending which may under certain conditions result in quite a notable reduction of the carrying capacity. In a steel column subjected in this way to axial compression and bending stresses there is also the danger of the occurrence of unstable equilibrium, at any rate in cases where the load has already increased to a value involving permanent deformations. It is on account of this statical condition that the problem of determining the carrying capacity differs fundamentally from that of the buckling of a straight bar.

This special problem of stability was first examined by *Kármán* both theoretically and experimentally in connection with his well known experiments on buckling with very small eccentricities of loading.[1] Here *Kármán*, beginning from the energy-line for a particular kind of steel, developed a powerful graphical method of integration for solving the differential equation of the bending moment line, but this portion of his work remained for a long time unnoticed. The approximate method, developed some thirteen years later by *Krohn*,[2] shared the fate of *Kármán*'s investigations on account of the difficulty of the calculations involved, and still more because its results would not admit of quantitative evaluation. Some years later *Roš* and *Brunner* developed an approximate graphical method, and showed the results of this in a diagram which, for the first time, expressed quantitatively the relation between carrying capacity, eccentricities and

[1] *T. von Kármán:* Untersuchungen über Knickfestigkeit. V.D.I., No. 81, 1910.
[2] *R. Krohn:* Knickfestigkeit. Bautechnik, 1923.

slenderness for a definite law of deformation.[3] Finally *Chwalla,* following up *Kármán*'s line of thought, gave the strict solution applicable to columns with any desired eccentricity of loading.[4]

All these investigations were exposed to the notable defect that results could be obtained from them only after very tedious calculation, and could be represented only in the form of a diagram or of a table of figures. When it is further remembered that a large number of types of steel and shapes of cross section exist which each require different diagrams, it will readily be understood that this circumstance not merely makes them difficult to apply in practice but may actually prevent such application; the clearest indication of this fact being that in the official regulations of nearly all countries (so far as the author is aware the only exception is the Swiss regulations) no account is taken of the new and fully established knowledge now available in reference to the design of eccentrically compressed columns of steel — obviously because no simple formula with a theoretical basis is available.

The author wishes briefly to indicate how a solution to this important problem in steel construction may be expressed as a formula. First of all, the law of deformation for the types of steel now in use may for the present purpose be replaced by an ideally plastic line, having regard to the circumstance that fixation can occur only in extremely short columns, such as hardly ever occur in practice, having a ratio of slenderness of $\lambda < 20$. The assumption that *Hooke*'s law is valid as far as the yield point is amply supported by careful compressive experiments carried out by the German Steelworks Committee.[5] If, further the line of bending moment is replaced by a sine curve, the author's solution by formula[6] is at once obtained for the simplest case of a rectangular cross section, and the results correspond closely with the values obtained from the exact bending moment diagram (the maximum error being 3 %).[7] From this the yield point may be derived in the usual way by means of a compression experiment.

Finally, the author has examined, under the same assumptions, the behaviour under load of eccentrically compressed steel columns in relation to the shape of cross section,[8] in reference to which some clarification is required, having regard to the thin walled sections used in steel work, concerning the conditions of stability both in and at right angles to the moment diagrams. Fig. 1 shows the critical axial stress σ_{kr} for the most frequently used type of St. 37 with an

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[3] Cf. reports of First Meeting and First Congress for Bridge and Structural Engineering, Vienna, 1928, and Paris, 1932.

[4] *E. Chwalla:* Theorie des außermittig gedrückten Stabes aus Baustahl. Stahlbau, 1934. (Summary statement of the strict graphical solution.)

[5] *W. Rein:* Versuche zur Ermittlung der Knickspannungen für verschiedene Baustähle. No. 4 of Reports of Ausschuß für Versuche im Stahlbau. J. Springer, Berlin, 1930.

[6] *K. Ježek:* Näherungsberechnung der Tragkraft exzentrisch gedrückter Stahlstäbe. Stahlbau, 1935. — Die Tragfähigkeit axial gedrückter und auf Biegung beanspruchter Stahlstäbe. Stahlbau, 1936.

[7] *K. Ježek:* Die Tragfähigkeit des exzentrisch beanspruchten und des querbelasteten Druckstabes aus einem ideal plastischen Stahl. Report of Akademie der Wissenschaften, Vienna, Math.-naturw. Kl., Abt. IIa, Vol. 143, No. 7, 1934.

[8] *K. Ježek:* Die Festigkeit von Druckstäben aus Stahl. Julius Springer, Wien 1937.

eccentricity of m = 1 (that is to say, the axial load coincides with the centre of gravity). Beyond this no equilibrium between external and internal forces is possible, depending on the slenderness λ and on the shape of cross section. It is recognised that the effect of the shape of section in small columns is important, but its importance is rapidly lost when the slenderness increases or the eccentricity decreases. The most favourable performance is given by columns of cruciform section, and the least favourable by columns of I and T cross section. In the last two cases, in particular, the critical stress is only slightly above the limiting stress σ_n for the elastic condition.

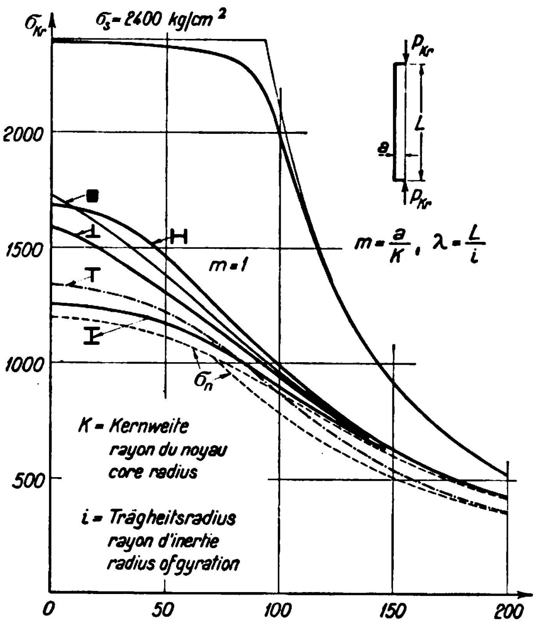


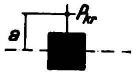
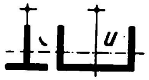
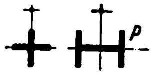
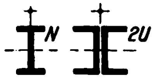
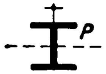
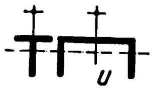
Fig. 1.

The line of critical stress drawn for m = 0,01 (the σ_k line) is almost independent of the shape of cross section, and in the case of columns of medium slenderness this clearly indicates the effect of exerted on the carrying capacity by a vanishingly small eccentricity of $^1/_{100}$ of the core radius.

Finally, it is possible, by considering columns of any desired cross section, to obtain approximate formulae as in Table 1 applicable to any type of steel, the coefficients μ_1 und μ_2 being dependent on the type of section. In the case of columns of T cross section below a particular value of axial stress a second formula, similarly derived, is to be used. This basis of calculation applies quite generally to steel columns subject to axial pressure and to bending, when the measure of eccentricity (in the sense explained) is taken as the relation between the bending moments (referred to the undeformed axis of the column) and the axial load. For further explanations the author would refer to his published papers. For $\mu_1 = 1$ and $\mu_2 = 0$ there is available the formula corresponding to the σ_n line (shown as a broken line in Fig. 1).

For the purpose of the practical design of columns which are loaded *concentrically in the "design" sense of the word*, the author would propose to adopt as a value of the unavoidable eccentricity $^1/_{100}$ of the core radius. There will then be obtained, with $m_o = 0{,}01$ and $\mu_1 = 1$, $\mu_2 = 0$, a simple *"buckling formula"* which will correspond to the fact (now accepted without question) that the limit of compression σ_s represents the limit of buckling stress, so that practically, in the case of very slender columns, we arrive back at the *Euler* formula. It should be stated that these formulae are well confirmed by the published experimental results, and that they ensure a safe estimate of carrying capacity while reducing the amount of calculation required. The problem may thus be regarded as having been satisfactorily cleared up, both from a theoretical and from a practical standpoint.

Table I. Bases of Calculation for Eccentrically Loaded Steel Columns.

Cross Section	Formula for Critical Slenderness	Region of Equilibrium	Co-efficients μ_1	Co-efficients μ_2	Remarks
	$\lambda^2 = \frac{\pi^2 E}{\sigma_{Kr}} \left[1 - \mu_1 \frac{m\,\sigma_{Kr}}{(\sigma_s - \sigma_{Kr})}\right]\left[1 - \mu_2 \frac{m\,\sigma_{Kr}}{(\sigma_s - \sigma_{Kr})}\right]$	Unlimited $0 \leq \sigma_{Kr} \leq \sigma_s$	0.5	0.5	L = Length of column F = Area of cross section $W_{1,2}$ = Resisting moment of bending compression or bending-tension edge i = Radius of gyration $\lambda = \frac{L}{i}$ = Slenderness a = Eccentricity $m = \frac{a\,F}{W_1}$ = Measure of eccentricity σ_s = Yield point E = Modulus of elasticity σ_{Kr} = Critical stress P_{Kr} = $F \cdot \sigma_{Kr}$ = Carrying capacity
			0.5	0.5	
			0.4	0.4	
			0.9	0.1	
			0.9	0.1	
		$\frac{\sigma_{Kr}}{\sigma_s} \geq \frac{W_1 - W_2}{W_1 + W_2}$	0.8	0.2	
	$\lambda^2 = \frac{\pi^2 E}{\sigma_{Kr}} \left[1 - \mu_1 \frac{W_1\,m\,\sigma_{Kr}}{W_2(\sigma_s + \sigma_{Kr})}\right]\left[1 - \mu_2 \frac{W_1\,m\,\sigma_{Kr}}{W_2(\sigma_s + \sigma_{Kr})}\right]$	$\frac{\sigma_{Kr}}{\sigma_s} \leq \frac{W_1 - W_2}{W_1 + W_2}$			