IIa. Influence of stationary and of repeated loading

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IIa

Influence of stationary and of repeated loading.

Einfluß dauernder und wiederholter Belastung.

Endurance — Résistance aux efforts répétés statiques ou dynamiques.

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IIa 1

Permissible Concrete Stresses in Rectangular Reinforced Concrete Sections under Eccentric Loading.

Zulässige Betondruckspannungen in rechteckigen Eisenbetonquerschnitten bei außermittigem Druck.

Contraintes de compression admissibles dans les sections de béton armé rectangulaires sollicitées excentriquement.

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Several investigators have raised objections to the usual method of designing reinforced concrete sections in bending or bending combined with compression, by the method based on the assumption of a straight line relation between stresses and strains in the concrete under compression. Nevertheless, the method is still in general use, and the Building Regulations of nearly all countries are based thereon.

In previous publications[1] and[2] the author has presented a method wherewith the *ultimate moments* or the *ultimate loads* of reinforced concrete members with rectangular cross-section may be computed in fair agreement with the results of actuals tests. On the basis of the ultimate carrying capacity of any rectangular section, determined in this way, the usual method of design may be tried out. Investigation will show how far the method meets the fundamental requirement that the same desired *factor of safety* should be maintained with different grades of concrete, different percentages of reinforcement and different eccentricities of load, and the most suitable working stresses may be determined. The case of simple bending has already been treated[3]; here the case of bending combined with compression will be investigated. Only short members with negligible deflections are considered.

1) Computation of Ultimate Loads.

The usual distinction must be made between *over-reinforced* and *normally reinforced* sections. Failure of the former starts on the compression side of the

[1] *A. Brandtzaeg:* „Der Bruchspannungszustand und der Sicherheitsgrad von rechteckigen Eisenbetonquerschnitten unter Biegung oder außermittigem Druck.“ Norges Tekniske Höiskole, Avhandlinger til 25-ars jubileet 1935, F. Bruns Bokhandel, Trondheim, page 677 to 764.

[2] *A. Brandtzaeg:* Det kgl. norske Videnskabers Selskabs Skrifter 1935, Nr. 31, F. Bruns Bokhandel, Trondheim.

[3] *A. Brandtzaeg:* „Die Bruchspannungen und die zulässigen Randspannungen in rechteckigen Eisenbetonbalken.“ Beton und Eisen, Vol. 35, No. 13, July 5, 1936, pages 219 to 222.

section; no yielding of the tensile reinforcement occurs during failure. With normally reinforced sections the failure starts with yielding of the tensile steel; through opening of the crack of failure the compression area is subsequently reduced and finally crushed. In intermediate cases the two types of failure overlap. While in the case of simple bending the type of failure depends only on the properties of the materials and the percentage of reinforcement, it is, in the case of bending combined with compression, dependent also upon the eccentricity of the load.

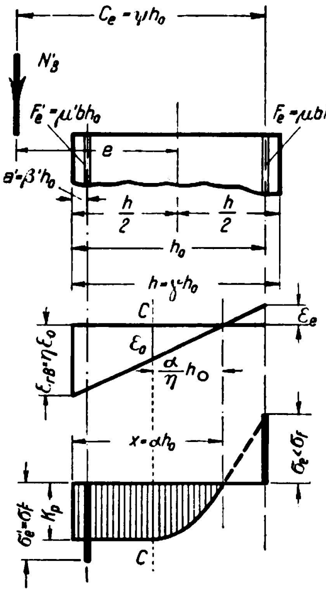

Fig. 1.

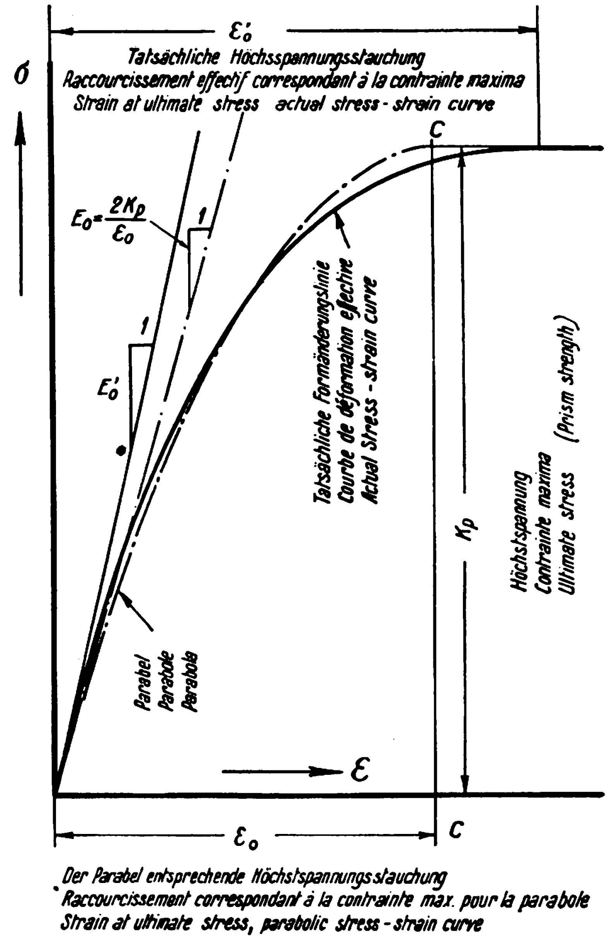

Fig. 2.

a) *Over-reinforced Sections.*

At the failure of a reinforced concrete member in bending or bending and direct compression the ultimate strain on the compression side of the member, ε_{rB}, is very much larger than the strain, ε_o, at which the same concrete under axial compression would reach its ultimate stress, the prism strength, K_P. The size of this *ultimate strain on the compression side* determines to some extent the ultimate carrying capacity of the member. It may be conveniently expressed by means of the *ultimate strain ratio,* $\eta = \frac{\varepsilon_{rB}}{\varepsilon_o}$.

In Figs. 1 and 3 is shown the distribution of stress which is assumed for a section at the stage of failure in bending or bending with compression. Where the compressive strain is smaller than ε_o (to the right of the lines C-C in Figs. 1

and 3) the stresses vary according to the stress-strain curve of the concrete in simple compression (Fig. 2). Where the strain is larger, the stress remains constant equal to the prism strength of the concrete, K_P. The steel stresses also correspond to the strains. Compression steel of mild or intermediate grade generally will have passed its yield point before the stage of failure is reached. No account is taken of tension in the concrete.

The above assumptions are in agreement with the author's own tests (See [1], pages 728 to 735 and [2], pages 54 to 61). *Saliger* has made similar assumptions on the basis of his tests.[4]

For the purpose of the analytical treatment the following equation, proposed by *Talbot,* is substituted for the actual stress strain curve of the concrete:

$$\sigma = E_0 \varepsilon \left(1 - \frac{1}{2} \frac{\varepsilon}{\varepsilon_0}\right) \tag{1}$$

Here σ is the compressive stress and ε the corresponding strain, ε_0 is the abscissa of the vertex of the parabola (Fig. 2) and E_0 defines the slope of the tangent to the parabola at zero stress. By suitable choice of the values of E_0 and ε_0 the parabola is fitted as well as possible to the actual stress-strain curve. Generally E_0 should then be chosen somewhat smaller than the actual modulus of elasticity of the concrete, E'_0, and ε_0 somewhat smaller than the strain, ε'_0, at which the concrete actually reaches its ultimate stress, K_p (Fig. 2). (See [1], pages 738—739 and [2], pages 64—65.)

Other curves, as for instance the one proposed by *von Emperger,*[5] agree somewhat more closely with the actual stress-strain curve. With the curve proposed by Talbot, however, the analysis is simpler, and the curve is sufficiently accurate for the present purpose. In 9 tests made by the author, the error arising from the use of Talbot's curve instead of the actual stress strain diagram of the concrete amounted for the ultimate loads to $\div$ 4.6 to + 1.0 per cent, average $\div$ 0.48 per cent, and for the ultimate moments to $\div$ 0.7 to + 0.7 per cent, average + 0.13 per cent (See [1], page 732 and [2], page 58, Table 8, Columns 13 and 14).

The computation should be made separately for the two cases, Fig. 1 und Fig. 3, with the neutral axis inside and outside the cross-section, respectively.

In the first case the distance to the neutral axis, defined by the ratio $\alpha = \frac{x}{h_0}$, is given by the equation:

$$\left[\frac{1}{2} - \frac{1}{3\eta} + \frac{1}{12\eta^2}\right]\alpha^3 - (1-\psi)\frac{3\eta - 1}{3\eta}\alpha^2 + [2n\eta\psi\mu - (1-\psi-\beta')m'\mu']\alpha - 2n\eta\psi\mu = 0 \tag{2}$$

[4] *R. Saliger:* „Versuche über zielsichere Betonbildung und an druckbewehrten Balken." Beton und Eisen, Vol. 34, No. 1 and 2, Jan. 5 and 20, 1935, pages 12 to 18 and 26 to 29.

[5] *F. v. Emperger:* „Die Formänderung des Betons unter Druck." International Association for Testing Materials, Congress in Zürich 1931, pages 1149 to 1159. — See also Beton und Eisen, Vol. 35, No. 10, May 20, 1936, page 179.

Here $n = \frac{E_o}{E_e}$ (E_e is the modulus of elasticity of the tensile steel) and $m' = \frac{\sigma'_F}{K_p}$ (σ'_F is the yield point of the compression steel). The other notation is shown in Figs. 1 and 3.

The ultimate load then is:

$$N'_B = \frac{1}{\psi}\left[\alpha\left(1-\frac{\alpha}{2}\right)-\frac{\alpha}{3\eta}\left(1-\alpha+\frac{\alpha}{4\eta}\right)+m'\mu'(1-\beta')\right]bh_oK_P \qquad (3)$$

The unit stress in the tension reinforcement is found to be:

$$\sigma_e = 2\,n\eta\,\frac{1-\alpha}{\alpha}K_P \qquad (4)$$

In the second case, $\alpha > 1$, Fig. 3, we obtain two equations for the ultimate load. Equilibrium of the axial forces requires:

$$N'_B = \left[\frac{3\eta-1}{3\eta}\alpha-\frac{\eta}{\alpha}(\alpha-\gamma)^2\left(1-\frac{\eta}{3\alpha}(\alpha-\gamma)\right)+m'\mu'+2\,n\eta\mu\frac{\alpha-1}{\alpha}\right]bh_oK_P \qquad (5a)$$

and the equilibrium of moments about the center of gravity of the tension reinforcement gives:

$$N'_B = \frac{1}{\psi}\left\{\alpha\left(1-\frac{\alpha}{2}\right)-\frac{\alpha}{3\eta}\left(1-\alpha+\frac{\alpha}{4\eta}\right)+m'\mu'(1-\beta') + \frac{\eta}{\alpha}(\alpha-\gamma)^2\left[\frac{\alpha+2\gamma}{3}-1+\frac{\eta}{3}\left(1-\frac{\alpha+2\gamma}{4}\right)-\gamma\frac{\eta}{\alpha}\left(\frac{1}{3}-\frac{\gamma}{4}\right)\right]\right\}bh_oK_P \qquad (5b)$$

From the equations (5a) and (5b) N'_B may be determined graphically.

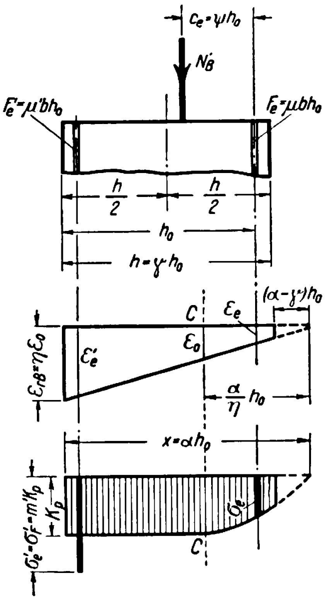

Fig. 3.

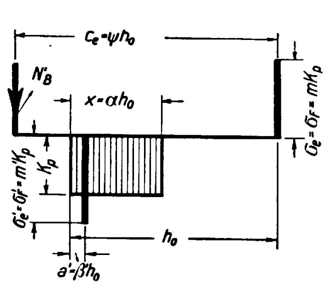

Fig. 4.

b) *Normally Reinforced Sections.*

In the vicinity of the crack that opens up at failure, the compressive stress in the concrete may be taken as constant over the entire compression area of the section (Fig. 4). The resulting error is very small, as is shown in [1], page 698 and [2], page 24. The stress in the tensile steel is assumed to be equal to the yield point, σ_F, as discussed in [3], Sections 4 and 6. After the steel has started to yield, there can be no influence of shrinkage or other tensile stresses in the concrete on the steel stress at the crack of failure.

With these assumptions we have:

$$\alpha = -(\psi - 1) + \sqrt{(\psi - 1)^2 + 2\,m\mu\psi - 2\,m'\mu'(\psi - 1 + \beta')} \tag{6}$$

and

$$N'_B = \frac{1}{\psi}\left[\alpha\left(1 - \frac{\alpha}{2}\right) + m'\mu'(1 - \beta')\right] bh_o K_P \tag{7}$$

where $m = \frac{\sigma_F}{K_P}$.

With the load acting inside the cross-section, the ultimate load, N'_B, according to Equation (7), is quite large, and it increases very rapidly with decrease of the eccentricity and increase of the percentage of reinforcement. In actual fact, therefore, with $\psi < 1$ nearly all cross-sections conforming to ordinary practice are to be classed as *fully* reinforced, as discussed below, Articles 5 and 6.

2) Values of the Constants K_P, n and η.

By means of the above equations we may compute the ultimate load on any rectangular reinforced concrete member under bending combined with direct compression, provided the constants K_P, n (E_o) and η are known for the particular concrete in question. To make the equations applicable in all cases, the constants should be known as direct functions of some known numerical criterion of the quality of the concrete, as for instance the cube strength, K_W. No such functions, correct under all conditions, are, however, available. The relation of the prism strength, the modulus of elasticity and the ultimate strain ratio to the cube strength varies with a series of conditions, as for instance with the moisture content and the porosity of the concrete, the properties of the cement and the aggregates, etc. Nevertheless it seems possible to state general relations which will be sufficiently accurate for the purpose of a general investigation of the variation of the ultimate load with the quality of the concrete, the percentage of reinforcement and the eccentricity of the load. Better agreement with actual tests in any particular case may, of course, be obtained by determining at least K_P and E_o experimentally. The following relations are based mainly on the tests described in papers [1] and [2]:

$$K_P = 0{,}77\,K_W \tag{8}$$

$$E_o = 95\,500 + 390\,K_W \ \text{kg/cm}^2 \tag{9}$$

$$\eta = 1{,}25 + \frac{400}{K_W} - \frac{K_W}{400} \tag{10}$$

These relations have been used in the computations to follow, for concretes with $K_W = 100\ \text{kg/cm}^2$ to $300\ \text{kg/cm}^2$.

Equation (10) represents fairly well the *lowest values of the ultimate strain ratio* found in tests by the author and by *Saliger.*[4] More extensive experiments are, of course, needed to determine what wider field of application the relation may be given. The fact that the ultimate strain ratio decreases with increase of the concrete strength is particularly important (see [3], page 221). One might, perhaps, expect η to decrease with the eccentricity of the load. The tests, however, have shown no regular variation of η with variation of the eccentricity (see [1], page 739 and [2], page 65, Table 9, Column 9).

3) Comparison of Computed with Actual Ultimate Loads.

The tests described in the papers [1] and [2] included the testing of 9 over-reinforced and 4 normally reinforced specimens with eccentrically applied axial loads, with $\psi = 0.661$ to 1.855. The specimens had 0.70 to 4.64 per cent of tensile reinforcement. The concrete used in the tests gave rather unusual values of the ratio K_P/K_W. When the actual values of K_P, as found in the tests, and also the test values of E_0 and η (which, however, are in fair agreement with equations (9) and (10)) are entered in the computations according to equations (2) to (5), ultimate loads are found, which for two of the three groups of over-reinforced specimens agree well with the test results. The greatest deviation is 12 per cent and the average deviation for the 6 specimens of these groups is 5 per cent. On account of differences in the compacting of the concrete in different kinds of specimens, the tests with the third group of over-reinforced specimens gave no basis for such comparison. Also for these specimens, however, the influence of variations in the eccentricity of load and the percentage of reinforcement seems to be well represented by the equations of Section 1 (see [1], page 744 and [2], page 70, Table 10, Column 8).

The actual ultimate loads of the four normally reinforced specimens were on the average 8,8 per cent greater than computed on the basis of the actual values of K_P. When the cube instead of the prism strength is entered in Equations (6) and (7), the actual ultimate loads are on the average 1.7 per cent smaller than the computed ones. It does, in fact, seem probable that during a local failure like that taking place in normally reinforced specimens, the compressive stress in the concrete may well reach a value equal to the cube strength. For the sake of safety, however, the prism strength is used in the computations.

The most complete series of tests of reinforced concrete in bending combined with compression, known to the author, is the one carried out by *Bach* and *Graf*.[6] In Table 1, Column 14, are given the average ultimate loads of the 15 groups of test specimens. The average dimensions, percentages of reinforcement and eccentricities of load are listed in columns 2 to 12, according to the report in paper.[6] The average cube strength of the concrete was $K_W = 225\ \mathrm{kg/cm^2}$, consequently $K_P = 0.77\ K_W = 173\ \mathrm{kg/cm^2}$, which agrees with the test results for plain specimens in centric compression (see [6], Table 24). According to the equations (9) and (10) we then have $n =$ about 11.5 and $\eta =$ about 2.5. The ratios m and m' have been determined from the values of the yield point of the steel shown in Table 3 of paper [6]. With the constants thus determined, the ultimate loads of the 15 groups of specimens have been

[6] *C. Bach* and *O. Graf*: „Versuche mit bewehrten und unbewehrten Betonkörpern, die durch zentrischen und exzentrischen Druck belastet wurden." Forschungsarbeiten auf dem Gebiete des Ingenieurwesens, No. 166 to 169, 1914.

computed, see Table 1, Column 13. As seen from Column 15 the agreement between the computed and the actual ultimate loads is good. For one of the groups of plain concrete specimens the computed load fell 15.3 per cent below the actual value, otherwise the deviations vary between — 3.98 per cent and + 5.15 per cent. The average deviation of the computed from the actual loads for all 15 groups amounts to — 1.13 per cent.

Slater and *Lyse* have tested two plain concrete prisms under eccentric loading.[7] The dimensions of the prisms were $20.3 \times 20.3 \times 30.5$ cm, the prism strength of the concrete was K_P 285 kg/cm² and consequently the probable cube strength about $K_W = 370$ kg/cm². When η is computed from Equation (10), α from Equation (2) and N'_B from Equation (3) with $\mu = \mu' = 0$, we obtain $N'_B = 74.4$ t. The actual average ultimate load was 70.5 t, that is 5.3 per cent smaller than computed.

In the above cases it is seen that the ultimate loads computed from the equations of Article 1 agree fairly well with the results of tests. It seems, therefore, that the equations may at least be used as the basis of a general investigation of the variation of the ultimate load of eccentrically loaded members with the eccentricity of the load and the percentage of reinforcement.

4) The Factor of Safety.

The permissible or *safe loads* may be computed by dividing the ultimate loads by the *factor of safety*. The proper choice of the factor of safety has been discussed at some length in previous publications (see [1], pages 688 to 693; [2], pages 14 to 19 and [3], pages 221 to 222). If an *actual* factor of safety of 2 is desired, the *nominal* factor of safety for simple compression should be raised to 3.3 or 3.4, on account of the influence of long-time or repeated loads and on account of the difference in strength due to difference in size between ordinary structural members and usual test specimens. *The proposed new Norwegian Building Regulations for Reinforced Concrete*, designated as NS 427, the first part of which was published for discussion in the autumn of 1935,[8] are based on factors of safety in simple compression of 4.13, 3.85, 3.65 and 3.60 respectively for the four Standard Concretes A to D with cube strengths of 290 kg/cm², 230 kg/cm², 180 kg/cm² and 140 kg/cm² respectively.

Certain differences in the manner of failure of concrete in simple compression and in bending or bending with compression, make it seem desirable to have a factor of safety 10 % higher for bending and bending with compression than for simple compression. (See [1], pages 751 to 754; [2], pages 77 to 80 and [3], page 222.) The factors of safety for bending and bending with compression to correspond with the above values then should be 4.54, 4.24, 4.02 and 3.96 respectively for the Standard Concretes A to D. These values are used in the computations referred to below.

For the reinforcement there is no such difference between the actual and the

[7] *W. A. Slater* and *Inge Lyse:* „Compressive Strength of Concrete in Flexure as Determined from Tests of Reinforced Beams." Proceedings, American Concrete Institute. Vol. 26. 1930. in particular pages 852 to 859.

[8] „Forslag til Norsk Standard: Regler for utførelse av arbeider i armert betong — NS 427, utarbeidet av Den Norske Ingeniørforening." Supplement to Teknisk Ukeblad No. 38, 1935.

Table I.

Actual and calculated rupture loads for eccentric loading according to tests by Bach and Graf 1914.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Test piece No.	Reinforcement	Eccentricity of load measured from the axis of the test piece	Mean dimensions					Mean percentage of reinforcement		Position of load (fig. 1, 3, 4)		Rupture load			Notes
			b	h	a	h_o	a'	μ	μ'	c_e	$\psi = \frac{c_e}{h_o}$	Calculated N'_B	Mean test result N_B	$\frac{N'_B - N_B}{N_B}$	
		cm	cm	cm	cm	cm	cm	%	%	cm		tons	tons	%	
75, 88, 142	0	10	40.1	40.2	0	40.2	0	0	0	30.1	0.749	138.0	136.0	+ 1.47	without reinforcement
76, 89, 143	0	15	40.1	40.1	0	40.1	0	0	0	35.05	0.874	69.3	81.8	— 15.30	„ „
82, 90, 97	4 ø 16	0	40.1	40.1	3.4	36.7	0	0.559	0	16.65	0.454	277.0	280.3	— 1.18	heavy reinforcement
85, 91, 94	„	20	39.9	40.1	3.6	36.5	0	0.564	0	36.45	0.999	93.6	93.0	+ 0.65	normal „
86, 92, 95	„	30	40.0	40.1	3.6	36.5	0	0.567	0	46.45	1.272	57.9	60.3	— 3.98	„ „
87, 93, 96	„	50	40.0	40.1	3.9	36.2	0	0.570	0	66.15	1.830	28.9	30.0	— 3.67	„ „
107, 108	8 ø 16	10	40.0	40.1	3.7	36.4	3.1	0.558	0.560	26.35	0.724	198.3	202.5	— 2.07	heavy reinforcement
99, 102, 118	„	20	40.1	40.1	3.6	36.5	3.3	0.558	0.556	36.45	0.999	119.3	124.0	— 3.79	normal „
119, 120, 121	„	20	40.1	40.2	3.6	36.6	3.3	0.558	0.555	36.50	0.998	119.0	123.3	— 3.49	„ „
100, 103	„	30	40.1	40.3	3.5	36.8	3.3	0.554	0.552	46.65	1.269	69.3	69.6	— 0.43	„ „
101, 104	„	50	40.2	40.2	3.6	36.6	3.3	0.558	0.552	66.50	1.818	33.3	32.4	+ 2.78	„ „
140, 141	8 ø 22	10	40.0	40.3	3.7	36.6	3.8	1.045	1.043	26.45	0.723	236.6	225.0	+ 5.15	heavy reinforcement
63, 122, 137	„	20	40.1	40.1	3.8	36.3	3.7	1.047	1.050	36.25	0.999	164.8	157.5	+ 4.63	„ „
123, 138	„	30	40.1	40.1	3.7	36.4	3.8	1.044	1.045	46.35	1.272	105.5	105.0	+ 0.48	normal „
65, 124, 139	„	50	40.1	40.1	3.8	36.3	3.7	1.050	1.048	66.25	1.825	55.1	53.5	+ 3.00	„ „

Constants of material: $\eta = 2{,}5$; $n = 11{,}5$. For rounds of 16 mm ø: $\sigma_F = 3773$ kg/cm², $\sigma'_F = 3680$ kg/cm², $K_P = 173$ kg/cm².
For rounds of 22 mm ø: $\sigma_F = 3672$ kg/cm², $\sigma'_F = 3754$ kg/cm².

nominal factor of safety, since for mild and intermediate steel the tensile strength which can be relied upon under such repetition of loading as occurs in most reinforced concrete structures, will come very close to the yield point of the steel, which is the stress used in computing the ultimate loads of normally reinforced members according to Equations (6) and (7). Consequently the nominal factor of safety may be chosen equal to the actual factor of safety desired. In the computations referred to below a factor of safety of 1.8 was used for normally reinforced sections. This should be fully sufficient for a uniform material like steel.

5) Safe Loads and Limiting Points.

In Fig. 5 are shown the safe loads, N_{zul}, computed as described above, for a section with tensile reinforcement only ond for a symmetrically reinforced section. The computation was made for the following case: Position of load 1.5 h_o from the tensile reinforcement (moment arm ratio, $\psi = 1.5$), $\gamma = 1.08$, $\beta = 0.08$ (See Figs. 1 and 3), cube strength of concrete, $K_W = 180$ kg/cm^2, $\eta = 3.03$, $n = 12.7$ (Standard Concrete C according to NS 427), yield point of steel $\sigma_F = \sigma'_F = 2000$ kg/cm^2,[9] $m = m' = 14.4$. With the percentage of reinforcement as abscissa the safe unit loads, $\frac{N_{zul}}{bh_0}$, have been plotted as well for over-reinforced sections [Equations (2) to (5)] as for normally reinforced sections [Equations (6) and (7)]. At any particular value of μ, the lower one of the two corresponding values of N_{zul} does, of course, represent the actual value of the safe load. (Heavily drawn lines in Fig. 5.)

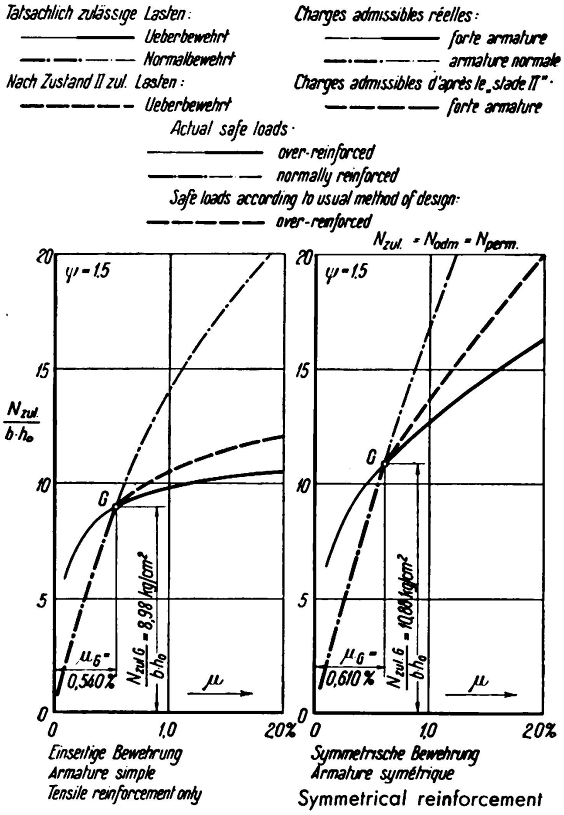

Fig. 5.

Safe loads for concrete C with $\psi = 1.5$ as actually obtained and according to usual method of design.

The point G, where the two lines for N_{zul} intersect, is the *limiting point* separating the two ranges of reinforcement, one range of *partly reinforced* sections, where the reinforcement determines the safe load, and one of fully *reinforced sections,* where the safe load is dependent mainly upon the strength of the concrete.

Lines like those in Fig. 5 might well be used as a means of designing eccentrically loaded rectangular reinforced concrete sections. However, the ordinary method of calculation may as well be used, *provided only that the working*

[9] This is considered as the lower limit for ordinary mild reinforcing steel as used in Norway.

stresses are so chosen that the ordinary calculation will in every case lead to the correct value of the safe load.

For partly reinforced sections, where the reinforcement determines the safe load, this can generally be attained by the use of one single value of the working steel stress for all percentages of reinforcement. For fully reinforced sections, however, the case is different. With one single value of the allowable concrete fibre stress the ordinary method of calculation gives nominal safe loads which increase far more rapidly with increase in the percentage of reinforcement than

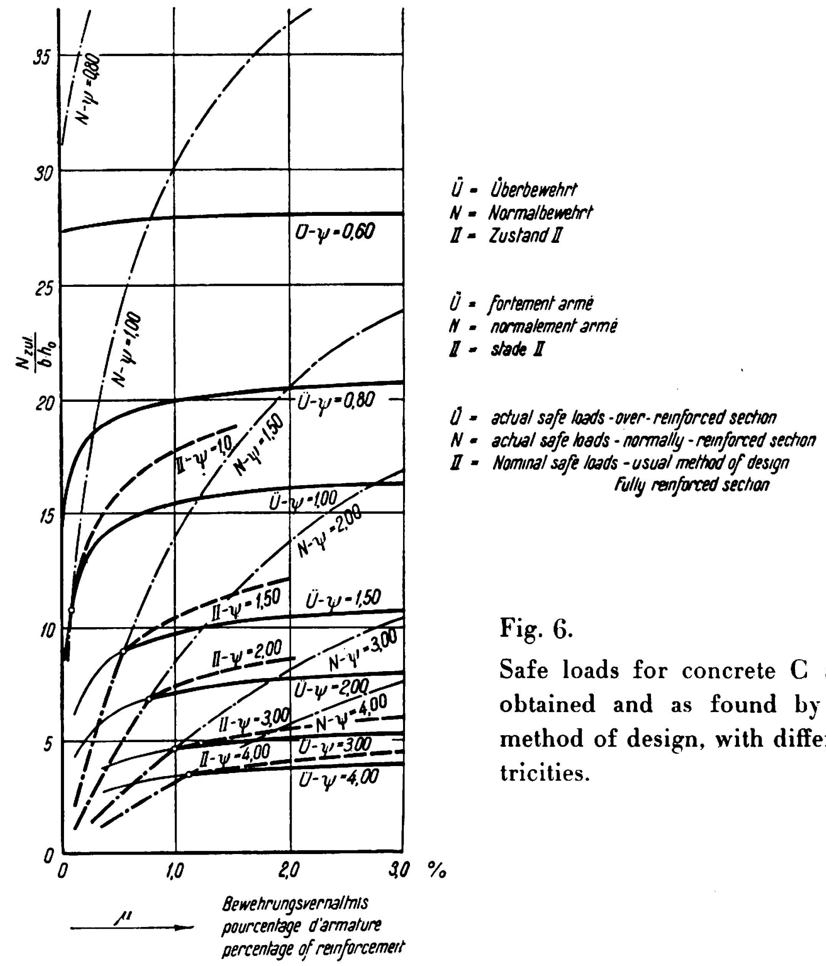

Fig. 6.

Safe loads for concrete C as actually, obtained and as found by the usual method of design, with different eccentricities.

actual safe loads, as determined according to the above analysis. This is shown in Fig. 5 and also in Fig. 6, where the actual and nominal safe loads are plotted for several values of ψ, the assumptions being as for Fig. 5. Consequently, one definite factor of safety can only be maintained throughout the range of fully reinforced sections if the working stress for concrete is varied with the percentage of reinforcement.

It has been shown previously that in the case of pure flexure the correct allowable fibre stress in concrete is the stress corresponding to the limiting point, G. (See [1], page 688, [2], page 14 and [3], page 222). The same applies to the case of bending with compression, provided that the eccentricity of load is large. With smaller eccentricities allowable concrete stresses other than those corresponding to the limiting points may be of practical interest. In the first place, when the load is acting inside the cross-section, there hardly is any limiting point to be found, since practically all sections are fully reinforced (See Article 1, b and Fig. 6). In the second place, even with the load acting well outside the cross-section, the percentages of reinforcement corresponding to the limiting points are so small, that in practice very often more reinforcement must be used (Fig. 6).

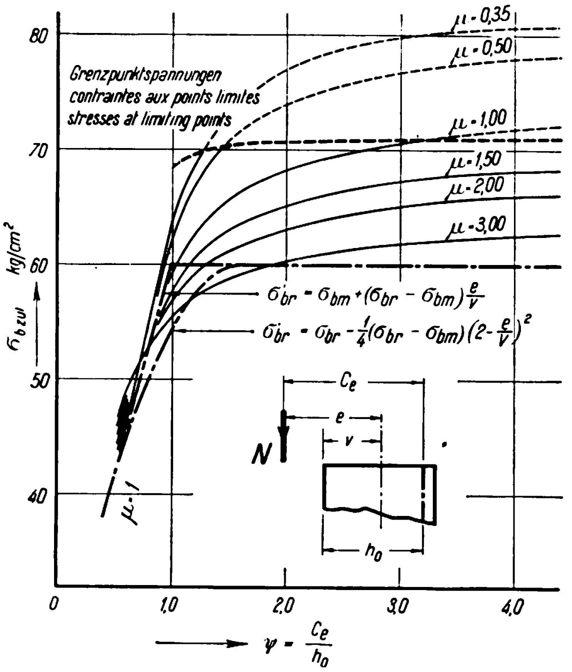

Fig. 7.

Correct permissible stresses for concrete C with different eccentricities, using reinforcement on one side only.

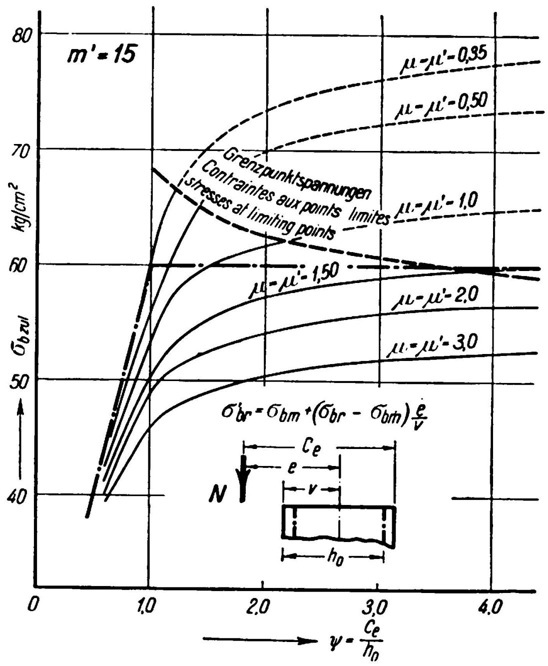

Fig. 8.

Correct permissible stresses for concrete C with different eccentricities, using symmetrical reinforcement (assuming m' = 15).

6) Correct Working Stresses for the Concrete.

From the actual safe loads, determined as described above, the corresponding correct working stresses for concrete, to be used with the ordinary method of calculation, can be computed for different eccentricities of load and different percentages of reinforcement. Working stresses, thus determined for Standard Concrete C with assumptions as in Article 5, have been plotted in Figs. 7 and 8 with $\psi = \frac{C_e}{h_0}$, as a measure of the eccentricity, as abscissa. In addition, the concrete stresses corresponding to the limiting points, discussed in Article 5, have been plotted in the diagrams. Concrete working stresses exceeding the stresses at limiting points, are of no significance, since they correspond to sections for which the steel, not the concrete stress determines the safe load (partly reinforced sections).

As one might expect, the diagrams show that the *correct working stresses for concrete decrease very rapidly with decrease in the eccentricity of the load. As the load approaches the centre of gravity of the cross-section, the correct working stresses approach those valid for simple compression.*

Thus, under the assumptions stated above, the allowable fibre stresses for standard concrete C with tensile reinforcement only should be as follows:

In pure bending, at limiting point $\sigma_{b\,zul\,1} = 71.0$ kg/cm².

In bending with compression, with 1 per cent of reinforcement:

With the load at the edge of the cross-section

($\psi = 1.0$) . . . $\sigma_{b\,zul} = 59.6$ kg/cm² $= 0.84\ \sigma_{b\,zul\,1}$.

With position of load so that the stress at the far edge of the cross-section is zero

($\psi = 0.63$) . . . $\sigma_{b\,zul} = 49.0$ kg/cm² $= 0.69\ \sigma_{b\,zul\,1}$.

With the load acting at a distance of 0,135 h_o from the centre of gravity of the section

($\psi = 0.54$) . . . $\sigma_{b\,zul} = 44.8$ kg/cm² $= 0.63\ \sigma_{b\,zul\,1}$.

It is seen that if the *same* working stress is used in actual design in all these cases, the factor of safety will actually be very much less with small eccentricities of load than in the case of pure flexure.

7) Effectiveness of Compression Steel.

As the Figures 7 and 8 show, the correct working stresses for concrete vary much with the quantity of reinforcement, and in particular with the quantity of compressive reinforcement. With symmetrical reinforcement the correct working stresses are appreciably lower than with tension reinforcement only. The same applies to the case of pure flexure, as the dotted line in Fig. 9 shows. The correct concrete working stress for Standard Concrete C at limiting points is about 21 per cent lower with symmetrical reinforcement than with tension reinforcement only.

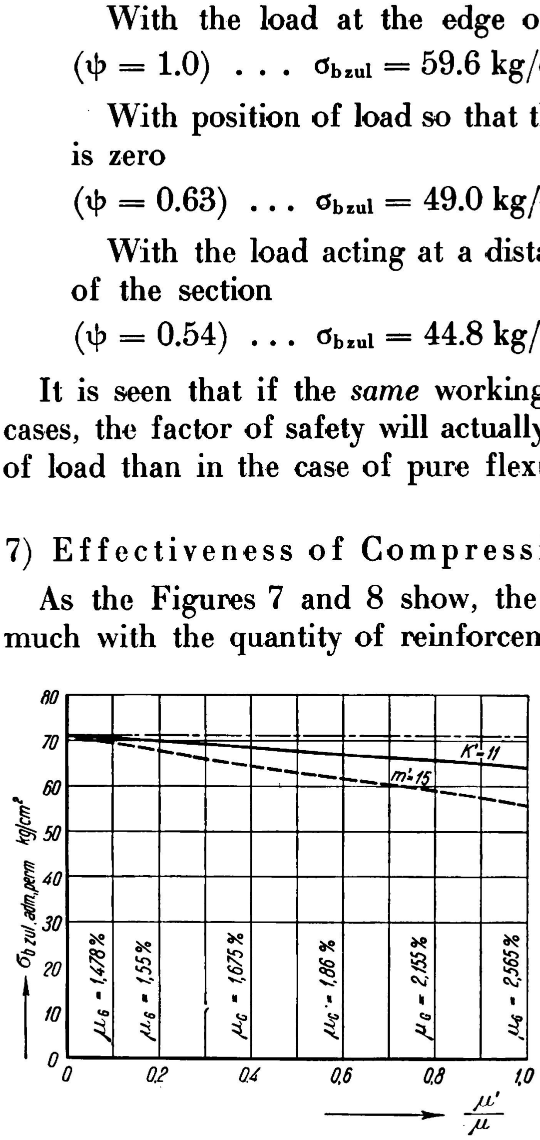

Fig. 9.

Correct permissible stresses for concrete C under pure bending with different amounts of compression reinforcement (calculated partly with m' = 15 and partly with k' = 11).

The correct concrete working stresses represented in Figures 8 and 9 have been computed from the safe loads by the ordinary method of computation, whereby the stresses in concrete and steel have been assumed to be distributed as indicated in Fig. 10. The stress in the compressive reinforcement has been computed from the equation:

$$\sigma'_e = m' \sigma_{br} \frac{\alpha - \beta'}{\alpha} \tag{11}$$

where σ_{br} is the allowable fibre stress in concrete in the case considered, and m' is equal to $\frac{\sigma'_F}{K_P}$, as defined in Article 1, a. For the concrete assumed here, with

$K_w = 180$ kg/cm² and $K_P \cong 138$ kg/cm² (cylinder strength at 28 days (f'_c) about 2000 lb. per sq. in.) and for steel with a minimum value of the yield point, $\sigma'_F = 2000$ kg/cm² (about 28400 lb. per sq. in.), we have m′ approximatively equal to 15, which is the value used in computing the curves of Figures 8 and 9.

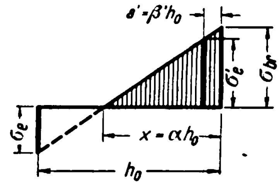

Fig. 10.

Now, while the actual stress in the concrete at failure is equal to K_P (Figures 1 and 3), the nominal stress corresponding to the load at failure according to the stress-distribution of Fig. 10, will be much larger than K_P. Correspondingly, at that load the nominal stress in the compressive reinforcement according to Equation (11) will be much larger than σ'_F. That equation thus leads to an exaggeration of the effect of the compression steel upon the ultimate load. To correct this, a factor k′, smaller than m′, should be used in Equation (11). The factor should be chosen so as to make the computed stress in the compression steel at failure equal to $m' \cdot K_P = \sigma'_F$. The result should be about the same if k′ is taken as given by the equation

$$k' = m' \frac{\sigma_{bm}}{\sigma_{br}} \frac{\alpha}{\alpha - \beta'} \tag{12}$$

where σ_{bm} is the allowable concrete stress in simple compression. At the working load, σ'_e will then be equal to $m'\sigma_{bm}$.

Most building regulations specify the use of the factor n instead of m′ in Equation (11). Usually, however, $n = 15$ is used, at least for the grade of concrete considered here, and since that was the value of m′ used in computing the curves of Figures 8 and 9, computation according to most building regulations would give the same results as are shown there, with the same exaggeration of the effect of the compression steel.

In that portion of the proposed new Norwegian Regulations, NS 427, which has not yet been published, values of k′ approximately in agreement with Equation (12) are specified for use in the cases of bending and bending with direct stress. In simple compression, the ratio between stresses in steel and concrete is taken to be $m' = \frac{\sigma'_F}{K_P}$. For the grade of concrete considered here, $k' = 11$ and $m' = 15$ are the specified values. For the stress in the tensile reinforcement, the ratio $n = 15$ is used in all cases.

The full line in Fig. 9 and the curves in Fig. 11 show the correct concrete working stresses obtained by using $k' = 11$ instead of $m' = 15$ in Equation (11). It is seen that there is still a difference between sections with and without compression steel. This is due mainly to the fact that according to NS 427, σ_{br} for the concrete considered is only 60 kg/cm², while according to our computations $\sigma_{br} = 71$ kg/cm² would be the correct value. However, much of the difference is eliminated with the use of k′ instead of m′.

8) The Working Stresses for Concrete as Specified in Building Regulations.

In the building regulations of most countries very little account is taken of the great influence of the eccentricity of load on the correct concrete working stresses for structural members in bending with compression, which is demonstrated in Figures 7, 8 and 11. According to the regulations of several countries, the full working stress for pure bending may be applied also in the case of bending with compression, provided only that the working stress for simple compression is not exceeded when the load is considered as acting centrally. If, for instance, the allowable fibre stress in flexure is 60 kg/cm² and in simple compression 38 kg/cm², as specified for Standard Concrete C in NS 427,[8] the full bending stress could in the case treated in Article 6, assuming 1 per cent of tensile reinforcement only, be applied with the load acting only 0.105 h_0 from the center of gravity of the section, that is, with $\psi = 0{,}508$. The correct working stress would in that case be about 43.5 kg/cm², as against 71.0 kg/cm² in pure bending. That is, the factor of safety would be about 39 per cent less than in the case of pure bending.

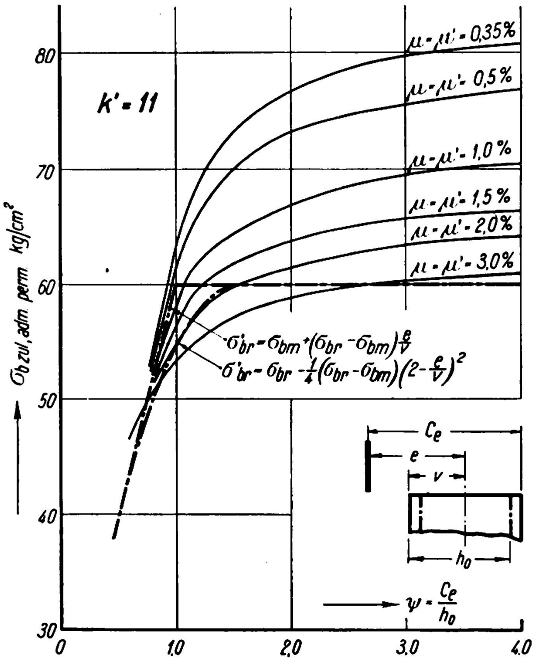

Fig. 11.

Correct permissible stresses for concrete C with different eccentricities, using symmetrical reinforcement (calculated with $k' = 11$ instead of m') (compare Fig. 8).

The latest American regulations[10] provide for an increase in the working stress for eccentrically loaded as compared to centrically loaded columns, trough multiplying the working stress for simple compression with a factor, which for instance with $\psi = 1.0$ and 1 per cent of reinforcement on either side of the cross-section, amounts to about 1.163. In a column without spirals the working stress would then be $0.154\ f'_c \cdot 1.163 \cong 0.18\ f'_c$. ($f'_c$ is the minimum ultimate compressive strength of test cylinders at 28 days, for the concrete considered about 2000 lb per sq. in.) Now, the allowable unit stress in pure flexure is specified to be $0.40\ f'_c$. From Fig. 8 we find the correct concrete working stress with $\psi = 1.0$ and $\mu = \mu' = 1.0$ per cent to be 53.2 kg/cm², or about 75 per cent of the correct working stress in pure flexure with no compression steel (71.0 kg/cm²) which has been determined with the same factor of safety. That means that the factor of safety in the case considered would be the same as in pure flexure, if the working stress were fixed at $0.75 \cdot 0.40\ f'_c = 0.30\ f'_c$. Actually only $0.18\ f'_c$ is allowed, and hence the American Concrete Institute's Regulations provide in this case for a factor of safety which is about 67 per cent greater than the factor of safety actually used in pure flexure.

As seen, the case of bending combined with compression is treated very differently in the building regulations of different countries. According to some

[10] *Building Regulations for Reinforced Concrete* (A.C.I. 501—36 T) tentatively adopted, Feb. 25, 1936, Journal American Concrete Institute, March-April 1936, Vol. 7, pages 407—444.

regulations, the factor of safety is much smaller in the case of bending with compression than in the case of pure flexure, according to others, it is larger.

In the proposed new Norwegian Regulations, NS 427[8], an attempt has been made to adapt the working stresses for concrete in bending with compression somewhat better to the correct values. The allowable unit fibre stress for concrete in bending with compression is specified as follows:

a) With the load acting inside the cross-section ($\psi < 1,0$):

$$\sigma'_{br} = \sigma_{bm} + (\sigma_{br} - \sigma_{bm}) \frac{e}{v}; \qquad \frac{e}{v} < 1 \tag{13}$$

where: σ_{br} = allowable unit fibre stress in pure flexure,
σ_{bm} = allowable unit fibre stress in simple compression,
e = eccentricity of load, measured from the gravity axis of the equivalent concrete section,
v = distance from gravity axis to extreme fibre in compression.

b)With the load acting outside the cross-section ($\psi \geqq 1$):

$$\sigma'_{br} = \sigma_{br}; \qquad \frac{e}{v} \geqq 1 \tag{14}$$

The allowable unit stresses according to Equations (13) and (14) have been plotted in Figures 7 and 11 for comparison with the correct values. It is seen that although the working stresses specified in the proposed NS 427 do not lead to the same factor of safety in all cases, nevertheless much of the variation implicit in other specifications has been eliminated.

The agreement between correct and specified working stresses would be improved, if the full allowable fibre stress for pure flexure were to be applied only with $\frac{e}{v} > 2$ or $\psi >$ about 1,6, and if a parabolic instead of a linear variation of the working stress for smaller eccentricities were specified, for instance as given by Equation (15):

$$\sigma'_{br} = \sigma_{br} - \frac{1}{4} (\sigma_{br} - \sigma_{bm}) \left(2 - \frac{e}{v}\right)^2; \qquad \frac{e}{v} < 2 \tag{15}$$

The corresponding curves are shown in Figures 7 and 11, they agree quite well with the smaller values of the correct working stresses as here determined.

IIa2

The Calculation of Reinforced Concrete Sections Subject to Bending.

Berechnungsverfahren von auf Biegung beanspruchten Eisenbetonquerschnitten.

Les méthodes de calcul des sections de béton armé sollicitées à la flexion.

Dr. techn. Ing. E. Friedrich,
Dresden.

A. The German and Austrian Regulations.

I. Decisions of the German Committee for Reinforced Concrete.

1) *Carrying capacity.*

According to the German Regulations of 1932, Paragraph 17, reinforced concrete sections subject to bending are to be calculated on the assumption that strain is proportional to distance from the neutral axis and that co-operation by the concrete on the tension side is entirely neglected. (Condition II b in

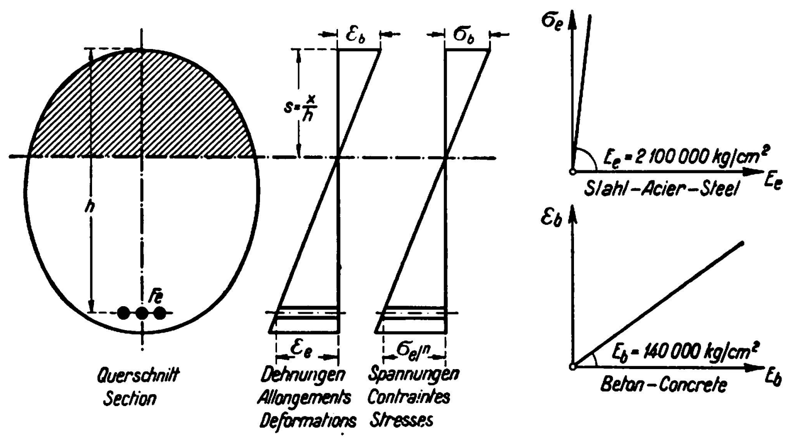

Fig. 1.
German Regulations.

Fig. 1.) The ratio of the moduli of elasticity of steel and concrete is taken as n = 15, and both in the steel and in the concrete a straight line relationship is assumed to hold good between stress and strain (*Hooke*'s Law).

In what follows below, the carrying capacity will be denoted by

$$T = \frac{M \cdot h}{J_i}$$

where J_i is the "ideal" moment of inertia. The expression $\frac{J_i}{h}$ is independent of the shape of the cross section of the beam, and in case of an homogeneous cross section it corresponds to the section modulus W and represents concrete stress plus $\frac{1}{n}$ times steel stress. A picture of the conditions governing the strength of a reinforced concrete section is obtained when T is expressed as a function of the distance from the neutral axis $s = \frac{x}{h}$. When plotting this function the abscissae are so divided that successive values $\frac{1}{s}$ occupy equal distances.

Following the method of calculation hitherto used (in accordance with Type IIb) the carrying capacity in the concrete portion of the beam becomes

$$T = \frac{W_b}{s}$$

(where W_b is the cube strength) and in the steel portion

$$T = \frac{\sigma_s}{n} \frac{1}{1-s}$$

(where σ_s is the yield point of the steel). If this system of co-ordinates is adopted the curve of carrying capacity becomes a straight line for the concrete and a hyperbola as regards the steel.[1]

2) *Comparisons with experimental results.*

Fig. 2 represents the results of experiments carried out on rectangular cross sections reinforced with St 37, wherein the cube strength of the concrete $W_b = 110$ kg per sq. cm as nearly as possible, and wherein the yield point of the steel was $\sigma_s = 2800$ kg per sq. cm. The cross sections were so chosen as to develop a carrying capacity corresponding to a considerable range of s. In Fig. 2 the calculated carrying capacities according to the German regulations (broken line) and the capacities determined by experiment are juxtaposed, and the following comparisons emerge:

a) In the region where failure depends on the yield point of the steel: —

  α) The experimental results always work out approximately 10 % higher than those found by calculation.

  β) The curves of carrying capacity obtained by calculation and by experiment are entirely similar. It is not possible, however, to justify an increase in the permissible stress in the steel region, and the risk of cracking would in itself be an objection to such a course, nor is there good cause for altering the method of calculation in the region α.

[1] *E. Friedrich:* „Über die Tragfähigkeit von Eisenbetonquerschnitten." Beton und Eisen, 1936, No. 9.

b) In the region where breakage is governed by the strength of the concrete: —

A) The first point that arises is that the curve of carrying capacity extends to values of much higher percentage of reinforcement than the steel carrying capacity line [or as in Fig. 2 up to much higher s-values].

B) In the whole of the second region the carrying capacity is much higher according to experimental values, than according to the calculated values.

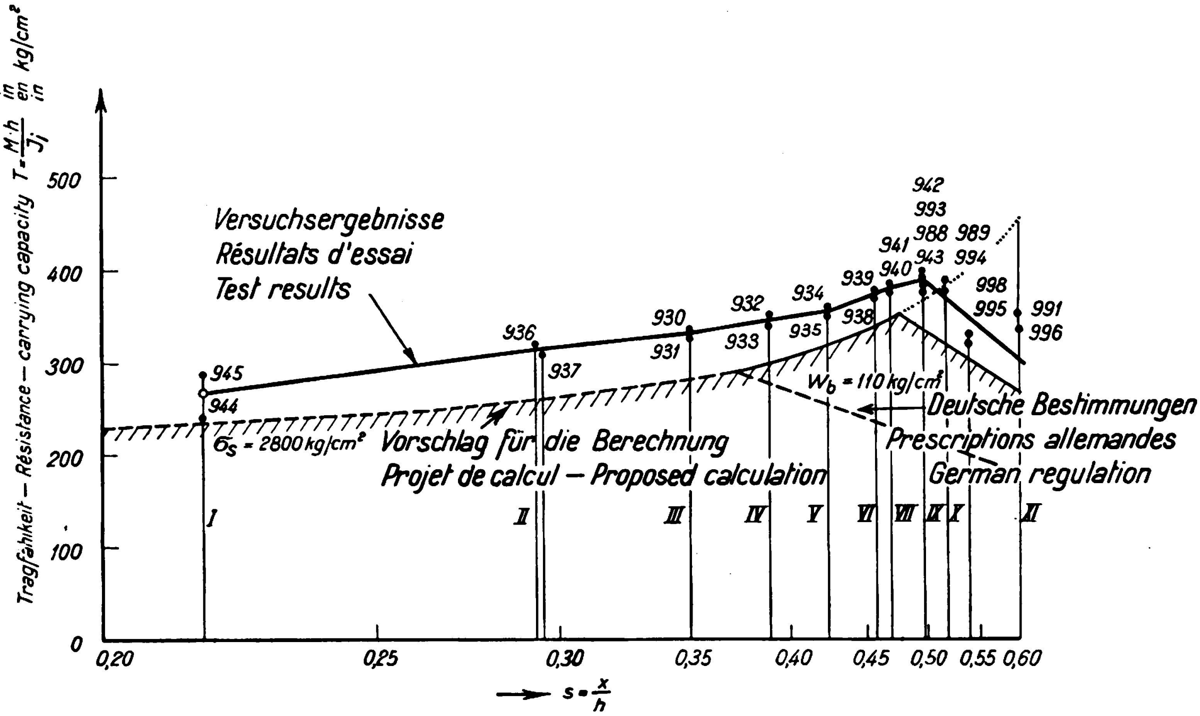

Fig. 2.

Curve of carrying capacity as determined by experiment (full line), by calculation (broken line) and as proposed for St. 37 (hatched border).

II. The Austrian regulations.

In the Austrian règulations an attempt has been made to overcome the defects of the method of calculation hitherto in use. According to the suggestion made by *von Emperger* and *Haberkalt* the limits for the steel and the concrete region is to be raised to an extent corresponding with an increase in the permissible concrete stress to 15 to 25% above that allowed hitherto. Since, however, the existing values of permissible stress have in fact been retained, it follows that the curve of carrying capacity shows a break at the point which marks the limit of reinforcement, and two disadvantages arise in consequence of this:

a) Cases may occur in which the calculated carrying capacity is reduced on an addition being made to the reinforcing steel.

b) Since the limit of reinforcement is made dependent on the percentage of reinforcement provided, the suggestion can be applied only to rectangular cross sections.

Fig. 3 shows the curve of carrying capacity in accordance with the Austrian regulations.

B. New suggestions for the calculation of reinforced concrete sections subject to bending.

The tendency in reinforced concrete design, both in buildings and bridge work, is to avoid both sloping undersides to the beams and compression reinforcement. A suggestion is now made whereby this tendency can be satisfied while retaining the same degree of safety as at present, and while conforming with the lessons of experiments.

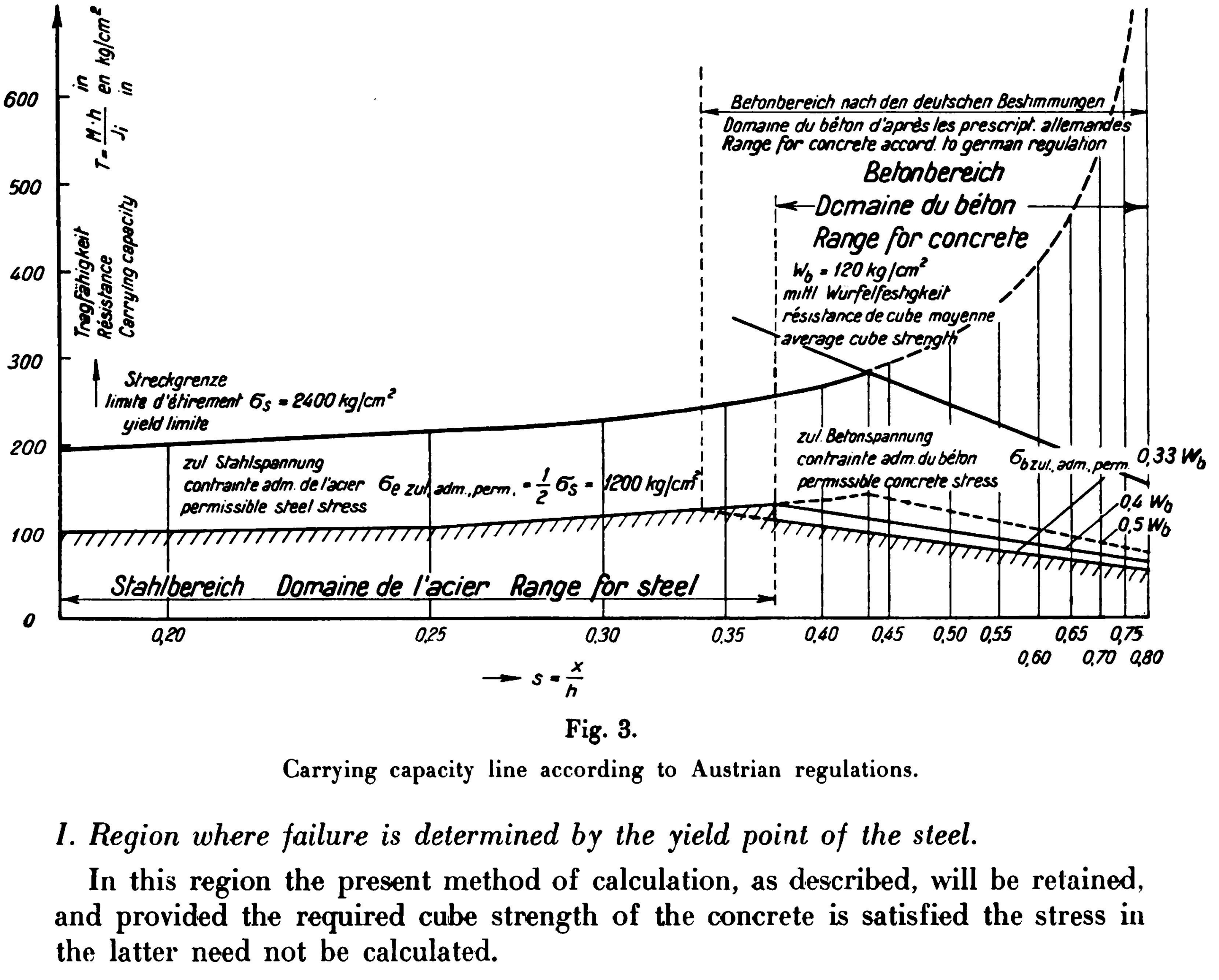

Fig. 3.

Carrying capacity line according to Austrian regulations.

I. Region where failure is determined by the yield point of the steel.

In this region the present method of calculation, as described, will be retained, and provided the required cube strength of the concrete is satisfied the stress in the latter need not be calculated.

II. Region where failure is determined by the strength of concrete.

1) Basis of calculation.

a) Determination of the neutral axis.

Where the bending moment is moderate, the condition indicated by IIb will obtain, as assumed by the method of calculation hitherto in use. That is to say the concrete will tend to crack in the tension zone once the stress in its outermost fibre becomes equal to the breaking stress (which is here equated to the cube strength) but, instead of the beam at once breaking as implied by the calculations hitherto in use, the condition IIb will give way to a new condition IIc, characterized by the fact that the concrete on the compression side becomes plastic. The neutral axis remains in its original position and distance from the neutral axis may, therefore, best be calculated on the same assumptions as hitherto.

$$s^2 + 2s\varphi - 2\psi = 0 \qquad (1)$$

$$\left(\text{where } \varphi = \frac{f}{b \cdot h}, \quad \psi = \frac{\gamma}{b \cdot h^2} \quad \text{and} \quad f = n\,F_e + n\,F'_e \right.$$

$$\left. \gamma = n\,F_e\,h + n\,F'_e\,h'\right).$$

b) Stress-strain curve for the steel.

To be calculated on the basis of *Hooke*'s Law (Fig. 4).

$$\sigma_s = E_e \cdot \varepsilon_e$$

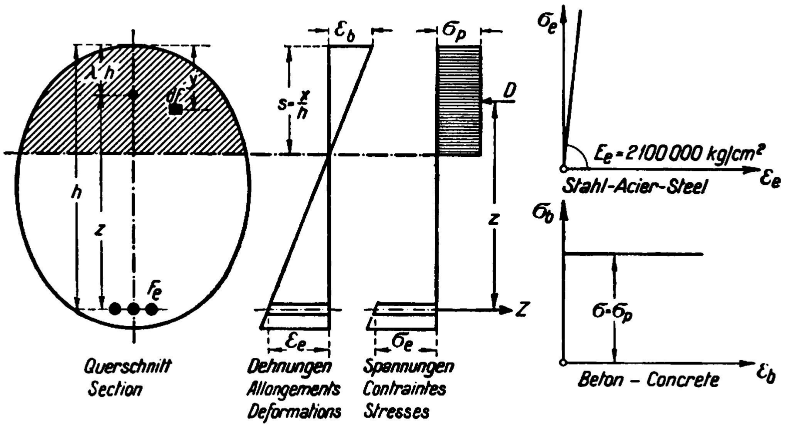

Fig. 4.
Basis of calculation for proposed calculation based on the strength of concrete.

c) Stress-strain curve for the concrete.

To be calculated on the basis of the law of plasticitiy:

$$\sigma_p = \text{const. (independent of E)}$$

d) Navier's assumption.

The calculation assumes that cross sections remain plane.

e) Equilibrium.

At every cross section the external and internal forces must be in equilibrium.

2) Details of calculation.

The stresses and carrying capacity of reinforced concrete sections can be calculated from these assumptions, the total compression D being obtained from the equation.

$$D = \int^{F_W} \sigma_p \cdot df = \sigma_p \cdot \int^{F_W} df = \sigma_p \cdot F_W$$

in which F_w is the effective area of concrete.

The total tension is

$$Z = F_e \cdot \sigma_e = F_w \cdot \sigma_p$$

and for equilibrium we have $Z = D$, or

$$F_e \cdot \sigma_e = F_W \cdot \sigma_p$$

whence

$$\sigma_e = \sigma_p \cdot \frac{F_W}{F_e}. \tag{2}$$

The statical moment of the effective concrete area about the extreme fibre is

$$S_W = \int^{\text{effec. concr. area}} y \cdot df,$$

and hence the distance $\lambda \cdot h$ to the centre of gravity of the effective concrete area is given by

$$\lambda \cdot h \cdot F_W = S_W.$$

The lever arm for the internal forces is

$$z = h - \lambda \cdot h = h \frac{h \cdot F_W - S_W}{h \cdot F_W}.$$

and since the internal and external moments must be equal we have

$$D \cdot z = M$$

$$\sigma_p \cdot F_W \cdot h \cdot \frac{h \cdot F_W - S_W}{h \cdot F_W} = M$$

$$\frac{S_W}{h} - F_W + \frac{M}{h \cdot \sigma_p} = 0. \tag{3}$$

Equation I serves to fix the neutral axis and Equation III enables the carrying moment M to be calculated.

3) Comparison with experimental results.

The formulae explained under (2) above will now be compared with the experimental results for rectangular beams given in Fig. 2, reinforced with St. 37.

For a rectangular cross section —

$$F_W = s \cdot b \cdot h$$

$$S_W = s^2 \cdot h^2 \cdot \frac{b}{2}.$$

In order to allow a comparison between the method of calculation now put forward and that hitherto in use the value of

$$T = \frac{M \cdot h}{J_i}$$

will now be calculated.

In the case of a simply reinforced rectangular section we have

$$\frac{J_i}{h} = b h^2 \frac{\left(1 - \frac{s}{3}\right) \cdot s^2}{2}$$

and it follows from Equation III that

$$T = \frac{M \cdot h}{J_i} = 2 \frac{\sigma_p}{s} \cdot \frac{1 - s/2}{1 - s/3}.$$

Fig. 2 also contains a line marked by hatching, which shows the results obtained from the proposed method of calculation.

In Fig. 5 a comparison is made between the experimental results as indicated in Fig. 2 and the newly suggested method of calculation.

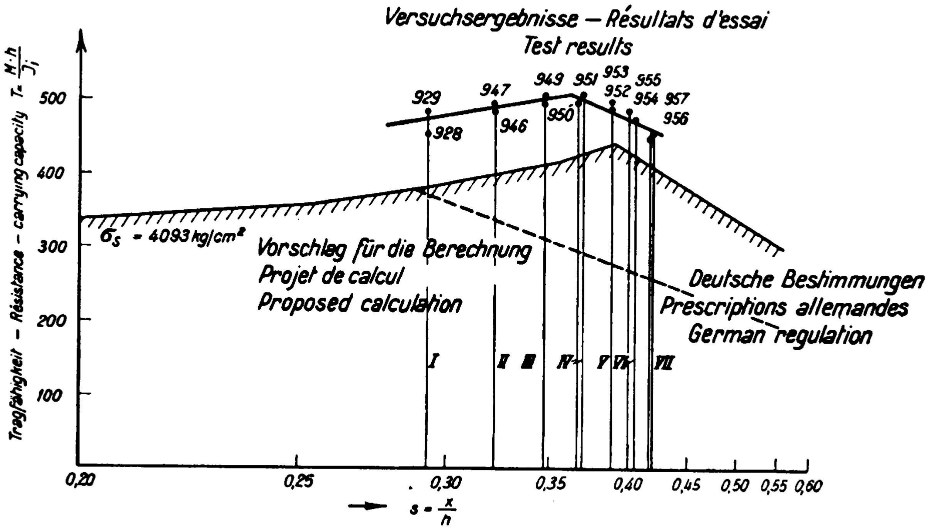

Fig. 5.

Carrying capacity line according to proposal for high grade reinforcing steel.

Fig. 5 shows a further series of experimental results obtained with constructional steel of high yield point (Isteg steel for which $\sigma_s = 4100$ kg per sq. cm). In these experiments the prism strength was ascertained to be $\sigma_p = 94$ kg per sq. cm. Comparison with the method of calculations hitherto in use indicates that the new suggestion gives much better agreement with the experimental results. Fig. 6 shows the fracture of the beam 957, which had taken place in the concrete region; Fig. 7 shows the fracture of beam 947 which has taken place in the steel region, and these two illustrations will enable the two separate regions to be clearly distinguished.

C. Suggestions in regard to the "Regulations".

It has now been shewn how the actual carrying capacity can be reconciled with the carrying capacity as calculated, and suggestions will be made for amending the regulations accordingly.

1) Stress in steel.

The permissible stress in the steel will be as hitherto $\sigma_{e\,perm} = \frac{\sigma_s}{2}$ unless the risk of cracking makes it desirable to prescribe a lower value.

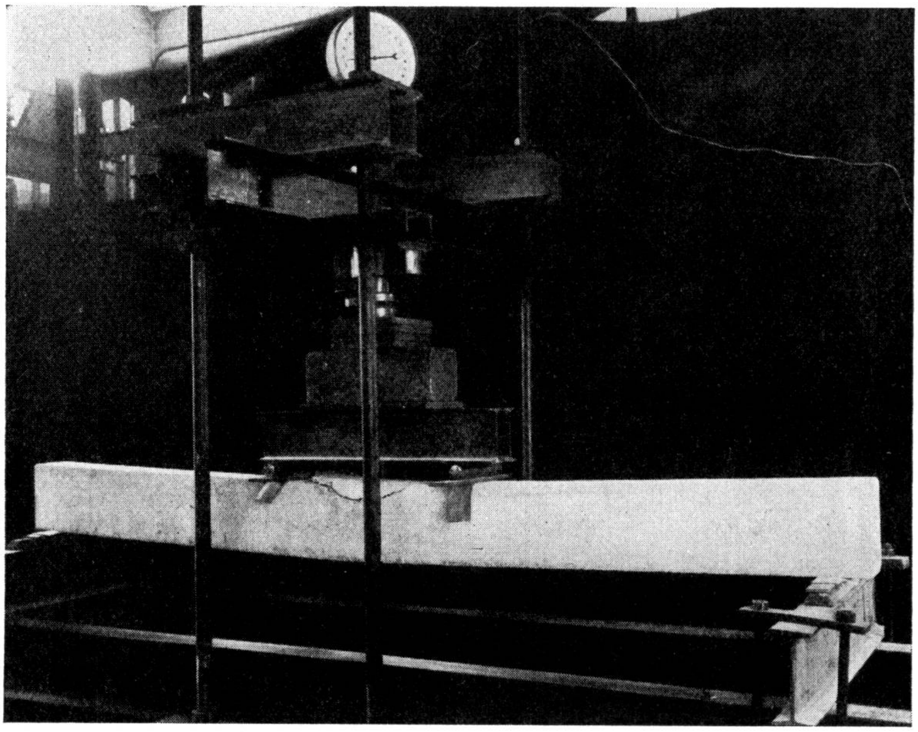

Fig. 6.
Fractured beam No. 957 (fracture due to reaching ultimate strength of concrete).

2) Stress in concrete.

At present a factor of safety of three in relation to the cube strength W_b is adopted. Since, however, the prism strength is to enter into the calculations,

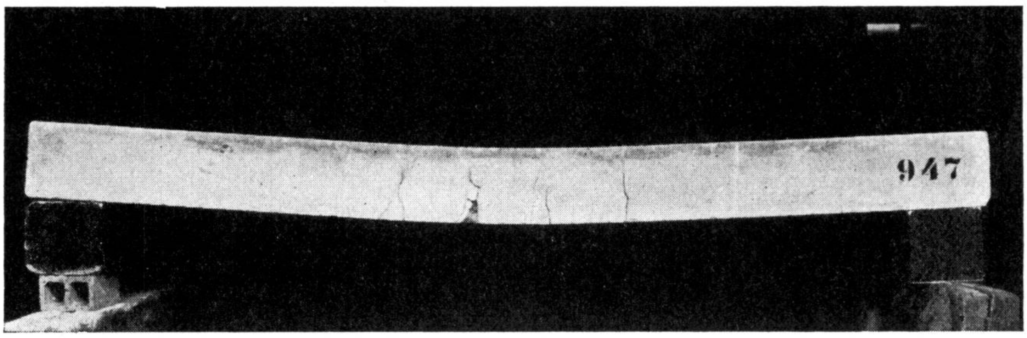

Fig. 7.
Fractured beam No. 947 (fractures due to reaching yield limit of steel).

the permissible concrete stress as hitherto allowed must be reduced. As a rule the prism strength can be taken as 0.75 of the cube strength, (conversion factor for cube strength assuming stationary loading) and on equating the permissible stress to one quarter of the cube strength we obtain ($\sigma_{b\,perm} = \frac{1}{4} W_b$)

permissible stress = 30 kg per sq. cm when cube strength = 120 kg per sq. cm
permissible stress = 40 kg per sq. cm when cube strength = 160 kg per sq. cm
permissible stress = 56 kg per sq. cm when cube strength = 225 kg per sq. cm

3) Limit of reinforcement in rectangular cross sections.

This limit can be determined by equating the carrying capacity in the steel region to that in the concrete region. The carrying capacity of the steel region is

$$\frac{M \cdot h}{J_i} = \frac{\sigma_s}{n} \cdot \frac{1}{1-s};$$

and that in the concrete region is

$$\frac{M \cdot h}{J_i} = 2 \cdot \frac{\sigma_p}{s} \cdot \frac{1-s/2}{1-s/3}.$$

Writing

$$k = \frac{\sigma_s}{n\,\sigma_p},$$

it becomes possible to obtain s_G from the equation

$$s_G = \frac{3}{2} - \frac{1}{2} \cdot \sqrt{\frac{3\,(1+3k)}{3+k}}. \tag{4}$$

Conclusion.

A great many suggestions for reconciling the results of calculation and of experiment have already been made but if these suggestions are to be embodied in actual regulations they must be perfectly open to experimental confirmation. The suggestion here put forward for determining the limit of reinforcement in rectangular cross sections reinforced with St. 37 is one which appears to be fully supported by experiment, and similar tests on high tensile reinforcing steel are in hand.

The series of experiments has further been extended to cover the case of beams with compression reinforcement, so as to investigate the change in carrying capacity that results from the use of the latter.

The suggested procedure favours a much more uniform utilisation of the material than is obtained by the existing methods, and since, to a considerable extent, it renders inclined soffits and compression reinforcement unnecessary, it offers improved possibilities of adapting reinforced concrete construction to modern requirements in design: in buildings, for instance, a flat under-surface can in this way be given to reinforced concrete floors covering several spans, and in bridge work the girders can be made of equal thickness throughout. At the same time the proposal is attended by economic advantages, in that shuttering and reinforcing steel are saved.

IIa 3

New Experiments on Reinforced Concrete Beams.

Neue Eisenbetonbalkenversuche.

Nouveaux essais effectués sur des poutres de béton armé.

Ministerialrat Dozent Dr. Ing. F. Gebauer,
Wien.

Comparative Experiments with Different Amounts of Cover to the Steel and Different Arrangements of Stirrups, and Experiments on Very Heavily Reinforced Beams.

The degree of safety possessed by a reinforced concrete structure cannot be correctly assessed by designing it on the ordinary "n" method.[1] The experimental results show great differences between the actual degree of safety and that assumed from calculation or that which it is desired to ensure.[2] If the stresses in the material are calculated from the breaking moment by the aid of the "n" method, values are obtained which differ considerably above or below the values which are to be regarded as governing the properties of the material, namely the cube strength of the concrete and the elastic limit of the steel.[3] In particular, consideration of the curves for extension of the steel and compression of the concrete indicates that no justification can be put forward for using the "n" method of calculation.[4]

The author, continuing his investigation of the correctness of his views, has carried out a further series of experiments on beams. In one set of these experiments beams with different amount of cover to the steel (from 2 to 5 cm) were made the subject of comparative tests, and beams with ordinary stirrups were compared with those having the stirrups inclined at 45°.[5]

The dimensions of the beams were b : h = 20 : 20 cm. The reinforcement consisted of three round bars of St. 37 of 10 mm dia. and the proportion of steel

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[1] *Stüssi:* The Safety of Simply Reinforced Rectangular Beams. Publications I.A.B.S.E., Vol. I, Zürich 1932.

[2] *Abeles:* Über die Verwendung hochwertiger Baustoffe im Eisenbetonbau. Beton und Eisen, 1935, Nos. 8 and 9.

[3] *Gebauer:* Berechnung der Eisenbetonbalken unter Berücksichtigung der Schwindspannungen im Eisen. Beton und Eisen, 1934, No. 9.

[4] *Gebauer:* Das alte n-Verfahren und die neuen n-freien Berechnungsweisen des Eisenbetonbalkens. Beton und Eisen, 1936, No. 2.

[5] *Gebauer:* Vergleichsversuche über den Einfluß der Dicke der Eisenüberdeckung und den Einfluß der Bügellage auf das Tragvermögen von Eisenbetonbalken. Beton und Eisen, 1937, No. 8.

was 0.59 %. The cube strength of the concrete was between 416 and 425 kg per sq. cm; the elastic limit of the round steel bars was between 2859 and 2959 kg per sq. cm, and the span of the beam 2.00 m. In the case of the beams of 22 cm total depth carrying two isolated loads at 80 cm centres, the average breaking load was 5.725 tonnes, and in that of the beam of 25 cm total depth it was 6.06 tonnes; taking account also of the shrinkage stresses in the reinforcement the calculated breaking loads work out at 5.70 and 5.93 tonnes respectively. Taking no account of the shrinking stresses in the steel and of tensile stresses in the concrete, but having regard only to the actual dimensions of the beams, the calculated breaking loads were between 4.50 and 4.57 tonnes. Whereas the actual breaking loads differ from those calculated by the first method by only 0.4 or — 2.1 %, using the latter method of calculation they differed by —21 and —25 % respectively. Calculating with the aid of the "n" metod, the elastic limit of the steel bars should be attained under a load of 4.05 tonnes regardless of the depth of cover, and the difference by comparison with the actual breaking load amounts in this case to between —29 and —33 %.

Using the "n" method the depth of the compression zone amounts to $x = 6.82$ cm, whereas in the trial beam the cracks extended to within about 1 cm of the compression face. According to the method of calculation which does not involve "n" the calculated compression depth works out $x = 0.82$ cm.

The Steuermann method of calculation,[6] which takes no account of the depth of cover to the steel and assumes a triangular compression diagram for the concrete, likewise implies considerably greater depths of the compression zone than appear from the bending test of the beam: for instance with $\sigma_{bz} = 25$ kg per sq. cm, $x = 2.66$ cm and the breaking load is 6.27 tonnes. Since in this instance the tensile strength of the concrete was not ascertained no more accurate comparison could be made.

In these cases[6] the shapes of the elongation curve for the steel and of the compression curve for the concrete show particularly well that the "n" method cannot be regarded as a proper method of calculating either the breaking condition or, still less, the stresses that arise under working loads.

The author carried out a further series of tests on experimental beams with a view to examining the effect of exceptionally heavy reinforcement.[7] Three pairs of beams were tested containing respectively 3.14, 4.91 and 6.53 % of steel. The dimensions were b : h = 20 : 20 cm, total depth 25 cm, span 2.00 m. The reinforcement was of St 37, namely four round bars of 20 mm dia, four of 25 mm dia, and, in the last example, three round bars of 30 mm with one of 25 mm dia. To prevent the beams failing prematurely through shear stresses their end portions were furnished with heavy inclined stirrups in addition to the bent-up main bars. The elastic limit of the reinforcing steel was 2.580 kg per sq. cm without any notable deviation. One beam from each pair was tested after four weeks and the other after six weeks. The concrete strengths at four weeks

[6] *Steuermann:* Das Widerstandmoment eines Eisenbetonquerschnittes. Beton und Eisen, 1933, Nos. 4 and 5.

[7] See also *Gebauer:* „Neue Balkenversuche zur Klärung der Schwindspannungsfrage und des Verhaltens von Balken bei außergewöhnlich starken Bewehrungen." Monatsnachrichten des österr. Betonvereins 1937, Heft 5.

amounted respectively to 466, 458 and 410 kg per sq cm and after six weeks to 473, 512 and 514 kg per sq cm. The breaking loads on the beams (stated in the same sequence as above) were 22.0 and 22.0 tonnes; 28.9 and 29.9 tonnes; and 32.9 and 36.0 tonnes. The decisive part played by the concrete strength is easily recognisable in the breaking loads.

Using the method of calculation which take no account of "n" but which is based on the elastic limit of the steel, on the cube strength of the concrete and on the assumption of a uniform distribution of compressive stress with (or without) taking account of the shrinkage stresses, the breaking loads in the several beams after hardening for four weeks work out at 21.5 (20.0) tonnes, 30.8 (28.7) tonnes and 33.1 (30.7) tonnes. The corresponding loads after six weeks hardening are 22.9 (20.4); 32.8 (29.7); and 40.4 (37.1) tonnes.

Comparison between the calculated and the experimental results shows that in the case of the beams reinforced with 3.14 % of steel, a better agreement is obtained when the shrinkage stresses are taken into account than when they are ignored, but generally speaking the discrepancies in the case of beams containing more than 4 % of reinforcement are not large, whether the shrinkage stresses have been considered or not. For beams containing 4.91 % and 6.53 % of reinforcement the experimental results approximate to those found by calculation regardless of the shrinkage stresses, though if the latter are taken into account the difference amounts to 12.2 % in the case of only one of the beams (N° 64). Hence the tolerance of 10 % which is usually regarded as acceptable is only slightly exceeded. This deviation of 12 % is easier to explain in view of the uncertainty which attends the calculation of shrinkage stresses in any case, and of the difficulty of constructing heavily reinforced beams in which the spaces between the reinforcing bars are very narrow. Moreover a yielding of the concrete at the end hook was observed to occur immediately before the actual breakage, so that the full resisting moment of the beam could not be developed.

From the experiments hitherto carried out it may also be inferred that where the reinforcement is particularly heavy the shrinkage stresses exert a smaller influence because of the smaller proportion between the circumference and the area of the cross section of the bars; on the other hand thinner reinforcing bars have a proportionately larger area of contact and with these the effect of shrinkage is consequently greater.

Supported by the experimental results explained above, the author has advocated the abandonment of the "n" method before the Second International Congress on Bridge and Structural Engineering in Berlin. It is to be noticed that Prof. *Saliger,* also, has taken up this point of view in the Preliminary Report of the Congress, though he has left the question of shrinkage stresses out of account and instead of using the cube strength has worked on the prism strength of the concrete which is about one quarter lower, with the result that calculation gives breaking loads somewhat lower than are determined in these experiments.

IIa 4

The Behaviour of Concrete and Reinforced Concrete under Sustained Loading.

Das Verhalten von Beton und Eisenbeton unter dauernder Belastung.

Comportement du béton et du béton armé sous l'action des charges permanentes.

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This short contribution will deal only with the effect of sustained loading on reinforced concrete constructions, on the basis of numerous data obtained experimentally in the laboratory regarding the behaviour of concrete and reinforced concrete under compressive, tensile and bending loads maintained in action over a period of two to three years. The slow changes of shape which result from shrinkage and expansion will be considered together with those attributable to sustained loading.

Special emphasis will be laid on the importance of the conditions under which the concrete is stored, for if the deformations and other properties of the concrete are to be recorded numerically account must be taken of whether the structure is immersed or buried, or is exposed to the weather and seasonal changes, or whether it is under cover or heated during a great part of the year.

The strength R_b, the modulus of elasticity E_b, the plastic strain under sustained loading, and the amount of shrinkage, all vary considerably according to these conditions of exposure. The following are relative average values for concrete stored under permanent conditions for three years.

	under water	in air: relative humidity 70 %	in air: relative humidity 45 to 50 %
R_b	1.00	0.75	0.60
E_b	1.00	0.80 to 0.85	0.65 to 0.70
Plastic strain	1.00	2.00 to 2.25	3.00 to 4.00
Shrinkage	+ 1.00	—3.50 to —4.50	—5.00 to —6.00

It must be understood that numerical values for the properties of concrete also vary a great deal in accordance with such well-known factors as the proportions of the mix, the granulation and the age of the concrete.

Once plastic deformation has taken place under the action of the dead load and permanent live load, everything proceeds as if the value of the modulus of elasticity were reduced, and (as is well known) one of the results of this condition is a corresponding change in the distribution of stress between the concrete and the reinforcement. This change takes place slowly, and, as is true of the strain, it tends in the course of time towards a limiting value.

In reinforced concrete *compression members,* stored in dry air and loaded to between 22 and 24 % of the cube strength of the concrete, the compressive stress in the reinforcing bars may reach 15 to 20 kg per sq. mm, or 19 to 27 kg per sq. mm if the compression due to shrinkage is added. When the stress in the concrete amounts to between 30 and 32 % of the cube strength, stresses of 20 to 30 kg per sq. mm may arise in the reinforcing bars under certain conditions of testing in dry air, and when augmented by the compression due to shrinkage they may considerably exceed the elastic limit of mild steel.

In *members subject to bending* the compression zone may behave in a similar way to the above, and in dry air the compressive stresses in the reinforcement, including those due to shrinkage, may in exceptional cases approach the elastic limit of mild steel. In the tensile zone, however, the increase in stress of the reinforcing bars is relatively small, and consequently the lever arm of the resisting couple is not greatly reduced despite the plastic strain undergone by the concrete.

It is of interest to note that the compression in tensile reinforcement due to initial shrinkage was found to have disappeared during the long period that the bending load was maintained in being, and a similar observation has been made on bars embedded in specimens of reinforced concrete exposed permanently to simple tensile and compressive loads.

In all the beams subjected to bending (concrete at 60 kg per sq. cm, steel at 12 kg per sq. mm, m = 15) while permanently exposed to dry air, the cracks in the concrete under tension appeared as a result of the shrinkage stresses of the concrete while the load was being applied, and the cracking continued to increase while the load was maintained, though the cracks did not open at all conspicuously.

After long periods under load, neither the compressive nor the tensile strength of plain concrete, nor the compressive nor the bending strength of reinforced concrete, was found to be less than the strength of the corresponding members stored under the same conditions without having been subjected to the loads. Once the permanent strains had taken place the elastic character of the reinforced members continued to be manifested after repeated loading and unloading which followed upon two or three years of maintenance under permanent load.

The conclusion may be drawn that the strength of reinforced concrete is not reduced by its being kept under heavy permanent loads during a very long period. It does not appear that a lower breaking stress in the concrete need be assumed to meet such conditions, nor does the usual coefficient of 28/100 need to be diminished. Less importance attaches to the elastic limit of the steel being exceeded in the case of compressive reinforcement than in that of tensile reinforcement, but it would, nevertheless, appear desirable to make use of high elastic limit steels in the compression zone of the concrete under special conditions, where the magnitude of the permanent load and the conditions to which the structure is exposed are liable to cause large plastic strains in the concrete with the passage of time, and where, consequently, there is a risk of excessive stresses in the reinforcing bars. In such a case special attention should be paid to the effectivences and the spacing of the cross stirrups, and the danger of cracking should be the object of special care.

IIa 5

Effect of Plasticity of Concrete and Steel on the Stability and Endurance of Reinforced Concrete.

Der Einfluß der Plastizität des Betons und des Stahles auf Stabilität und Dauerhaftigkeit des Eisenbetons.

Rôle de la plasticité du béton et de l'acier sur la stabilité et la durée du béton armé.

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Before investigating how the endurance of structures is affected by the plasticity of their materials — particularly as regards reinforced concrete — it may be well to consider exactly what is meant by "plasticity" and what influence this property has on variations in stress. In an earlier paper the author has attempted, by means of a simple mathematical theory, to show that the magnitude of elastic and plastic strains depends not only on the momentary load but also on antecedent conditions other than loading. The theory cannot pretend to finality, but consists merely of a series of syllogisms which derive from certain simple experimental premises and which lead to conclusions that are difficult to establish experimentally.

If a sample of steel be subjected to a load in excess of its elastic limit the steel will suffer a permanent deformation and if the load remains constant the deformation will continue to increase, to a greater or less extent, with the passage of time, according to a law of creep which depends on the quality of the steel and on the temperature. This creep, while extremely small in the case of loads close to the creep limit, is nevertheless not zero. If the load fluctuates between two definite limits the permanent deformation grows very appreciably in course of time, and if the upper limit of the varying load exceeds the critical fatigue load (or natural limit of elasticity as defined by *Bauschinger*) this phenomenon takes place even below the limit of elasticity (yield point]: it is the result of an exchange of energy which arises from the elastic and plastic hysteresis between the elastic and plastic strains. A rapidly applied load may, therefore, produce contrary effects according to its mode of application: thus a single shock may cause diminution of the plasticity, whereas repeated loading, sustained vibration or slowly applied loads may bring about an increase in this quality.

The practical importance of this phenomenon appears when alternating or pulsating loads are applied to reinforced concrete members which have been pre-stressed in accordance with the system of *Freyssinet*. When this occurs the steel bars may be observed gradually to extend, and at the same time the pre-imposed compression in the concrete is reduced; in certain cases, where the amount of the pre-stressing is small by comparison with the load applied, cracks may appear in the concrete under tension. The destructive effects produced by the repeated loadings on the steel bars becomes greater in proportion as the bars are irregular, embrittled or rusted; and the fatigue limit is much lower for the hook at the end of a bar than for the straight portion.

There appears to be no method, other than experiment, of determining the stress-strain curve for a sample of concrete beforehand, for this material has no fixed elastic limit; the latter varies according to age and depends on the rate at which loads are applied. Everything which has been said above regarding the plasticity of steel is applicable even more strongly to concrete. The constants of hysteresis which define the plastic and elastic viscosity are small, and the amount of such hysteresis large. Hence, in calculations relating to reinforced concrete, the concept of a "coefficient of elasticity" has no meaning unless it is associated with constants which define the conditions of plasticity, creep and hysteresis. That is why agreement has never been reached as to the proper value for the modular ratio m.

The effect of accelerated creep under repeated loading is found also in concrete, to a very high degree. The phenomenon of plastic creep is attended by phenomena of irreversible friction; these are additive, and tend to hasten adaptation because of the effect of bond on the elastic strain, to which the author has referred in his earlier paper.[1] Moreover this process of adaptation is associated with all those factors which usually accompany ageing: increased stiffness, increased strength, and reduced shrinkage, etc. The concrete is liable to exhibit all those phenomena of fatigue after repeated loading which occur in the metal. For instance, a concrete which has a breaking strength of 350 kg per sq. cm and is subjected to loads varying 500 times a minute between 50 and 300 kg will break at the end of one hour; during this time its modulus of elasticity will have changed and the length of the specimen will have decreased. For any such specimen there is a fatigue limit which determines the pulsating load that will cause breakage after a limited number of alternations; but below this limit, on the other hand, the effect of repeated loading is to bring about an increase in the statical strength.

A number of experiments on the behaviour of bent beams subjected to repeated loads have been carried out in the laboratoires[2] in Paris to which the author is attached, and it has been observed that even in these cases there exists a characteristic fatigue limit, so that by making successive experiments on a series of similar beams it becomes possible to construct a *Wöhler* curve in which the first limb is much more steeply inclined than would be the case for concrete or steel by itself. Finally, it has been noticed that the principal effect produced by successive loadings was to accelerate the occurence of plastic strain, and it has been possible to devise a method of accelerated experiment for the study of the adaptation that takes place in a reinforced concrete member under load, the effects

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[1] See Theme I.

[2] Laboratoires du Bâtiment et des Travaux Publics.

of the repeated loadings being practically equivalent to an artifical ageing of the work in question. This has led to the observation that adaptation does not take place equally in the parts under tension and those under compression. Again, it appears that the fatigue limit in relation to the static breaking load is much lower for concrete in tension than for concrete in compression. Account must also be taken of those mutual forces between steel and concrete which constitute the bond: the experiments go to show that bond is very sensitive to repeated loadings, and a large number of beams failed through slipping of the bars, probably because the latter had been unable to adapt themselves to the deformations caused by plasticity. In yet other cases the stabilisation of the bar, after its first slip, brought about a considerable degree of cracking in the concrete without actually leading to failure of the member.

The upshot of these considerations is that any attempt to calculate the strain in a piece of concrete by referenec to elementary data must be a very complex matter, and can, in the present state of our knowledge, be attempted only as a rough approximation. When all is said and done, the scope for the occurrence of adaptation appears very great, and however rough this approximation may be its effect is to suggest that when the earliest designers of reinforced concrete introduced the idea of partial continuity they came nearer to the truth than do all the hyperstatical calculations which have since been developed.

IIa 6

The Behaviour of Reinforced Concrete Framed Structures at Incipient Failure.

Das Verhalten von Eisenbeton-Rahmenkonstruktionen bei beginnender Zerstörung.

Comportement des portiques en béton armé à l'amorce de la rupture.

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and

F. G. Thomas, B. Sc., Assoc. M. Inst. C. E., Garston.

At working stresses it is probable that the distribution of moments throughout a reinforced concrete framework is reasonably well given by calculations according to the elastic theory. It has been shown by prolonged loading tests at the Building Research Station[1*] and in America[2] that the creep of concrete at working stresses has no important effect on the moment distribution in a frame.

When incipient failure is reached at any part of the structure, however, the inelastic movements of either the steel at its yield point or of the concrete near its ultimate strength are so large that the elastic theory no longer holds. Deformation of the affected part is limited by the movements of the rest of the structure so that collapse of this part may not occur until considerable elastic deformation has occurred elsewhere. That is, further load can be carried by the structure without collapse, the maximum stress at the affected part tending to remain practically constant whilst the moments and stresses at other parts increase. For convenience the change in the distribution of bending moments from that in a purely elastic framework will be called "redistribution of bending moments" in this paper.

Tests on two-span continuous beams by *Kazinczy*[3] have shown that when steel is the deciding factor for failure, variation of the amount of steel in the span or over the central support from that required by the elastic theory leads to redistribution of moments such that the full strengths of both the span and support sections are reached. Similar results were obtained for built-in beams by the German Reinforced Concrete Committee[4] for the condition of failure due to steel yield. Such redistribution is to be expected because of the large inelastic deformation of steel at its yield, but the extent to which it can be relied upon without causing concrete failure is unknown. No previous tests are known in which the effect of inelastic deformation of the concrete at incipient failure on the ultimate strength of a framework has been studied.

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* These figures relate to the list of references at the end of the paper.

The tests described in this paper were part of an investigation undertaken at the Building Research Station in conjunction with the Reinforced Concrete Association to obtain definite information on the importance of inelastic deformations at highly stressed parts of a reinforced concrete framework. The ivestigation included tests to destruction on, A, two span continuous beams and B, portal frames.

A. Tests to Destruction on Two-span Continuous Beams.

Tests were carried out on two-span continuous beams designed as follows:

1) With weakness over the central support due to the use of a low percentage of tension steel.
2) With weakness over the central support due to the use of a low strength concrete without compression reinforcement.
3) As (2) except that compression reinforcement was provided.
4) As (2) but with an increased span length in order to reduce shear stresses.
5) As (2) except that a low strength concrete was used at an age of about 6 months instead of 7 days.

All tests were made in duplicate, and river aggregates were used throughout.

1) *Primary Failure in Tension Steel.*

Details of the beams and the positions of the loading used in the tests to determine the effect of using insufficient steel when calculated according to the ordinary elastic theory are given in Figure 1.

The notation used in the table of Figure 1 and subsequent tables of stresses is as follows:

t denotes the stress in longitudinal tension reinforcement.

t' denotes the stress in longitudinal compression reinforcement.

M denotes the bending moment.

n denotes the depth of neutral axis.

a denotes the arm of the resistance moment.

S denotes the total shear.

s denotes the shear stress.

t_w denotes the stress in web reinforcement.

s_b denotes the bond stress.

W denotes the Load.

ξ_B denotes the Distance of point of inflexion from B.

ξ_F denotes the Distance of point of inflexion from column face.

s_E denotes the Bond at E (lower bars).

R_A, R_B, R_C denotes the reactions at A, B and C.

It will be seen that over the central support where the moment is normally greatest, there are only two $^3/_8$ in. diameter bars whereas in the span four $^5/_8$ in. diameter bars are provided. At quite a low load therefore, the yield point stress of the $^3/_8$ in. diameter bars would be expected; it would be anticipated that yield of these bars would lead to a redistribution of moments whereby the section over the central support would be continuously relieved, enabling the load carried by the system to be further increased until failure in the span.

The actual moments during the tests were determined by measuring the strain in the supporting steel joist at a fixed distance from the end supports and hence

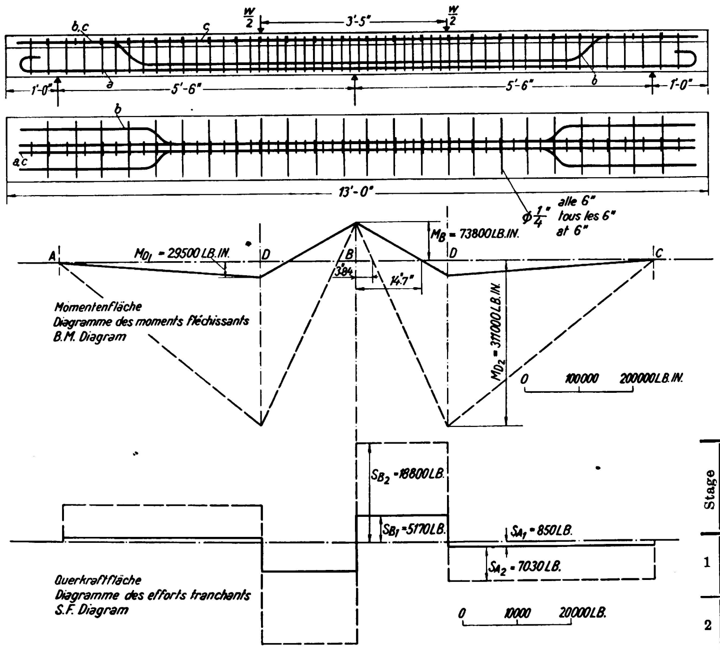
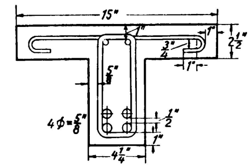


Fig. 1.

Redistribution of moments in continuous beams.
Steel failure.



Eisenliste – Liste des Fers – Bar schedule			
Eisen / Armature / Bar	φ	No.	Form – Forme
a	5/8"	2	12'-6"
b	5/8"	2	1'-9" 9" 7'-6" 9" 1'-9"
c	3/8"	2	12'-6"

Calculated Stresses (lb. inch units)

Stage	at B t	at B c	at B M	at B n	at B a	at B S	at B s	at B t_w	at B s_b	at D t	at D c	at D M	at D n	at D a	at D s_b	at A S	at A s	at A t_w	at A s_b	W	ξ_B	R_B	$R_A = R_C$
1	44700	2545	73800	1.85	7.47	5170	162	6255	298	3740	285	29500	2.09	6.40	103	852	28	2170	30	10980	14.7	10340	852
2			73800			18800	592	22700	1073	39400	3000	311000			374	7030	231	17800	251	50750	3.94	37600	7030

Dead Load Effects

$M_B = 3800$ lb in $R_A = 204$ lb
$M_D = 0$ lb in $R_B = 476$ lb

$u = 8000$ lb/in² $m = 5.5$

calculating the end reactions from a previous calibration of the joist. The results for one of the two beams tested are shown in Figure 2.

Incipient failure over the central support is clearly indicated by a sudden decrease in moment at that point, after which the moment increased somewhat.

On the assumption that the central support moment remains constant after yield begins, the subsequent moments in the span have been calculated and the theoretical curves are shown on the diagram. It is evident that the assumption leads to a very fair estimate of the actual span moments for the present test.

The concrete used for this test was made in the proportions 1:1:2 (by weight) using rapid hardening Portland cement, and the beam was tested at an age of 44 days. For the second beam a high alumina cement 1:2:4 concrete (by weight) was used, and the beam was tested at an age of 6 days. In the second beam, as a result of the higher tensile strength of the high alumina cement concrete, the help afforded by the concrete in tension was such that the stress in the continuity steel increased from a very low value to its yield point value at the occurrence of the first crack over the support. Apart from this effect, there was apparently no important difference in behaviour, resulting from the use of the two types of cement.

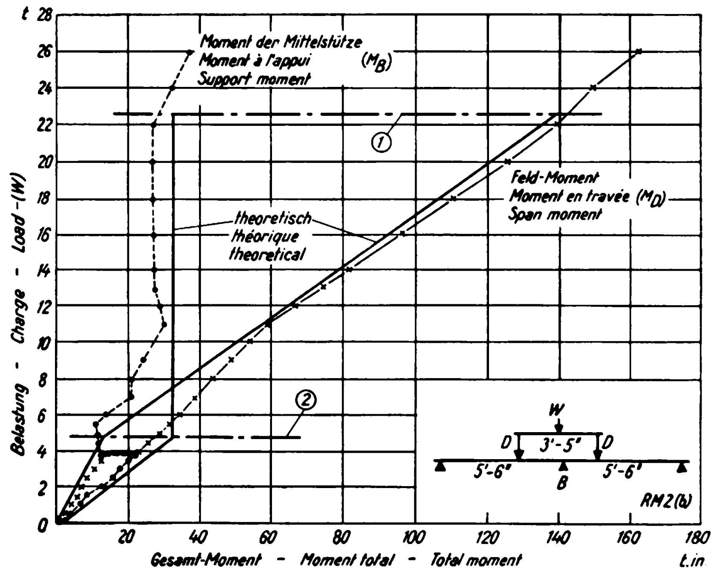


Fig. 2.

Tests on continuous beams. Steel failure (b).
Rapid-hardening portland cement 1 : 1 : 2 concrete (by wt.)
Water/cement ratio = 0.44 (by wt.). Age at test — 44 days.
Cube strength of concrete = 6660 lb. per sq. in.
① Theoretical load for general failure.
② Theoretical load for support failure.

The deflections at midspan relative to the central support were measured throughout by means of dial gauges. There was no appreciable difference between the deflections of the two beams, and at three-quarters of the failing load the maximum deflection was only about 0,1 in. The supporting steel joist deflected during the test and the sinking of the end supports relative to the central support was therefore also measured. This sinking affects the moments during the elastic stage of the test and has therefore been taken into account in calculating the theoretical curves and stresses given in Figures 1 and 2.

The maximum crack widths, measured with a portable microscope, are given in Table 1. The cracking over the central support increased considerably during the second part of the test, i. e. after the steel had commenced to yield, and shortly before final failure the cracks were from 0.06 to 0.08 in. wide. These cracks are approximately ten times the width usually observed just before the commencement of steel yield.

The loads calculated for failure, (1) on the elastic theory, and (2) on the basis that both the support and span sections develop their full strengths after redistribution, are given in Table 2 together with the actual failing loads. It will be seen that the effect of the redistribution of moments on the load carrying capacity of a continuous beam may be considerable in cases of steel weakness over the central support. However, the cracking accompanying the increased load is very marked, so that in practice, advantage can be taken of moment redistribution due to steel yield only in cases where the increased cracking is not a matter of importance.

Table 1.

Maximum Crack Widths in Continous Beams.

Series	Load — tons	Maximum Crack Width — inch × 10−3: Over Central Support						In Span					
		5	10	15	20	25	Yield[1]	5	10	15	20	25	Yield[1]
1. Steel Failure	(a)	0	15	30	42	60	0	0	1.3	2.3	2.6	3.8	0
	(b)	6	15	34	55	79	5	0	1.5	2.6	4.6	6.6	0
2. Concrete Failure	(a)[2]	0	1.5	2.4	3.1	3.3	0.5	0	1.9	3.5	6.0	10.0	0.6
(No compression steel)	(b)	0	1.3	2.2	2.6	2.6	1.2	0	1.3	2.2	3.3	3.9	0.7
3. Concrete Failure	(a)	1.0	3.1	3.7	4.6	5.5	3.4	0	0.9	1.6	2.4	3.5	1.3
(With compression steel)	(b)	1.6	4.0	5.2	9.2	10.5	4.8	0	1.3	1.7	2.6	5.2	1.5
4. Concrete Failure	(a)	3.3	3.7	—	—	—	1.6	1.5	4.0	—	—	—	0.8
(Increased span length)	(b)	0.1	1.0	—	—	—	0	1.3	4.2	—	—	—	0
5. Concrete Failure	(a)	0	1.6	2.7	2.6	1.5	—	0	1.3	2.5	3.6	5.0	—
(Weak concrete at about 6 months)	(b)	0	0.7	1.0	1.1	1.2	—	0	1.4	2.4	3.5	7.2	—

[1] The yield load is the theoretical load for support failure according to the elastic theory (see Table 2).

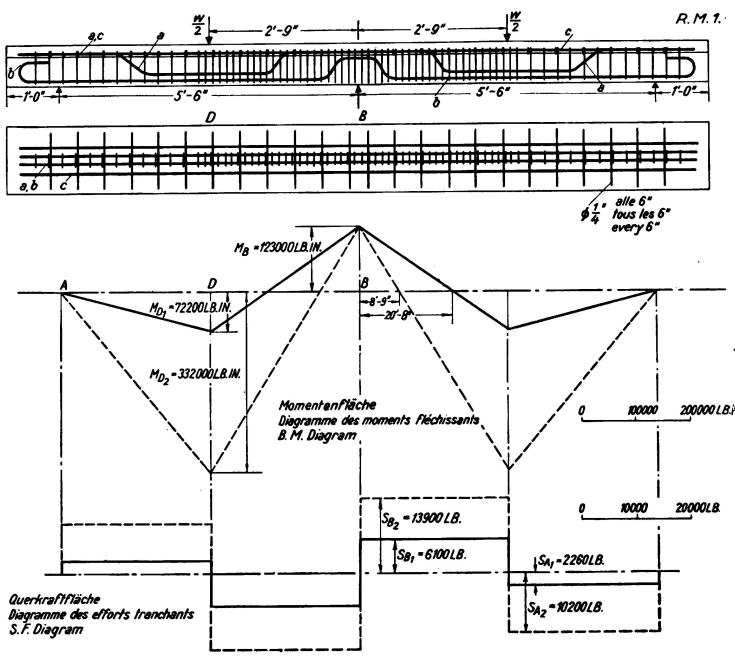
[2] The maximum crack widths in beam (a) of series (2) was measured at the depth of the most highly stressed edge of the tension steel; in all other beams the measurements were at the depth of the centre of the most highly stressed bar.

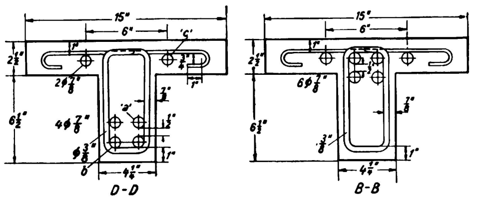
2) *Primary Concrete Failure. No Compression Steel Provided over Central Support.*

In the beams designed to fail by crushing of the concrete, all the tension reinforcement in the span was taken up over the support so that the compression at that point was taken wholly by the concrete in the rib. Details of the beams, spans and positions of loads are given in Figure 3. The concrete was made with ordinary Portland cement using a $1:2^1/_2:3^1/_2$ mix (by weight) and a water-cement ratio of 0,66 (by weight). Te tests were made at an age of 7 days, the strength aimed at being the lowest (2250 lb. per sq. in.) allowed by the Rein-

Table 2. Failing Loads of Continuous Beams.

Basis of Bending Moment Calculations	Basis of Resistance Moment Calculations		Failing Loads — tons: 1. Steel Failure		2. Concrete Failure (No compression steel)		3. Concrete Failure (Compression steel)		4. Concrete Failure (Increased span length)		5. Concrete Failure (age $5^1/_2$ months)	
		Test No: —	RM 2 (a)	RM 2 (b)	RM 1 (a)	RM 1 (b)	RM 3 (a)	RM 3 (b)	RM 4 (a)	RM 4 (b)	RM 5 (a)	RM 5 (b)
Elastic theory: i. e. No redistribution of moments. Loads are for support failure	No Stress Redistribution	True "instantaneous" modular ratio used	4.9	4.9	7.0	7.2	13.0	14.2	2.7	2.3		
	Stress Redistribution	$m = \frac{40000}{\text{cube strength}} = \frac{40000}{u}$	5.0	4.9	7.6	7.8	19.5	19.8	3.0	2.5		
		Steel Failure: Maximum concrete stress reaches cube strength. Concrete Failure: $m = \frac{80000}{u}$	7.8	6.5	8.0	8.2	25.4	26.2	3.2	2.7		
Theory of Redistribution of Moments: i. e. Simultaneous failure at central support and in span	No Stress Redistribution	True "instantaneous" modular ratio used	22.7	22.6	20.8	21.4	25.7	28.1	9.8	8.6		
	Stress Redistribution	$m = \frac{40000}{u}$	23.0	22.6	27.8	28.5	35.0	36.3	13.0	11.8		
		Steel Failure: Maximum concrete stress reaches cube strength. Concrete Failure: $m = \frac{80000}{u}$	26.1	24.0	32.6	32.8	40.1	40.5	14.2	13.9		
Actual load at which signs of distress were first noticed in the concrete			—	—	20.8	24.0	23.0	24.0	9.0	9.5	18.8	16.5
Actual ultimate load carried by beam			29.1	28.7	27.5	28.6	27.6	28.9	13.4	13.0	33.0	27.5





Eisenliste - Liste des fers - Bar schedule			
Eisen / Armature / Bar	ϕ	No.	Form - Forme
'a'	7/8"	2	1'-9" · 2'-3 1/2" · 3 1/4" · 2'-8" · 3 1/4" · 2'-3" · 1'-9" · 7" · 3 1/4" · 7"
'b'	7/8"	2	5'-8" · 2" · 9" · 2" · 5'-8" · 6 1/4"
'c'	7/8"	2	12'-6"

Fig. 3.

Redistribution of moments in continuous beams.
Concrete failure.

Calculated Stresses (lb. inch units)

Stage	at B									at D						at A				W	ξ_B	R_B	$R_A = R_C$
	t	c	M	n	a	S	s	t_w	s_b	t	c	M	n	a	s_b	S	s	t_w	s_b				
1	6400	2050	123000	5.35	5.32	6100	270	7800	70	5180	445	72200	3.09	5.80	35	2260	80	4660	68	15800	20.8	12200	[illegible]
2			123000			13900	615	17700	159	23800	2050	332000	3.09	5.80	159	10200	366	21000	283	47300	8.9	27800	[illegible]

Dead Load Effects

$M_B = 4120$ lb in $R_A = 200$ lb

$M_D = 910$ lb in $R_B = 480$ lb

$u = 2050$ lb/in² $m = 9{,}5$

forced Concrete Code of Practice. Actually the strength was about 10 per cent. less than this value (see Appendix 1).

In order to reduce the shear stresses with this weak concrete, the loads were applied at midspan, instead of nearer to the central support as in the case of the previous beams.

The results are given in Figure 4. It will be noticed that there is not such a well defined point at which failure over the support commences as in the case of the previous beams in which the steel yielded, but rather a gradual change from the elastic to the inelastic stages of the test.

The concrete at the support continued to carry load in an apparently undistressed condition long after the load calculated to produce a fibre stress equal to the cube strength had been reached. In fact there was no evidence of crushing over the central support until the load was more than twice this value.

The measured span moments were again in fair agreement with those calculated on the assumption of a constant support moment after passing the elastic stage.

Throughout the test the crack widths were small (see Table 1) so that redistribution of moments in the case of concrete weakness may be considered without reference to cracking. The beam deflections were of the same order as those measured in the previous series.

3) *Primary Concrete Failure. Compression Steel Provided over the Central Support.*

In tests designed to show weakness in compression in the presence of a limited amount of compression steel the reinforcement was the same as in the previous beams except that the lower bars were continuous throughout the beam, thus providing help in compression over the central support. The concrete mix used was again $1 : 1^1/_2 : 3^1/_2$ (by weight) using ordinary Portland cement, and the tests were made at an age of 7 days; the strength (see Appendix 1) was a little higher than that obtained in the previous tests.

The moments throughout the system were measured, and it was again found that there was a gradual change between the two stages of the test, and it is interesting that the final loads attained (see Table 2) were almost exactly the same as for the beams in which no compression reinforcement was provided.

There was no evidence of compression failure over the central support until just before final collapse of the system. The main tensile crack at that section gradually closed towards the end of the test until it extended only about 2 in. from the top surface of the beam, indicating that the whole of the rib and even some of the flange was bearing compression forces.

The maximum crack widths are given in Table 1.

4) *Primary Concrete Failure. Beams with Increased Span Length.*

The beams in series (2) were provided with closely spaced stirrups over the central support in order to avoid shear failure with the weak concrete used. It was suggested that this reinforcement gave lateral support to the concrete, thus increasing its ability to carry longitudinal compression. In order to show whether this was the case, two further beams were prepared similar to those of series (2) except that the span length was increased to 12 ft. so that the failing moments would be reached at a lower load, hence reducing the amount of shear reinforcement required.

The results showed conclusively that the central support section was not weakened by the wider spacing of the stirrups. The percentage increase in load due to redistribution was approximately the same as before (see Table 2), and the support moment carried at failure was actually greater than had been obtained in the previous tests of series (2).

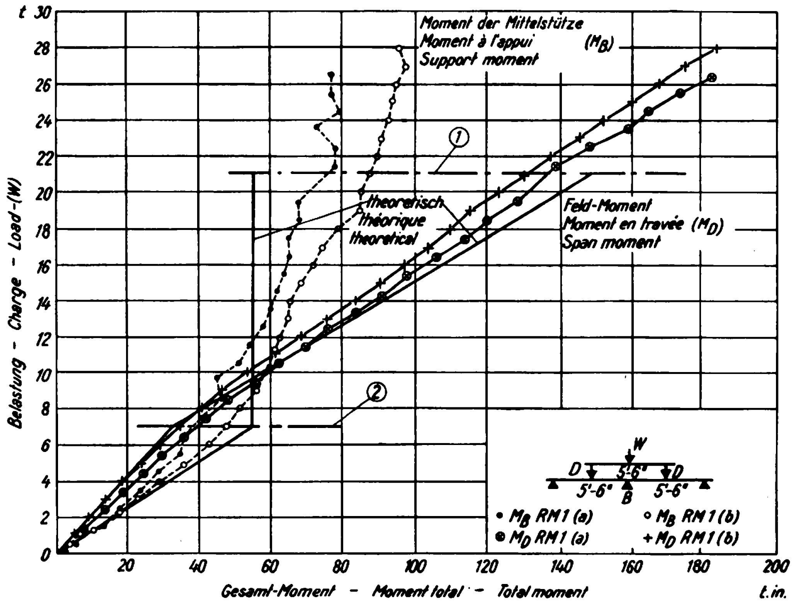


Fig. 4.

Tests on continuous beams. Concrete failure (No compression steel). Normal Portland cement $1:2^1/_2:3^1/_2$ concrete (by wt.) Water/cement ratio = 0.66 (by wt.) Age at test — 7 days. Cube strength of concrete = 2050 lb. per sq. in.

① Theoretical load for general failure.

② Theoretical load for support failure.

5) *Primary Concrete Failure at an Age of $5^1/_2$ Months.*

The tests previously carried out with weak concretes were made at an age of 7 days in all cases, and although it seemed probable that the amount of redistribution that occurred as a result of inelastic deformation of the concrete would depend on the strength of the concrete rather than its age, it was thought advisable to test two beams similar to those of series (2) (no compression reinforcement) at a greater age. In order to obtain a low strength at about 6 months an ordinary Portland cement was used in a mix of proportions 1:4:7 (by weight) for the first beam; this was changed to 1:5:6 for the second to give a better mix with the same water-cement ratio of 1,05.

The failing loads, given in Table 2, were as great and in one case greater than those obtained previously. The concrete strength was, however, not known very accurately as the cubes cast with the beams could not be relied upon to give a fair estimate of the concrete quality in the beam itself for such a poor quality concrete. Samples were cut from the ends of the beams and tested, and the results indicated that if anything the concrete was somewhat weaker than that used for the earlier tests. There is no doubt, therefore, that the redistribution obtained with the richer concrete was not attributable to the fact that it had hardened for only a comparatively short period.

B. Tests on Portal Frames.

Tests were made to determine to what extent the load-bearing capacity of a simple reinforced concrete portal frame may be increased as a result of redistribution of stress and moment when high stresses are reached at the column head.

The conditions tested were:

1) Primary failure of the tension steel in the column.

2) Primary failure of the concrete in compression in the column.

For each condition two frames were tested.

1) *Primary Failure of the Tension Steel in the Column.*

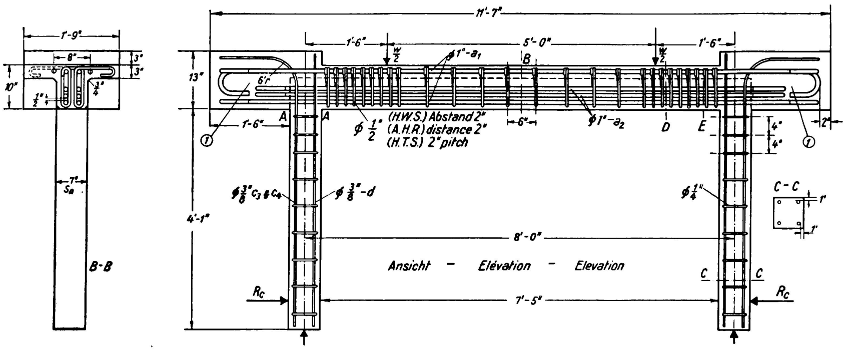
Details of the frames and the positions of loading are given in Figure 5. The design of the reinforcement and the method of loading was such that the beam was considerably stronger than the column. At incipient failure of the weak column there was a considerable reserve of strength in the beam.

In order to ensure that the frame should fail by bending, and not in shear or by slip of the bars, it was necessary to give special attention to the design of the shear reinforcement and the anchorage of the bars. It is clear that redistribution of moment can increase the ultimate load of a structure only when the conditions of bond and shear that result from such redistribution are amply provided for. The large blocks at the column-beam junctions were provided solely for the purpose of giving ample anchorage to the reinforcement of the beams and columns in order that the yield point of the steel could be reached.

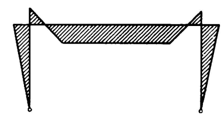
A high strength high alumina cement concrete was used for these tests; details of this are given in Appendix 2.

The horizontal load was applied by two helical tension springs stretched between the column feet, the load being transmitted to the column faces through knife edges. The load on the beams was applied through cylindrical bearings and rollers to allow free rotation and translation of the beam. In the first test the column feet were supported on similar bearings but it was found that the frictional force due to the rollers was sufficient to affect appreciably the horizontal spring load required to prevent outward movement of the feet, and a special knife edge link system was used for subsequent tests.

During the test, gauges were set up at the column feet to measure the movement outwards, and the horizontal load due to the springs was continually adjusted so that the feet were brought back to their original position. That is,



R.M.F.2. & R.M.F.3.



Momentenfläche (nicht masstäblich)
Diagramme des moments (non à l'échelle)
Moment diagram (not to scale)

Eisenliste Liste des fers Bar schedule			
Eisen Armature Bar	ϕ	Masse - Dimensions	Anzahl Nombre No. Off.
a_1	1"	7" — 11'-3" — 6"	6
a_2	1"	7" — 9'-11" — 6"	4
c_3	3/8"	4" — 2 1/4" — 18" — 5'-1"	2
c_4	3/8"	4" — 2 1/4" — 18" — 5'-1"	2
d	3/8"	5'-1"	4

All main bars in beam 1″ dia.
Vertical cover 1″.
① All hooks of internal dia. 4″.
Length of straight 4″.

Calculated Stresses (lb. inch units) R.M.F.3

Stage		Column						Beam at B			Beam at D				ξ_F	s_E	W	R_C
		c	t	M_A	S	s	s_b	c	t	M_B	S	s	s_b	t_w				
1.	I_1	4200	47300	102000	2380	65	190	920	7000	206000	17300	415	155	7600	2.6	190	34500	2380
	I_2	4800	47300	114000	2650	70	210	1200	9000	266000	21900	525	200	8700	2.3	242	43700	2650
2.		11000	47300	283000	6590	175	520	11000	40600	1240000	86000	2120	775	37600	0 2	—	172000	6590

u = 11000 lb. in² m = 5

I_1 Moments of inertia, for moment distribution calculations, based on whole area of Concrete Ignoring Steel.
I_2 Ditto, based on whole area of Concrete Including Steel.

Fig. 5.

Redistribution of moments in frames. Steel failure.

the conditions of restraint were those of a portal, position fixed and pin-jointed at the feet of the columns.

A view of one of the frames whilst the test was in progress is given in Figure 6. A special framework was arranged to prevent any rotation or lateral movement of the supporting beam relative to the upper loading beam, so that no torsional or lateral bending stresses should be set up in the columns.

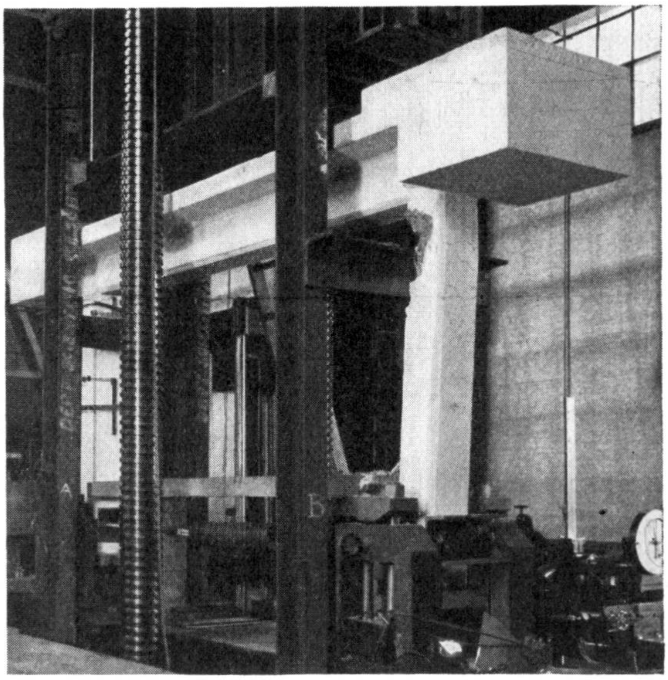


Fig. 6.
Test on reinforced concrete portal frame (Concrete failure).

The main results for the second test are shown in Figure 7. In this figure the applied loads are plotted against the horizontal reactions which are proportional to the moments at the column head, and on the same figure some theoretical curves are also given. One of these curves shows the load-reaction relationship expected for the frame from calculations based on the elastic theory; a series of curves are given for the relationship between the loads and reactions which produce steel yield on the following assumptions:

1) The "instantaneous" modular ratio determines the stress distribution,

2) the modular ratio is taken to be $m = \frac{40\,000}{u}$, and

3) the maximum concrete stress is assumed to reach the cube strength (u).

The point where the first mentioned curve intersects each of the steel failure curves determines the load at which the frame should have failed according to the elastic theory, with or without allowance for stress redistribution according to which assumption the curve represents. These loads are given in Table 3.

On the simplest theory of moment redistribution (i. e. assuming that the column tension steel remains continuously at its yield point) the horizontal reactions and therefore the moments will, after yield of the column steel, conform to the relationship shown by one of the steel failure lines in Figure 7, according to the amount of stress redistribution that occurs. The experimental results gave horizontal reactions which were initially somewhat lower than expected, redistribution beginning at quite a low load, soon after the appearance of cracks at the column head. The curve showing the experimental results gradually approaches the steel failure lines as the load is increased and crosses the line based on $m = \frac{40\,000}{u}$. Incipient concrete failure caused a sudden drop in the rate of increase in moment, and finally failure was reached as a result of concrete crushing.

The beam failure lines indicate the values for the applied load at which beam failure would occur for the degrees of fixity afforded by the various horizontal reactions and it will be seen that if the concrete in the column had not failed a slight increase in load could have been obtained before beam failure.

Throughout the test, measurements were made of the strains at the column heads. The strains were measured on the faces of the column; no direct readings were taken on the steel itself, the steel strain being deduced on the usual assumption that plane sections remain plane. This assumption will probably not

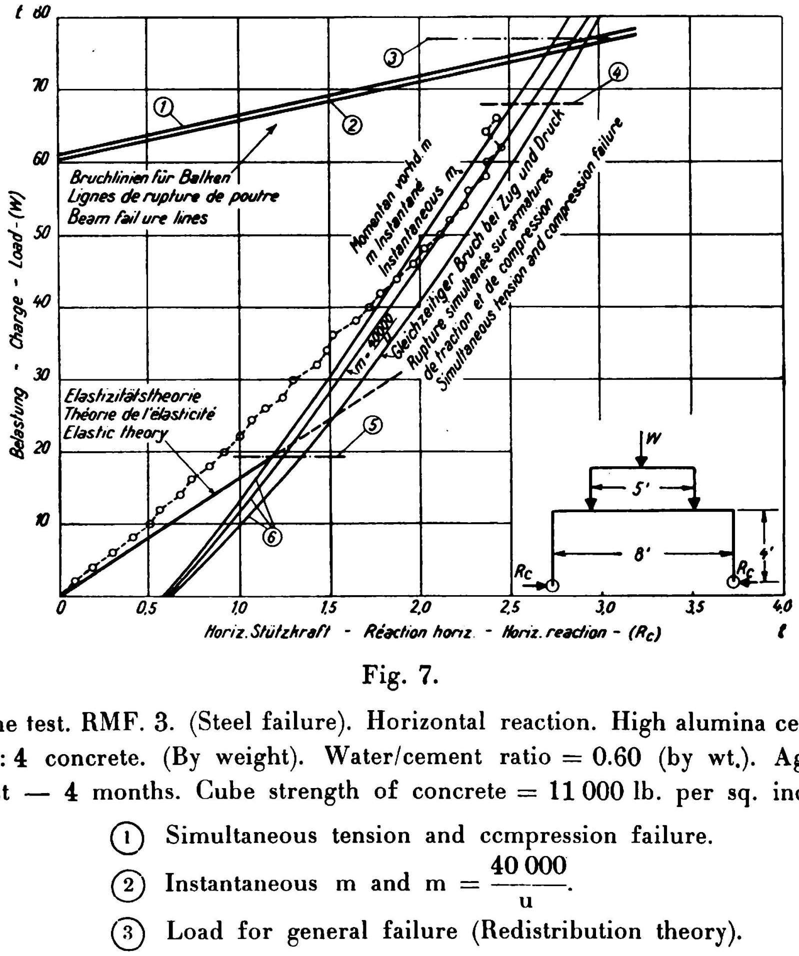


Fig. 7.

Frame test. RMF. 3. (Steel failure). Horizontal reaction. High alumina cement 1 : 2 : 4 concrete. (By weight). Water/cement ratio = 0.60 (by wt.). Age at test — 4 months. Cube strength of concrete = 11 000 lb. per sq. inch.

(1) Simultaneous tension and ccmpression failure.

(2) Instantaneous m and $m = \frac{40\,000}{u}$.

(3) Load for general failure (Redistribution theory).

(4) Actual failing load.

(5) Load for column failure (Elastic Theory).

(6) Column tension failure lines.

lead to very great error except in the final stages of the test. The strain for a steel stress of 47.300 lb. per sq. in. (the yield stress, see Appendix 2) was reached at a load of just over 20 tons, and the strain increased to over four times this value before collapse became imminent. The concrete strain at first signs of crushing was about 32×10^{-4}.

The beam deflection was measured relative to the loading points by means of dial gauges. This deflection was only one-thousandth of the span at about three quarters of the failing load. The overall longitudinal extension of the beam soffit was also measured; as failure of the frame was approached this

Table 3.

Failing Loads of Portal Frames.

Basis of bending Moment Calculations	Basis of Resistance Moment[1] Calculations		Failing Loads — tons: Steel Failure		Failing Loads — tons: Concrete Failure	
		Test No.: —	RMF 2	RMF 3	RMF 4	RMF 5
Elastic theory: i. e. No redistribution of moments, Loads are for column head failure.	No Stress Redistribution	True "instantaneous" modular ratio used	19.5	19.5	21.2	15.0
	Stress Redistribution	$m = \frac{40000}{\text{cube strength}} = \frac{40000}{u}$	21.3	21.3	24.0	18.3
	Stress Redistribution	Steel Failure. Maximum concrete stress reaches cube strength. Concrete Failure: $m = \frac{80000}{u}$	25.0	25.0	27.5	21.4
Theory of Redistribution of Moments: i. e. Simultaneous failure at column head and in beam span.	No Stress Redistribution	True "instantaneous" modular ratio used	75.0	75.0	46.0	41.7
	Stress Redistribution	$m = \frac{40000}{u}$	75.5	75.5	46.8	42.6
	Stress Redistribution	Steel Failure. Maximum concrete stress reaches cube strength. Concrete Failure: $m = \frac{80000}{u}$	77.0	77.0	47.8	43.6
Actual load at which signs of distress were first noticed in the concrete			65.0	64.0	40.0	38.0
Actual ultimate load carried by frame			65.0	67.8	47.1	43.2

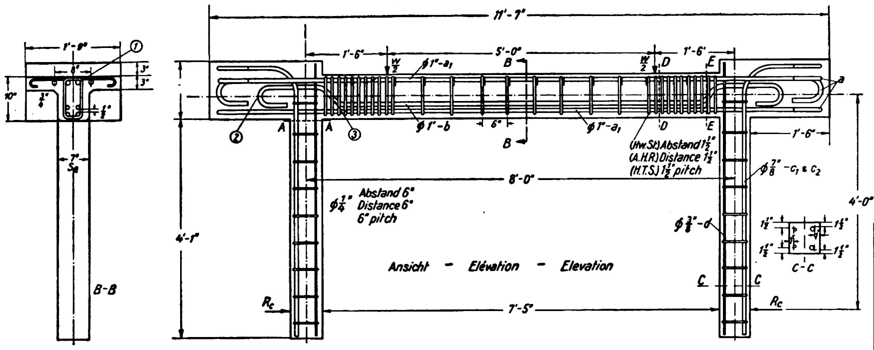
[1] By resistance moment in these tables is meant the ultimate moment the section can carry.

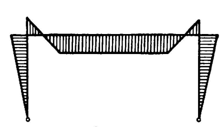
movement was about one-twelfth of an inch at each column head. This movement is insufficient, as an added eccentricity, to have an appreciable effect on the stress at the column head.

Cracks at the column head appeared at a load of about 5 tons, widened steadily throughout the test, and just before failure were about twice as wide as the cracks usually obtained when steel reinforcement reaches its yield point.

2) *Primary Failure of Concrete in the column.*

Details of the reinforcement used for the second type of frame are given in Figure 8. Again the design was arranged to give a reserve of strength in the beam. The tension steel in the column was increased to two $^7/_8$ in. diameter bars instead of $^3/_8$ in. bars and the concrete used was an ordinary Portland





Eisenliste - Liste des fers - Bar schedule

Eisen Armature Bar	ϕ	Masse - Dimensions	Anzahl Nombre No. Off
a_1	1"	11'-3"	6
b	1"	5', 9'-9"	2
c_1	7/8"	5'-1"	2
c_2	7/8"	5'-1"	2
d	3/8"	5'-1"	4

Calculated Stresses (lb. inch. units) R.M.F.4

Stage		Column						Beam at B			Beam at D				ξ_F	s_E	W	R_C
		c	t	M_A	S	s	s_b	c	t	M_B	S	s	s_b	t_w				
1.	I_1	2850	15000	127000	2960	90	115	860	12300	245000	21500	465	260	12400	3.1	460	43000	2960
	I_2	2850	13000	124000	2890	89	114	1010	14500	289000	23800	515	285	13800	2.3	510	47500	2890
2.		2850	7500	131000	3050	94	120	2850	41100	817000	53500	1160	645	31000	—	1150	107000	3050

I_1 Moments of inertia for moment distribution calculations based on whole concrete area ignoring steel.

I_2 Ditto, based on whole concrete area and including steel.

$u = 2850$ lb/sq. in.
$m = 9$ (Stage 1)
$m = \frac{80000}{u}$ (Stage 2)

① Stirrups welded.

② Hook internal radius = 3", all other hooks of internal radius = 4 × bar dia.

③ Internal radius of bend for main bars, 3 1/2".

(All main bars in beam 1" Dia. Vertical Cover 1").

Fig. 8.

Redistribution of moments in Frames. Concrete failure.

cement of $1:2^1/_2:3^1/_2$ mix (by weight). Details of the strengths of the steel and concrete are given in Appendix 2.

The method of test was identical with that used in the second frame of the previous series, and the values for the horizontal reaction for the first frame are given in Figure 9. It will be seen that the initial relationship between vertical load and the horizontal reaction is in good agreement with that expected from the elastic theory. According to this theory the concrete should crush at a load of about 21 tons, i. e. at the load when the initial line in Figure 9

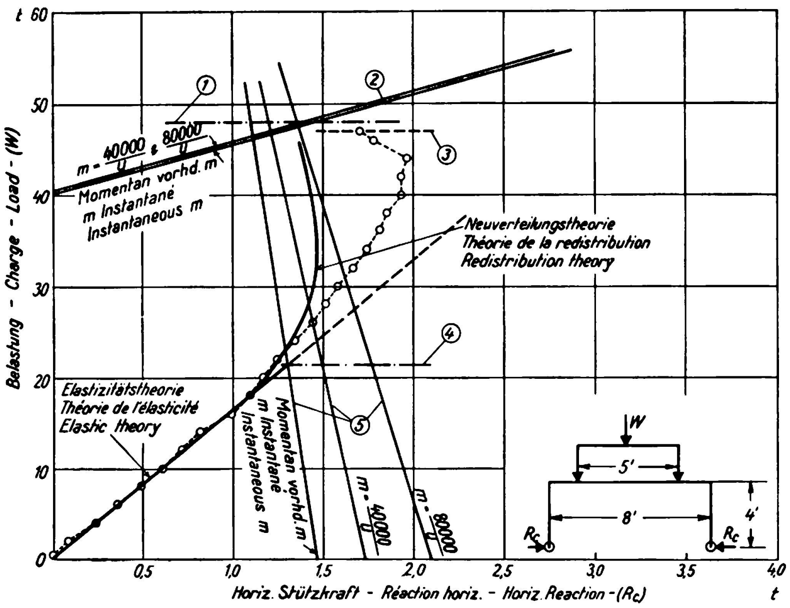


Fig. 9.

Frame test. RMF. 4. (Concrete failure). Horizontal Reaction. Normal Portland cement $1:2^1/_2:3^1/_2$ concrete (by weight). Water/cement ratio = 0.66 (by wt.). Age at Test = 9 days. Cube strength of concrete = 2850 lb. per sq. inch.

(1) Load for general failure (Redistribution theory).
(2) Beam failure lines.
(3) Actual failing load.
(4) Load for column failure (Elastic theory).
(5) Column compression failure lines.

reaches the compression failure line for a modular ratio of m = 9, which is the true value for the concrete used when inelastic deformations are disregarded. Curves representing compression failure are also given based on modular ratios of $\frac{40.000}{u}$ and $\frac{80.000}{u}$. The redistribution of stress in the column head section was even more favourable than is assumed by this last line, probably due to the increased stresses taken by the concrete above those assumed by a linear distribution of stress from the neutral axis to the compressed face. However, assuming this last compression failure line as a safe guide, it is seen that unless moment redistribution occurs there will be signs of distress in the concrete at a load of about 28 tons. If moment redistribution does take place, then the load will increase with a reduction of horizontal reaction until beam failure is reached at a load of about 48 tons. Actually redistribution will start before signs of distress can be seen and the approximate theoretical change in load and moment is indicated in Figure 9. The actual curve shows that the theory is on the safe side, the moments increasing more than expected from the simple theory of redistribution, with a sudden drop in moment after signs of crushing first appeared. The failing load, 47.1 tons, agrees well with the expected value (see Table 3) and was the result of simultaneous crushing of the concrete in the column and yield of the steel in the beam.

The strains at the column head were measured as before; the interpolated steel strains showed that the tension stresses were low throughout, but that the compression bars were working at their yield load towards the end of the test. The deflection of the beam and the extension of the soffit were again small; the column cracking was also of little importance whilst the beam cracks increased to a width of about 6 or 7 thousandths of an inch, a width usually associated with a steel stress of about 40.000 lb. per sq. in.

In the case of the second frame of this series, the concrete strength was somewhat less than that used for the first frame (see Appendix 2) but apart from the reduced values of load and moment due to this cause the results were very similar to those already discussed. Again the use of a modular ratio of $\frac{80.000}{u}$, together with the assumption that the column will continue to deform so as to redistribute the moments until beam failure occurs, leads to an accurate estimate of the failing conditions (see Table 3).

Discussion of Results.

A. Continuous Beam Tests.

The actual failing loads of the continuous beams, together with values calculated on various assumptions are summarised in Table 2. It is apparent that with all the beams, the ultimate load carried before failure of the system was greater than the theoretical load for support failure calculated on the elastic theory. The increased load can be considered as due to two factors, both resulting from inelastic deformations of either concrete or steel:

1) Redistribution of moments throughout the system tending to give simultaneous failure both at the central support and in the span.
2) Redistribution of stress at the highly stressed sections, increasing the moments these sections are capable of taking above the values as calculated by the ordinary theory.

In Table 2 the calculated loads are based on three sets of resistance moments. The first is obtained by the use of the true or "instantaneous" modular ratio, that is, the ratio which neglects all inelastic deformation of the concrete. The second is obtained by assuming that inelastic deformation of the concrete will lead to an increase of the modular ratio to a value $m = \frac{40.000}{\text{cube strength}}$, the value suggested for design purposes in the Code of Practice for the Use of

Reinforced Concrete in Buildings.[5] The third set of resistance moments were calculated on the following assumptions:

a) In the case of primary tension steel failure the steel will yield until the maximum concrete stress reaches the cube strength of the concrete.

b) In the case of primary concrete failure, the modular ratio will effectively increase to a value given by $m = \frac{80.000}{\text{cube strength}}$. If, however, tension steel yield occurs when this higher value is used, the resistance moment is calculated as for (a). If the calculated stress in the compression steel exceeds its yield value when the higher modular ratio is used, the calculations are modified so that the compression bars do not exceed thier yield value.

From Table 2 it will be seen that if the elastic theory is used for calculating the moments at failure, the theoretical failing loads are less than the actual ultimate loads, even when allowance is made for redistribution of stress.

On the other hand, if redistribution of moments is allowed for, the theoretical loads for simultaneous failure at the central support and in the span, when no redistribution of stress is taken into account, are also less than the actual loads carried, though the margin of safety is not so great.

If allowance is made for both moment and stress redistribution the use of a modular ratio of $\frac{40.000}{u}$ leads to theoretical loads which are not greatly different from the actual ultimate loads except in the case of the beams in which compression reinforcement was used over the central support with a weak concrete [series (3)]. The use of the third method of allowance for stress redistribution, when moment redistribution is also allowed for, is clearly unsafe except in the case of primary steel failure, for which it must be remembered that the redistribution of moments is accompanied by widening of the tension cracks, see Table 1.

The results of the tests on the beams in which compression reinforcement was provided are important. The use of a very high modular ratio for estimating the resistance moment of a section leads to increased computed stresses in the compression bars and it does not appear advisable to rely upon this. In order to investigate this aspect more fully, some simple beam tests were carried out to measure the resistance moments of the sections similar to those used over the central support in the main tests. From these tests, it was found that the use of the highest modular ratio $\frac{80.000}{u}$ is reasonable in all cases of concrete failure except those in which compression reinforcement was provided. In these cases the simple beam tests indicated that redistribution of stress may occur to the extent indicated by the use of the lower modular ratio of $\frac{40.000}{u}$, whereas the support moments measured in the continuous beam tests are not appreciably greater than those calculated on the basis of the "instantaneous" modular ratio. It is possible, however, that the higher shear stresses in the continuous beams with compression reinforcement may have been the reason for the low moment carried over the central support. It appears therefore that when compression reinforcement is provided at the support its effect should be ignored in making calculations taking moment redistribution into account. If this is done for the present beams of series (3), the calculated loads $\left(\text{using a modular ratio of } \frac{40.000}{u}\right)$ are 28.9 and 31,6 tons, 5 and 9 per cent. greater respectively than actually obtained. If the effect of the compression reinforcement in the span is also ignored, the calculated loads are 23.4 and 25.2 tons respectively and these are on the safe side.

B. Portal Frame Tests.

It is clear from the tests that there may be considerable divergence between the actual ultimate load-carrying capacity of a frame and the load which, according to calculations based on the elastic theory, produced a stress in the concrete or steel, at the column head, equal to the ultimate strength of the concrete or the yield strength of the steel. It is important to note that in the tests special precautions were taken to prevent shear failure, closely spaced high tensile steel stirrups being provided in the beams, and special anchorage blocks at the beam-column junctions. Redistribution of moments cannot occur unless the secondary reinforcement and the anchorage of the steel are sufficient for the conditions resulting from the redistribution.

In the case of primary steel failure, the increase in load due to redistribution of moment and stress was over 200 per cent. However, in this case complete moment redistribution did not occur, beam failure not being reached, owing to the earlier crushing of the concrete in the column, even though the cube strength was 11 000 lb. per sq. in. In such cases it is not at present possible to calculate accurately the load at which the concrete will fail as it depends on the deformation of the column after yield of the tension steel. Since the extent to which redistribution can take place as a result of steel yield is not clearly defined and redistribution leads to increased cracking it would be wise to ignore it until further experimental evidence has been obtained.

In the case of primary concrete failure, there are again considerable increases in the ultimate loads carried by the frames as a result of redistribution of stress and moment. If we consider that the useful limit of load increase is when signs of crushing first appear on the column faces it will be seen from Table 3 that the load increase above the value calculated on the elastic theory was 90 per cent. for the first frame and 150 per cent. for the second frame.

In both cases the increase in beam load-carrying capacity as a result of the column moment was less than 20 per cent. whereas the columns would, if loaded axially, have been able to withstand about twice the load that they took in the frame test. The need for taking bending in columns into account is evident.

It would appear that an estimate of the effects of redistribution can be made in simple cases where concrete failure is the deciding factor on the following assumptions:

1) The modular ratio can be taken as $\frac{80.000}{u}$.
2) Both column head and span develop their full strengths before failure of the system occurs.

In any cases where the use of the higher modular ratio leads to calculated stresses in the tension steel greater than the yield point of the steel, the particular section should be calculated on the assumption that both steel yield and the full concrete strength are developed.

It is clear from Figure 9 that stress redistribution occurred in the column head section to a greater extent than that indicated by the use of a modular ratio of $\frac{80.000}{u}$ and from this figure and Table 3, it is seen that the effect of stress redistribution, if moment redistribution is ignored, is to increase the failing load by about 30 per cent. for the particular section used. The increase may not be so great in other cases. For example in the continuous beam tests described earlier in this report the increase in resistance moment due to stress redistribution was only about 13 per cent. for the central support section of the beams of series (2) and (4). In the columns of the portal frames designed for concrete failure, the compression steel used was much less than the tension steel whereas normally the section would be symmetrically reinforced. In view of the smaller amount of stress redistribution that occurred in beam sections reinforced in compression, it would therefore be unwise to use the higher modular ratio, and a value of $m = \frac{40.000}{u}$ is likely to lead to more satisfactory results.

General.

It has been shown that as a result of inelastic deformation of either the steel or the concrete at incipient failure, moment redistribution will usually occur in reinforced concrete structures before final collapse.

The amount of moment redistribution that can occur depends on many factors but to a large extent on the amount of deformation possible at weaker sections. Where weaker sections are capable of developing sufficient deformation, redistribution will be complete and failure simultaneous at principal sections. Further investigation is necessary to fix the safe limits of deformation. Until this is done it would appear wise not to deviate greatly in design from the requirements of the elastic theory.

Design of reinforced concrete structures on the basis of redistribution of moments must take into account the higher bond and shear stresses that accompany redistribution.

List of References.

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2 *F. E. Richart, R. L. Brown* and *T. G. Taylor:* „The effect of Plastic Flow in Rigid Frames of Reinforced Concrete." Journal Am. Conc. Inst., Vol. 5, pt. 3, 1934, pp. 181—95.

3 *G. von Kazinczy:* „Das plastische Verhalten von Eisenbeton." Beton und Eisen, Vol. 32, pt. 5, 1933, pp. 74—80.

4 *C. Bach* and *O. Graf:* „Versuche mit eingespannten Eisenbetonbalken." Deutscher Ausschuß für Eisenbeton, Heft 45, 1920.

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Appendix 1.

Quality of concrete and steel used in connection with continuous beam tests.

a) *Concrete.*

Series	Beam	Concrete Mix (by wt.)	W/z Ratio.	Age at Test	Cube Strength — lb/sq. in.	True "Instantaneous" Modular Ratio.
1. Steel Failure	RM2 (a)	H.A. 1 : 2 : 4	0.60	6 days	10.140	5.0
	RM2 (b)	R.H.P. 1 : 1 : 2	0.44	44 days	6.660	6.0
2. Concrete Failure (No compression steel)	RM1 (a)	P. 1 : 2 1/2 : 3 1/2	0.66	7 days	2.020	10.0
	RM1 (b)	P. 1 : 2 1/2 : 3 1/2	0.66	7 days	2.070	10.0
3. Concrete Failure (With compression steel)	RM3 (a)	P. 1 : 2 1/2 : 3 1/2	0.66	7 days	2.250	9.5
	RM3 (b)	P. 1 : 2 1/2 : 3 1/2	0.66	7 days	2.470	9.1
4. Concrete Failure (Increased span length)	RM4 (a)	P. 1 : 2 1/2 : 3 1/2	0.66	7 days	2.130	9.7
	RM4 (b)	P. 1 : 2 1/2 : 3 1/2	0.66	7 days	1.830	10.4

P. = Ordinary Portland Ccment. H.A. = High Alumina Cement.
R.H.P. = Rapid-Hardening Portland Cement.

b) *Steel.*

Series	Bar diameter — inch.	Yield Stress — lb/sq. in.[1]	Failing Stress — lb/sq. in.[1]
1. Steel Failure	$\frac{5}{8}$	39.400	—
	$\frac{3}{8}$	44.700	62.200
2. Concrete Failure (No compression steel)	$\frac{7}{8}$	40.200	56.500
	$\frac{3}{8}$	46.100	61.500
3. Concrete Failure (With compression steel)	$\frac{7}{8}$	39.800	53.800
	$\frac{3}{8}$	46.700	62.700
4. Concrete Failure (Increased span length)	$\frac{7}{8}$	37.900	53.300
	$\frac{3}{8}$	46.700	61.800
5. Concrete Failure (Weak concrete at about 6 months)	$\frac{7}{8}$	36.600	51.500
	$\frac{3}{8}$	45.800	61.400

[1] The stresses are in all cases based on the nominal original area of the bar.

Appendix 2.

Quality of concrete and steel used in connection with portal frame tests.

a) *Concrete.*

Series	Beam	Concrete Mix (by wt.)	W/z Ratio	Age at test	Cube Strength lb/sq. in.
Steel Failure	RMF 2	H.A. 1 : 2 : 4	0.60	48 days	10.500
	RMF 3	H.A. 1 : 2 : 4	0.60	4 months	11.000
Concrete Failure	RMF 4	P. 1 : 2 1/2 : 3 1/2	0.66	9 days	2.850
	RMF 5	P. 1 : 2 1/2 : 3 1/2	0.66	7 days	1.850

P. = Ordinary Portland cement. H.A. = High Alumina Cement.

b) *Steel.*

Series	Beam	Bar diameter inch.	Yield Stress — lb/sq. in.[1]	Failing Stress — lb/sq. in.[1]
Steel Failure	RMF 2	$\frac{3}{8}$	49.200	60.800
		1	41.500	63.700
		1/2[2]	66.900	106.000
	RMF 3	$\frac{3}{8}$	47.300	59.700
		1	40.600	65.700
		1/2[2]	63.800	107.000
Concrete Failure	RMF 4 et RMF 5	$\frac{7}{8}$	38.600	53.800
		1	41.100	63.000
		1/2[2]	64.700	107.000
		$\frac{3}{8}$	48.300	60.300

[1] The stresses are in all cases based on the nominal original area of the bar.
[2] High tensile steel used for web reinforcement of beam.

IIa 7

Stressing and Factor of Safety of Reinforced Concrete Trussed Girders.

Beanspruchung und Sicherheitsgrad der Eisenbeton-Fachwerke.

Sollicitations et degré de sécurité des poutres réticulées en béton armé.

Dr. sc. techn. S. Mortada,

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It was found in the author's own experiments on reinforced concrete trusses that structures of this type offer exceptional resistance to impact and dynamic stresses.

The tests were carried out in the laboratory for testing materials at the Swiss Federal Institute of Technology at Zürich,[1] the specimens being two reinforced concrete trusses such as are used in bridge work (Fig. 1). The span of these girders was 6 m, their height 1.50 m and they were subject to an isolated load of 50 tons at the centre.

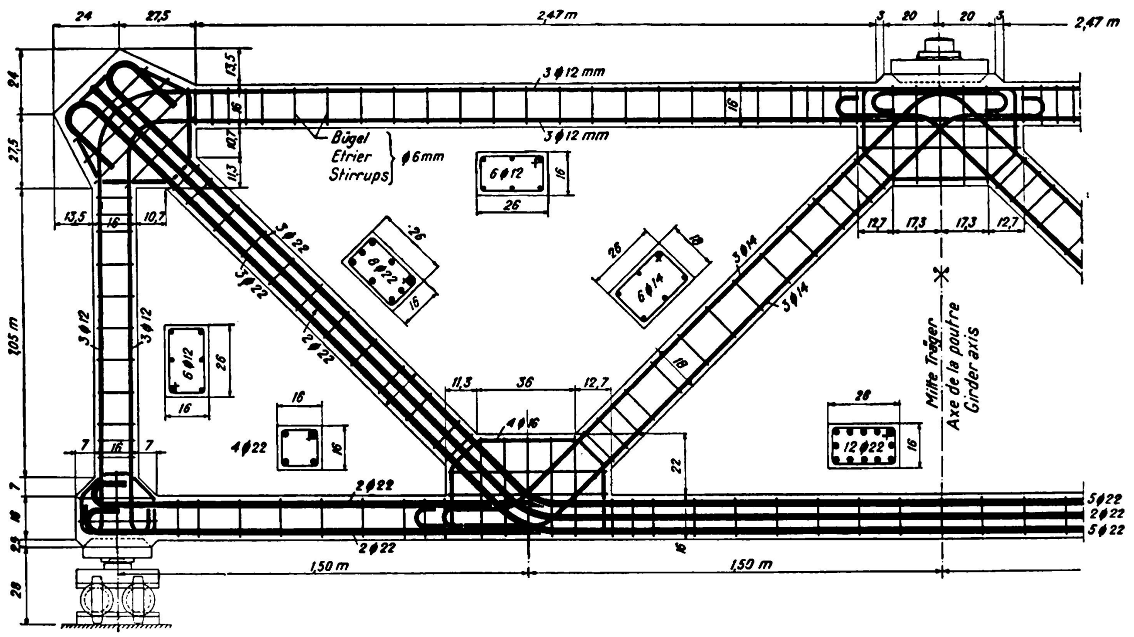

Fig. 1.

Details of the Test-Girder.

[1] *Mortada:* Beitrag zur Untersuchung der Fachwerke aus geschweißtem Stahl und Eisenbeton unter statischen und Dauerbeanspruchungen. Dissertation, Zürich, 1936.

The age of the concrete at the time of the experiment was 90 days, its prism strength $_{p}\beta_{d}$ was 360 kg/cm² and its fatigue strength $\sigma_{u} = 220$ kg/cm² amounting to approximately 0.6 of the prism strength. The reinforcement consisted

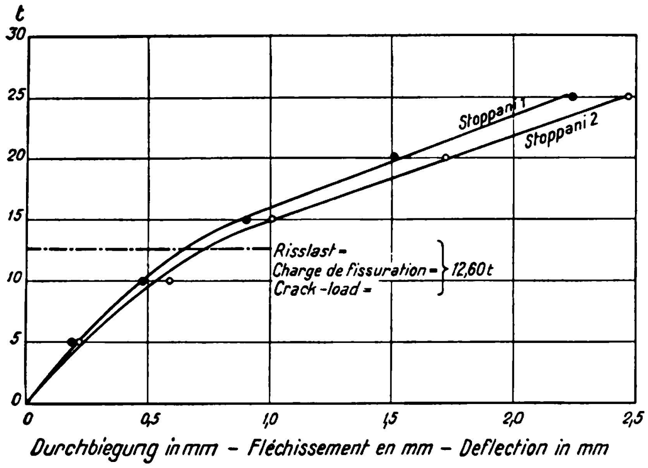

Fig. 2.

Determination of crack-load.

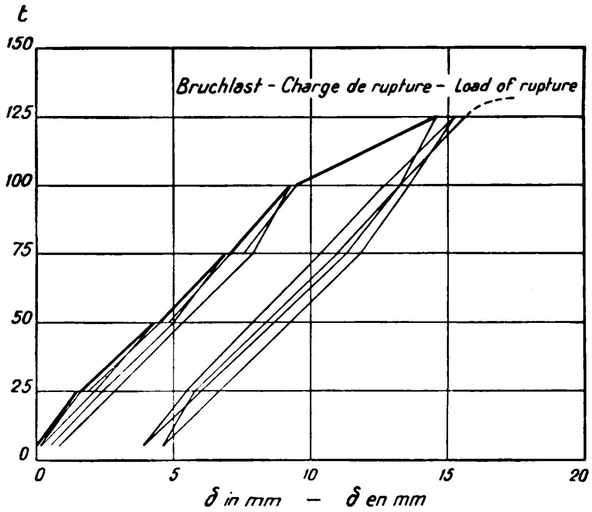

Fig. 3.

Break-test: Loading-Unloading-Deflection Diagram.

of round bars of ordinary steel having a yield point of 2700 kg/cm², a tensile strength of 4200 kg/cm² and a fatigue strength of 2500 kg/cm².

One of the two girders was subjected only to statical tests, in order to study its behaviour under static loading and finally its static breaking load. The second girder, however, was subjected to fatigue tests before being tested statically in exactly the same way as the first. In this way it was possible to ascertain how far the effect of fatigue influenced the statical behaviour and carrying capacity of a structure of this kind.

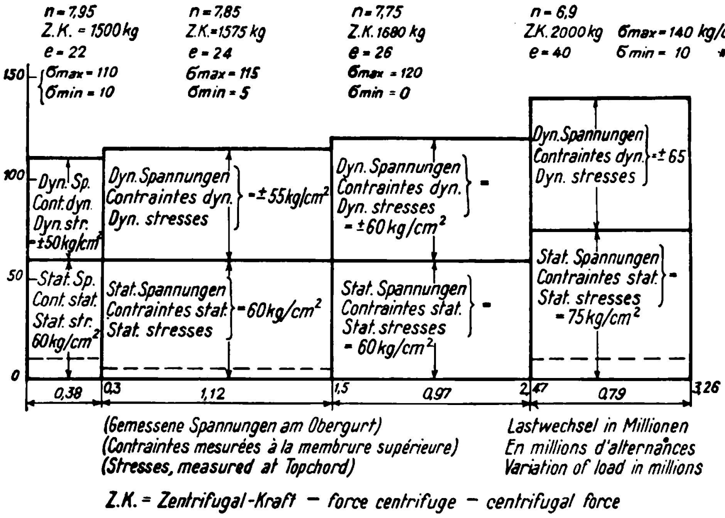

Fig. 4.

Fatigue-test: Measured stresses at different test-phases.

Preliminary experiments were made to ascertain the cracking load and the amount of permanent deformation which occurred after the concrete had cracked. The effect of cracking at various points in the concrete is to introduce irregularities in the stress-strain curve and this enables the cracking load to be determined (Fig. 2), amounting in this case to approximately $^{1}/_{4}$ of the calculated live load. The average breaking stress of the concrete in tension corresponding to the cracking load amounted to 17 kg/cm² though the tensile strength of the concrete itself was 40 kg/cm². The large difference between these two values is to be explained on the following grounds:

a) Pre-stressing of the concrete in tension as the result of shrinkage.
b) Incompleteness of the cracking of the concrete when related to the whole of the cross section.

The cracking of the concrete naturally resulted in large permanent deformations, amounting to about 25 % of the elastic deformation under live load. In structures of this kind the secondary stresses (especially those in the compression members) are exceptionally high, amounting to as much as 110 % (on the average they may be put at 70 %) while at the same time the bending stresses in tensile members are very low.

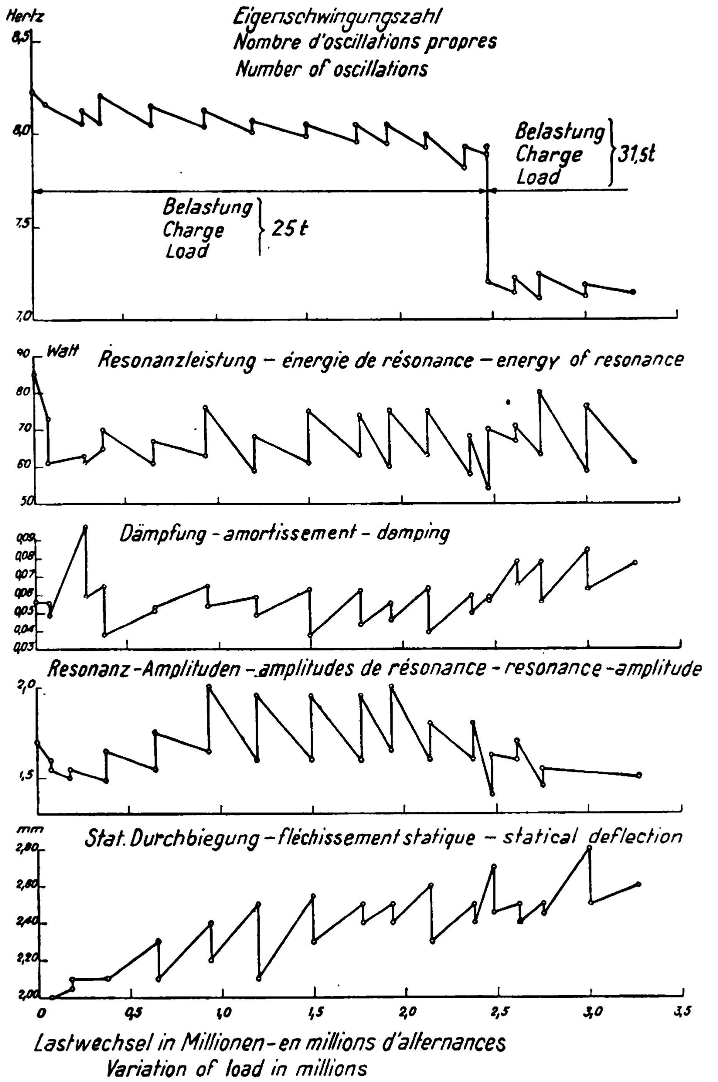

Fig. 5.

Fatigue-test: Change of dynamic-values with the Fatigue.

It was found that at the moment when the stress in the reinforcing steel reached the yield point the maximum compressive stress in the concrete was of the order of 220 kg/cm², which is equal to the fatigue strength of the concrete. The load corresponding to this was twice the live load. Since the fatigue strength of the concrete and the yield point of the steel are the criteria for resistance to repeated loading, it follows that in reinforced concrete trusses there exists a factor of safety of 2 against fatigue effects. Under stresses of this order the permanent deformations amount to 5.5 % of the total (Fig. 3) and may, therefore, be taken as accurate for practical purposes.

The factor of safety against statical breakage amounted to 2.6. The ratio between the respective factors of safety against repeated and statical loading is, therefore, that of $2/2.6 = 77\,\%$.

The ranges of stress and corresponding numbers of changes of load in the fatigue tests are indicated in Fig. 4. After a very large number of repetitions of load ($3^1/_4$ millions) within the permissible limits of stress (or even slightly in excess of those limits) no appreciable change in the statical or dynamical properties of the test girders could be observed and neither their carrying capacity nor their factor of safety were in any way affected.

A number of notable observations were made in the process of the fatigue tests (Fig. 5). Thus it was found that in the course of the process the damping and the static deflection increased, while at the same time, the restoring force and natural frequency of vibration decreased. Resonance tests carried out under the same experimental conditions showed that the power consumed by the vibration testing machine (Fig. 6), as well as the amplitude and the magnification, decreased with the fatigue, to be followed by subsequent recovery of stiffness. During certain periods of the tests a state of inertia was found to be established after a certain number of repetitions of load.

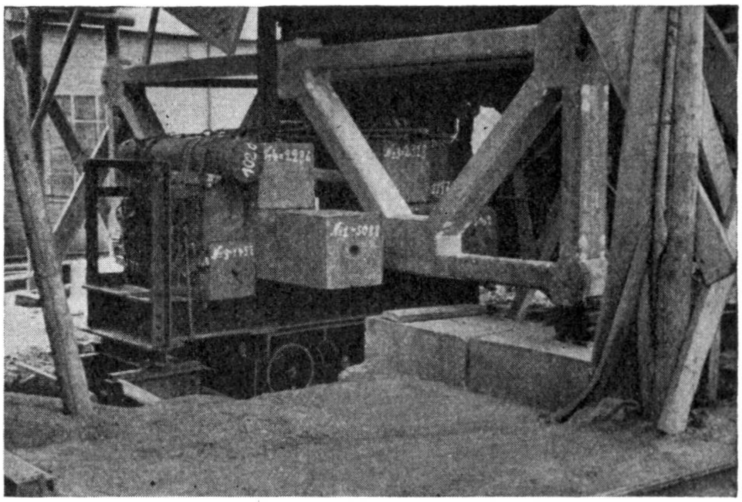

Fig. 6.
Arrangement of Fatigue-test.

From a practical point of view the most important conclusions to be drawn from these experiments on reinforced concrete girders are the following:

Frequently repeated stress within the observed limits (fatigue strength) does not adversely affect either the elasticity or the carrying capacity or the dynamical properties of reinforced concrete trusses, and in structures of this kind safety as regards static loading may also be taken to imply adequate safety against the effects of repeated loading.

IIa8

The Factor of Safety of Reinforced Concrete Structures.

Über die Sicherheiten der Eisenbetonbauten.

La sécurité des ouvrages de béton armé.

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1) *Present definition of factor of safety.*

According to present usage the factor of safety of static structures is defined by reference to the permissible stresses, being, as a rule, the proportion between the breaking stress or yield point of the material and the permissible stress.

This definition, however, is not an adequate one, and in the course of time it has gradually been found necessary to supplement it by statements of special requirements. For instance in the case of retaining walls it is necessary to insure not only against excessive pressure on the foundations but also against the risk of overturning; much the same thing applies to cantilever slabs; and brick chimneys are made subject to the special requirement that the theoretical tensile stresses must not occur beyond the centre of gravity of the cross section. In all these instances the special requirements are those which have reference to stability.

An even more notable fact is that the concept of permissible strength has no meaning in application to columns. It is true that the permissible stress is now prescribed as a function of the buckling length, but this practice amounts to no more than a restatement (or, as it were, a tabular solution) of the column formulae, and the crux of the matter is that columns are in fact dimensioned to carry a load which has already been multiplied by a factor of safety, there being no regular relationship between load and stress. Thus in the design of columns the whole idea of a breaking load is abandoned, and this is contrary to the practice followed in the design of tie bars wherein the cause of failure may always be taken as some defect in the material independent of any increase in load.

To sum up, it is impossible to give any concise definition of what is meant by the factor of safety in static structures as the term is used at present. It may be noted, also, that at the present time safety against dynamic stresses is partly covered by the introduction of an impact coefficient, and this again implies a different idea than the original one of depending on permissible stresses.

2) *Disadvantages of the present factor of safety as applied to structures.*

A general disadvantage of using the factor of safety in its present form is that it does not admit of concise definition. A further disadvantage is that the principal criterion of safety, namely the permissible stress, should be one which in many cases (such as problems relating to stability, column design and dynamic stresses) is of little or no importance. It is again a disadvantage that this principal criterion should need to be supplemented by a variety of extra requirements which have no relation to one another, the factor of safety being made to refer now to the loading, now to the conditions of fracture of the material, and now to the yield point of the latter. As materials increase in strength, problems of stability will tend to become ever more important, and this may entail an even greater variety of special conditions than at present. It must be counted a defect that the main criterion of safety should not be one which in itself guarantees stability from every aspect, and that the form of guarantee should not allow of different weight being attached to different kinds of stress and loading: thus certain stresses due to the dead weight of the structure and to the process of erection might reasonably be treated in a different way from the stresses that will arise when the completed structure is in service. It is a disadvantage, further, that the "own weight" of the structure should have to be multiplied by the same maximum value of the factor of safety whether its action is favourable or unfavourable to stability.

The most frequent occasion for special conditions, tending in this way to take the place of the permissible stress as the criterion of safety, is found in the absence of proportionality between load and stress. In columns this lack of proportionality is due to buckling, but in most other cases it is due to the fact that the dead load and the live load produce stresses which have no common measure: that is to say the dead load stresses and the live load stresses cannot directly be added together.

3) *Special disadvantages attending the application of the present form of the factor of safety to reinforced concrete structures.*

The disadvantages noted above are of general application to most forms of construction and to all kinds of material, but reinforced concrete possesses certain characteristics which render the present criteria of safety especially unsuitable.

In the first place, reinforced concrete is a heterogeneous material: as a rule the steel reinforcement is arranged, with the greatest possible nicety, to carry the tensile stresses, and the result is that when such stresses arise at unintended places the material is particularly ill suited to resist them. That is to say, in the case of reinforced concrete the lack of proportionality between load and stress is particularly marked, and reinforced concrete is much more sensitive than homogeneous materials to changes in the proportion between stationary and moving loads. Changes in the proportion between dead and live loads are particularly dangerous in the case of reinforced concrete arches, and from this point of view, indeed, the arched form of structure is at some disadvantage compared with the beam, whatever the material used.

As an example, the following are the stresses that arise in a two-hinged roof arch of 24 m span, 4 m rise and 15 cm thickness, reinforced on each side with five rods of 10 mm diameter per metre width, subject to a dead load of 400 kg/m^2 and a live load of a 100 kg/m^2:

Steel stress $\sigma_j \sim 943$ kg/cm².
Concrete stress $\sigma_b \sim 44.8$ kg/cm².

If the live load be increased by 50 % to 150 kg/cm² the stresses become

$$\sigma_j \sim 1770 \text{ kg/cm}^2$$
$$\sigma_b \sim 65.9 \text{ kg/cm}^2$$

In other words, σ_j is increased by 87.5 % and σ_b by 47.2 %.

On the other hand, in a simply supported reinforced concrete slab designed for the same dead and live loads, the increase both of σ_j and σ_b when the live load is increased by 50 % is only 10 %.

These figures speak for themselves. Structures which have been designed with special reference to the characteristics of a stationary load are particularly sensitive to changes in the relationship between stationary and moving loads, and generally speaking reinforced concrete structures are less favourably conditioned in this respect than either steel or timber structures — partly because reinforced concrete is in itself heavier, and partly on account of its heterogeneity.

A further reason for abandoning the present criteria of safety as applied to reinforced concrete constructions is to be found in the greater importance attaching to the conditions of breakdown of this material. In concrete and reinforced concrete *Hooke*'s Law is not valid, and for economic reasons the principles followed in dimensioning the cross section are those derived from breaking tests. Also in calculating shear forces (such as those due to moments, transverse loads, etc.) the tendency is always in the direction of laying emphasis on the conditions of fracture. This makes if all the more important that breakdown should be logically defined, which cannot be done by reference to the usual permissible stresses.

Yet a third reason for abandoning the permissible stress as a criterion of safety lies in the great dead weight of concrete structures. It may be observed that structures in which the dead weight is large may more safely be overloaded than structures in which it is relatively small. That is to say, a stationary load which is incapable of increasing above its assumed amount, which cannot vary, and which can exercise no dynamic effect, may be regarded more favourably than a moving load and from the point of view assessing the degree of safety possessed by the structure the former should be regarded in a different category from the latter. Indeed, so far as dynamic effects are concerned, this difference is already recognised by the introduction of impact coefficients, but otherwise the customary method of design by reference to permissible stresses is too severe in its treatment of stationary loads. This statement applies generally to all constructional materials, but the disadvantage is greatest in the case of mass structures and from this point of view reinforced concrete is prejudiced by comparison with steel and timber.

The customary method of calculation is illogical in yet another sense. In most countries, if any noticeable defects appear during the course of construction the work in question is not immediately pulled down, but a test is made under load, and if the defects seem to be serious the test loads are increased so as to produce an overload of perhaps 50 % at the most dangerous places; if the structure succesfully withstands these test loads it is regarded as acceptable for use. Dependence is, therefore, placed on a construction because it has been found amply safe to resist live load: but no regard is taken of its untested degree of safety to resist dead load. It should, however, here be observed that a structure which has been designed to carry a particular ratio of live to dead load may be dangerous when subjected to a form of loading in which this proportion is noticeably increased.

The heavy dead weight possessed by a reinforced concrete structure is a valuable characteristic, and one which ought not to be needlessly penalised.

4) *What should the factor of safety cover?*

The following points will be briefly mentioned:

a) Errors and inaccuracies in the assumed basis of design.
b) Defects of material.
c) Inaccuracies of execution.
d) Inaccuracies of the imposed loading.

In other words, the following items should all be covered: secondary stresses, internal stresses, certain fluctuating stresses, imposed stresses, erection stresses, inaccuracies of calculation, faulty material, inaccuracies in sections (such as steel bars) as delivered from the workshops, inaccuracies in erection and workmanship, inaccuracies in the "own weight", divergences of the live load from that assumed in the design, exceptional overloads such as test loads, and other contingencies.

It is not possible, however, to fix a factor of safety of ordinary magnitude which will cover all these contingent errors and inaccuracies individually: the most that can be done is to take account of their *probable* combinations.

It is true that the latter may equally well be expected to consist of a few high values as of a larger number of small or medium values, but it may be shown that several of the categories of defects named above can only be covered — or can most economically be covered — by the assumption of an increase in the live load. Generally speaking, it may be said that a stationary load can always be assumed to be replaced by a moving load, but a moving load cannot be taken as replaced by a stationary load.

Certain defects in material form an exception to this statement, in that the best way to allow for them is to assume a reduced value for the breaking stress or yield point. Here it is necessary to be clear what purpose is actually served by the use of a factor of safety. In the author's opinion what matters most is safety against breakage, whereas safety against cracking — important as it may be — is secondary.

5) *Proposed new form of the factor of safety for practical use.*

The factor of safety in its present form is expressible as follows:

$$(1) \quad \sigma_p + \sigma_g + \sigma_w + \sigma_t \leqq \sigma_{zul} = \frac{1}{n} \sigma_B$$

In the case of colums:

$$(2) \quad P_{zul} \leqq \frac{1}{n} P_{breakage}.$$

In application to problems of stability:

(3) $M_{favourable} \geqq n' \cdot M_{unfavourable}$

where zul. *(zulässig)* denotes "permissible"

p refers to live load
g „ „ dead load
w „ „ wind load
t „ „ temperature stresses, etc.
σ_B is the nominal breaking stress or yield point.
n and n' are factors of safety.

The first and most general rule can be re-written

(4) $n \cdot \sigma_p + n \cdot \sigma_g + n \cdot \sigma_w + n \cdot \sigma_t = \sigma_B$

or (5) $\sigma_{(n \cdot p)} + \sigma_{(n \cdot g)} + \sigma_{(n \cdot w)} + \sigma_{(n \cdot t)} = \sigma_B$

referring to the stresses caused by the loads multiplied by n. Equation (5) gives the nominal breaking condition which agrees with Equation (2) but is contradictory to Equation (3), seeing that n' is usually smaller than n. In other words, the definition of breakdown is not consistent; moreover it is impossible really to imagine the "own weight" as being multiplied by n, which, in the case of columns, is an abstraction that has to be made.

In the proposal now put forward, the three conditions numbered (1), (2) and (3) above are combined as follows:

$$\sigma_{(n_g \cdot g)} + \sigma_{(n_p \cdot p)} \leqq n_B \cdot \sigma_B = \sigma'_B \qquad \text{(I)}$$

where n_g is the factor of safety for dead load, and
n_p is the factor of safety for live load, while
n_B, which is less than unity, is the factor of safety of the material as such.

If, now, the coefficients n_p and n_g are so chosen that the ratio n_p/n_g is sufficiently great — for instance, 1.5 — then safety against overturning (the problem of stability) is automatically assured and no additional requirements need be stipulated. σ_B is the breaking stress or yield point as determined by experiment, as, for instance, the compressive strength of concrete at 28 days. The lower value $\sigma'_B = n_B \cdot \sigma_B$ is *defined* as the nominal breaking stress; it may, therefore be used as a definite basis for subsequent calculations.

Similar proposals have already been put forward by *Gerber* and others, but have never been fully worked out.

The nominal breaking load is definitely given by $n_p \cdot p + n_g \cdot g$ etc. and the nominal conditions for the breakdown of a structure are determined from the nominal breaking stresses and nominal breaking loads. If *Hooke*'s Law is to be abandoned as a basis for design — as has already been done in many respects for reinforced concrete — it must be replaced by other working principles, and since it is known that the properties of materials as determined experimentally cannot be directly applied to materials as used in actual structures it is better to distinguish certain safe "nominal" properties which can be attributed to the materials, so as to serve as a consistent and logical basis for design, rather than to rely on an arbitrary factor of safety.

When a number of external loads are present, such as, for instance, a vertical live load, a wind load, and additional loads due to shrinkage, temperature, settlement of the supports, etc., probable combinations can be allowed for in the following way:

$$\sigma_{(n'_g \cdot g)} + \sigma_{(n'_p \cdot p)} + \sigma_{(n_w \cdot w)} + \sigma_{(n_x \cdot x)} = n_B\, \sigma_B \quad \text{(II)}$$

wherein n'_g and n'_p are given lower values than n_g and n_p in Equation (I).

This principle can, of course, be carried further, but for practical purposes it is sufficient to lay down conditions (I) and (II). Additional stresses resulting from statical indeterminacy are less dangerous, from the point of view of breakage, than stresses due to loads, and generally speaking these are smaller than as calculated by *Hooke*'s Law because in the constructional materials adopted the line of stress is bent towards the axis of deformation, and moreover the additional stresses become smaller when the deformation is permanent. Thus n_x may be given a lower value than n_p and n'_p.

Where one particular moving load predominates over the others, as for instance, where the horizontal live load is much greater than the wind and braking loads, it is sufficient to satisfy one condition of form (II), and this is in fact the general case.

The present practice of requiring two separate conditions to be satisfied — one with and one without the additional loads — is inconsistent. In the case of statically indeterminate structures the usual requirement that $\sigma_g + \sigma_p \leqq \sigma_{zul}$ is apt to be applied in conjunction with certain assumed additional loads, instead of $\sigma_g + \sigma_p + \sigma_{addnl} \leqq \sigma'_z$ (wherein σ'_z denotes an increased permissible stress) being taken as the criterion which governs the dimensions, and as a result the degree of safety possessed by a statically indeterminate structure is often made to appear smaller than that of a structure which is determinate.

It is preferable, as here proposed, to adopt a lower factor of safety in respect of the additional loads than in respect of principal loads, seeing that the former cannot by themselves cause breakage, and the probability of maximum additional loads occurring simultaneously with maximum live load is smaller than that of the occurrence of maximum live loads by themselves.

Two different groups of factors of safety may be used according as the calculations are required to be more or less accurate: for instance $n_{g,1}$ — $n_{p,1}$ — $n_{x,1}$ and $n_{B,1}$ will give a higher degree of accuracy than $n_{g,2}$ — $n_{p,2}$ and $n_{B,2}$.

Considerations of this kind can be practically applied in structural designing. There is good justification for equating certain stresses, such as the erection stresses in the completed structure, to the "own weight" stresses, and in many cases if this is done the calculations are still further simplified — as for instance in the case of *Melan* structures where it is required to take account of the preimposed stresses in the rigid reinforcement. The general effect of the conditions of safety here proposed is to make it possible to take special account of special stresses without complicating the calculations. This fact is very important, for with the old method of calculation there was no way of making allowance for differences in liability to increase as between the different kinds of stress.

A further peculiarity of the *Melan* system of construction will now be mentioned. If, for instance, the pre-existing stress in the rigid steel reinforcement amounts to two-thirds of the permissible stress, then, according to the usual method of calculation, the total cross section may only be stressed up to $\frac{\sigma_{j, zul}}{3 \cdot 15} \cdot (F_b + 15\, F_j)$ — but this limitation is unjustified, for it would imply that if the pre-stress has been equal to $\sigma_{j, zul}$ the total cross section (concrete + rigid reinforcement + round reinforcement) is unable to carry any further load at all.

Using the old method of calculation, very arbitrary distortions have to be introduced in order to avoid an increase in the pre-imposed stress, and still more exception made from the ordinary rules of design. By the proposed method, however, the calculations are simplified as follows:

$$n_g \cdot \sigma_{j, \text{pre-stress}} + n_g \cdot \sigma_{j, g, \text{completed}} + n_p \cdot \sigma_{j, p, \text{completed}} \leqq n_B \cdot \sigma_B$$

(and similarly as regards the concrete stress): that is to say the calculations may be based upon the separate loadings which will actually arise, and finally all the stresses may be added together. Care need only be taken not to fix the ratio n_p/n_g too small.

It may happen that the dead load is imposed in the form of a live load, either as regards the proportion it bears to the assumed values, or because it is actually movable. There might, therefore, be a temptation to assume part of the fixed load as being movable, but such a procedure is unpractical because it introduces an unnecessary complication into the calculations by implying that there are two movable loads, differently constituted, instead of one, and because there is a limit to the movability of the "fixed" load. Moreover it is difficult to look upon large cross girders as movable. On the other hand it is easy to visualise a slab of varying thickness, so that the dead load will not be uniformly distributed over its area as assumed.

It is better to allow this freedom of movement of the "fixed load" to be covered by the factor of safety applied to the moving load, but where the fixed loads are very large in proportion to the moving loads such an assumption becomes insufficient. To meet this exceptional case it is both logical and practical to require that the total movable load must be taken as not less than a certain fraction of the total stationary load (for instance 10 %) in each structural member. This question, however, will only arise as regards the principal members of large structures subject to small live loads.

6) *The principal advantages of using the new proposals.*

a) The scope of the new proposals is more general than that of the usual methods of calculation.

b) The two main groups of defects which should be covered by the factor of safety — namely defects in material and defects in load — are each covered by their separate coefficients.

c) Safety as regards stability is automatically assured without the need for stating special requirements.

d) The existence of a large dead weight, which in general is to be looked upon as an advantage (being, for instance a protection against explosion risks, dynamic effects, noise, etc.) will not needlessly be penalised.

e) Where a structure is subsequently found, by accurate investigation, to have been particularly well built, it may without risk be more heavily loaded.

f) Test loadings, involving the imposition of increased live loads at the most dangerous places, may be carried out without undue risk.

g) It ought to be possible to identify the true factor of safety of any given structure with the ratio between the absolute maximum live load that can be brought to bear thereon at the moment of breakage and the live load that has been assumed in the calculations. This definition is not of course an entirely satisfactory one, but the nominal factor of safety against breakdown ought not to be too different from the true value in this sense. Such consistency can be obtained by the method of calculation now proposed, but not by that ordinarily followed.

h) The wide significance here attributed to the factor of safety can, in this way, at least be given a logical basis, and need not merely be regarded as a vague and unpractical symbol, as it must be when the ordinary method, based on permissible stresses, is used.

i) The nominal breaking stresses, the nominal breaking load and therefore the nominal breaking conditions can all be worked out.

k) The deviations from *Hooke's* Law, etc., which are admissible in approximate calculations, can be made subject to definite and consistent rules.

l) Safety against cracking, against repeated loading etc., may be attained by the same means, and more convincingly than by the usual methods.

m) The basis of calculation is rendered more consistent, and the statical calculations themselves are made simpler and more reliable, especially as regards structures in which questions of stability, pre-stressing, etc. are involved. The values finally adopted for the safety coefficients must be consistent with the rules governing both design and erection.

IIa 9

Tests on the Slow Buckling of Concrete Sticks.

Versuche über das langsame Knicken an Betonkörpern.

Essais de flambement lent de baguettes en béton.

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When a prism-shaped body is loaded at the end its condition is one of unstable equilibrium if the load equals or exceeds the limit given by *Euler's* expression $\frac{\pi^2 E I}{l^2}$.

The mechanism of buckling can in fact be visualised as follows: any slight eccentricity of the load gives rise to a bending moment which causes an initial deformation, this deformation has the effect of increasing that moment so as to cause a second deformation, and so on. If the deformations thus obtained constitute a divergent series the piece buckles. This is the meaning of *Euler's* equation, which further implies that the limit of stability is independent of the amount of initial eccentricity.

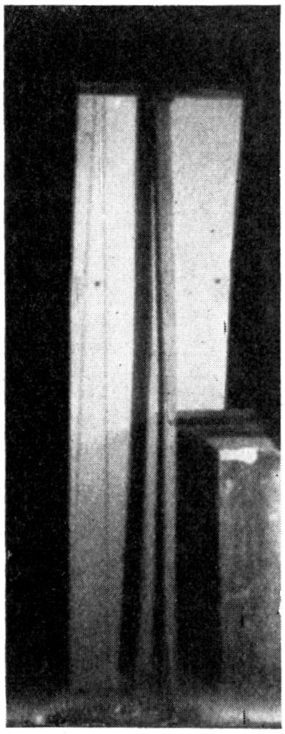

Fig. 1.
Photograph showing the deflection assumed by a test bar 135 · 3 · 3 cm the day before its collapse.

According to the customary rules governing the strength of materials these deformations occur immediately the load is applied, and as a corollary to this the series of phenomena described below take place almost instantaneously, and fracture occurs suddenly without any warning.

Concrete, however, behaves in a different way: the initial deformation is almost instantaneous, but its subsequent growth is slow. There exists, therefore, an *a priori* possibility that under certain values of load the piece may assume a condition of stable equilibrium under the action of the first deformation, and that it is the subsequent of slow deformations which constitutes the divergent series characteristic of buckling. In other words, the reasoning on which *Euler's* formula is based is independent of time, and what takes place is exactly as if the modulus of elasticity E were to decrease as the stress and its duration increased. In order, then, to arrive at the true criterion of buckling, it becomes necessary to introduce the final value of E into *Euler's* formula.

It was thought that the matter might be illustrated in a particularly striking way if it could be reproduced in the laboratory, and attempts were accordingly made to bring about slow buckling in concrete pieces. These pieces took the form of sticks measuring 135 cm by 3 cm by 3 cm, made from fine gravel concrete using 350 kg per cu. m of "superciment" or of aluminous cement (Photograph N° 1). The sticks were loaded by means of a lever-operated press, and the results are given in the following table:

No. of Specimen	Kind of Cement	Age of Concrete when tested (days)	Strength of concrete as measured on 20 cm cubes (kg per sq. cm)	Load Applied kg	kg per sq. cm	Results of Tests	Deflection	Modulus of elasticity as calculated from buckling
1	Artificial	130	260	780	86	Instantaneous buckling		210.000
2	do.	130		580	64	A deformation begins to occur, but after 6 days shows no perceptible further increase; the load is then increased:		
				650	72	Buckling at 14 days	3 mm	175.000
3	Superciment	19		1120	124	Instantaneous buckling		300.000
4	do.	19		720	80	Buckling after 15 minutes		195.000
5	Aluminous	3	430 at 3 days	1520	170	Instantaneous buckling		410.000
6	do.	8		1070	118	Buckling after 5 days	4 mm	290.000
7	Aluminous	4	360 at 4 days	1140	126	Instantaneous buckling		310.000
8	do.	4		960	106	do.		260.000
9	do.	4		900	100	do.		240.000
10	do.	4		780	86	Buckling after 5 minutes		210.000
11	do.	5		650	72	Buckling after 7 days	3 mm	175.000

It was found that the piece sometimes broke at once and sometimes resisted the load indefinitely, but between these two extremes it was found possible, after a few attemps, to bring about the desired phenomenon in sticks Nos. 2 (second test) 4, 6, 10 and 11.

These experiments amount to no more than a first approach to the study of a problem which deserves closer attention, and however incomplete the results given above it seemed worth while to publish them as evidence of the existence of this phenomenon of slow buckling and as forming a broad outline of its nature.

It is impossible to lay too much emphasis on the danger that may arise, in practice, from this cause, and on the necessity for adopting a very low value of the modulus of elasticity in applying *Euler's* formula. At the same time, it should be observed that the deformation of a member placed in this condition of unstable equilibrium becomes apparent a short time after the load is applied and then increases progressively to a high value. Since, therefore, failure through slow buckling is preceded by these visible phenomena, it is less dangerous than failure by instantaneous buckling, though the actual occurrence of fracture is in fact equally sudden in both cases.