The factor of safety of reinforced concrete structures

Autor(en): **Moe, A.J.**

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The Factor of Safety of Reinforced Concrete Structures.

Über die Sicherheiten der Eisenbetonbauten.

La sécurité des ouvrages de béton armé.

A. J. Moe,
Beratender Ingenieur, Kopenhagen.

1) *Present definition of factor of safety.*

According to present usage the factor of safety of static structures is defined by reference to the permissible stresses, being, as a rule, the proportion between the breaking stress or yield point of the material and the permissible stress.

This definition, however, is not an adequate one, and in the course of time it has gradually been found necessary to supplement it by statements of special requirements. For instance in the case of retaining walls it is necessary to insure not only against excessive pressure on the foundations but also against the risk of overturning; much the same thing applies to cantilever slabs; and brick chimneys are made subject to the special requirement that the theoretical tensile stresses must not occur beyond the centre of gravity of the cross section. In all these instances the special requirements are those which have reference to stability.

An even more notable fact is that the concept of permissible strength has no meaning in application to columns. It is true that the permissible stress is now prescribed as a function of the buckling length, but this practice amounts to no more than a restatement (or, as it were, a tabular solution) of the column formulae, and the crux of the matter is that columns are in fact dimensioned to carry a load which has already been multiplied by a factor of safety, there being no regular relationship between load and stress. Thus in the design of columns the whole idea of a breaking load is abandoned, and this is contrary to the practice followed in the design of tie bars wherein the cause of failure may always be taken as some defect in the material independent of any increase in load.

To sum up, it is impossible to give any concise definition of what is meant by the factor of safety in static structures as the term is used at present. It may be noted, also, that at the present time safety against dynamic stresses is partly covered by the introduction of an impact coefficient, and this again implies a different idea than the original one of depending on permissible stresses.

2) *Disadvantages of the present factor of safety as applied to structures.*

A general disadvantage of using the factor of safety in its present form is that it does not admit of concise definition. A further disadvantage is that the principal criterion of safety, namely the permissible stress, should be one which in many cases (such as problems relating to stability, column design and dynamic stresses) is of little or no importance. It is again a disadvantage that this principal criterion should need to be supplemented by a variety of extra requirements which have no relation to one another, the factor of safety being made to refer now to the loading, now to the conditions of fracture of the material, and now to the yield point of the latter. As materials increase in strength, problems of stability will tend to become ever more important, and this may entail an even greater variety of special conditions than at present. It must be counted a defect that the main criterion of safety should not be one which in itself guarantees stability from every aspect, and that the form of guarantee should not allow of different weight being attached to different kinds of stress and loading: thus certain stresses due to the dead weight of the structure and to the process of erection might reasonably be treated in a different way from the stresses that will arise when the completed structure is in service. It is a disadvantage, further, that the "own weight" of the structure should have to be multiplied by the same maximum value of the factor of safety whether its action is favourable or unfavourable to stability.

The most frequent occasion for special conditions, tending in this way to take the place of the permissible stress as the criterion of safety, is found in the absence of proportionality between load and stress. In columns this lack of proportionality is due to buckling, but in most other cases it is due to the fact that the dead load and the live load produce stresses which have no common measure: that is to say the dead load stresses and the live load stresses cannot directly be added together.

3) *Special disadvantages attending the application of the present form of the factor of safety to reinforced concrete structures.*

The disadvantages noted above are of general application to most forms of construction and to all kinds of material, but reinforced concrete possesses certain characteristics which render the present criteria of safety especially unsuitable.

In the first place, reinforced concrete is a heterogeneous material: as a rule the steel reinforcement is arranged, with the greatest possible nicety, to carry the tensile stresses, and the result is that when such stresses arise at unintended places the material is particularly ill suited to resist them. That is to say, in the case of reinforced concrete the lack of proportionality between load and stress is particularly marked, and reinforced concrete is much more sensitive than homogeneous materials to changes in the proportion between stationary and moving loads. Changes in the proportion between dead and live loads are particularly dangerous in the case of reinforced concrete arches, and from this point of view, indeed, the arched form of structure is at some disadvantage compared with the beam, whatever the material used.

As an example, the following are the stresses that arise in a two-hinged roof arch of 24 m span, 4 m rise and 15 cm thickness, reinforced on each side with five rods of 10 mm diameter per metre width, subject to a dead load of 400 kg/m^2 and a live load of a 100 kg/m^2:

Steel stress $\sigma_j \sim 943$ kg/cm^2.

Concrete stress $\sigma_b \sim 44.8$ kg/cm^2.

If the live load be increased by 50% to 150 kg/cm^2 the stresses become

$$\sigma_j \sim 1770 \text{ kg/cm}^2$$
$$\sigma_b \sim 65.9 \text{ kg/cm}^2$$

In other words, σ_j is increased by 87.5% and σ_b by 47.2%.

On the other hand, in a simply supported reinforced concrete slab designed for the same dead and live loads, the increase both of σ_j and σ_b when the live load is increased by 50% is only 10%.

These figures speak for themselves. Structures which have been designed with special reference to the characteristics of a stationary load are particularly sensitive to changes in the relationship between stationary and moving loads, and generally speaking reinforced concrete structures are less favourably conditioned in this respect than either steel or timber structures — partly because reinforced concrete is in itself heavier, and partly on account of its heterogeneity.

A further reason for abandoning the present criteria of safety as applied to reinforced concrete constructions is to be found in the greater importance attaching to the conditions of breakdown of this material. In concrete and reinforced concrete *Hooke*'s Law is not valid, and for economic reasons the principles followed in dimensioning the cross section are those derived from breaking tests. Also in calculating shear forces (such as those due to moments, transverse loads, etc.) the tendency is always in the direction of laying emphasis on the conditions of fracture. This makes if all the more important that breakdown should be logically defined, which cannot be done by reference to the usual permissible stresses.

Yet a third reason for abandoning the permissible stress as a criterion of safety lies in the great dead weight of concrete structures. It may be observed that structures in which the dead weight is large may more safely be overloaded than structures in which it is relatively small. That is to say, a stationary load which is incapable of increasing above its assumed amount, which cannot vary, and which can exercise no dynamic effect, may be regarded more favourably than a moving load and from the point of view assessing the degree of safety possessed by the structure the former should be regarded in a different category from the latter. Indeed, so far as dynamic effects are concerned, this difference is already recognised by the introduction of impact coefficients, but otherwise the customary method of design by reference to permissible stresses is too severe in its treatment of stationary loads. This statement applies generally to all constructional materials, but the disadvantage is greatest in the case of mass structures and from this point of view reinforced concrete is prejudiced by comparison with steel and timber.

The customary method of calculation is illogical in yet another sense. In most countries, if any noticeable defects appear during the course of construction the work in question is not immediately pulled down, but a test is made under load, and if the defects seem to be serious the test loads are increased so as to produce an overload of perhaps 50% at the most dangerous places; if the structure succesfully withstands these test loads it is regarded as acceptable for use. Dependence is, therefore, placed on a construction because it has been found amply safe to resist live load: but no regard is taken of its untested degree of safety to resist dead load. It should, however, here be observed that a structure which has been designed to carry a particular ratio of live to dead load may be dangerous when subjected to a form of loading in which this proportion is noticeably increased.

The heavy dead weight possessed by a reinforced concrete structure is a valuable characteristic, and one which ought not to be needlessly penalised.

4) *What should the factor of safety cover?*

The following points will be briefly mentioned:

a) Errors and inaccuracies in the assumed basis of design.
b) Defects of material.
c) Inaccuracies of execution.
d) Inaccuracies of the imposed loading.

In other words, the following items should all be covered: secondary stresses, internal stresses, certain fluctuating stresses, imposed stresses, erection stresses, inaccuracies of calculation, faulty material, inaccuracies in sections (such as steel bars) as delivered from the workshops, inaccuracies in erection and workmanship, inaccuracies in the "own weight", divergences of the live load from that assumed in the design, exceptional overloads such as test loads, and other contingencies.

It is not possible, however, to fix a factor of safety of ordinary magnitude which will cover all these contingent errors and inaccuracies individually: the most that can be done is to take account of their *probable* combinations.

It is true that the latter may equally well be expected to consist of a few high values as of a larger number of small or medium values, but it may be shown that several of the categories of defects named above can only be covered — or can most economically be covered — by the assumption of an increase in the live load. Generally speaking, it may be said that a stationary load can always be assumed to be replaced by a moving load, but a moving load cannot be taken as replaced by a stationary load.

Certain defects in material form an exception to this statement, in that the best way to allow for them is to assume a reduced value for the breaking stress or yield point. Here it is necessary to be clear what purpose is actually served by the use of a factor of safety. In the author's opinion what matters most is safety against breakage, whereas safety against cracking — important as it may be — is secondary.

5) *Proposed new form of the factor of safety for practical use.*

The factor of safety in its present form is expressible as follows:

$$(1)\quad \sigma_p + \sigma_g + \sigma_w + \sigma_t \leqq \sigma_{zul} = \frac{1}{n}\sigma_B$$

In the case of colums:

$$(2)\quad P_{zul} \leqq \frac{1}{n} P_{\text{breakage}}.$$

In application to problems of stability:

(3) $M_{\text{favourable}} \geqq n' \cdot M_{\text{unfavourable}}$

where zul. *(zulässig)* denotes "permissible"

p refers to live load

g „ „ dead load

w „ „ wind load

t „ „ temperature stresses, etc.

σ_B is the nominal breaking stress or yield point.

n and n′ are factors of safety.

The first and most general rule can be re-written

(4) $n \cdot \sigma_p + n \cdot \sigma_g + n \cdot \sigma_w + n \cdot \sigma_t = \sigma_B$

or (5) $\sigma_{(n \cdot p)} + \sigma_{(n \cdot g)} + \sigma_{(n \cdot w)} + \sigma_{(n \cdot t)} = \sigma_B$

referring to the stresses caused by the loads multiplied by n. Equation (5) gives the nominal breaking condition which agrees with Equation (2) but is contradictory to Equation (3), seeing that n′ is usually smaller than n. In other words, the definition of breakdown is not consistent; moreover it is impossible really to imagine the "own weight" as being multiplied by n, which, in the case of columns, is an abstraction that has to be made.

In the proposal now put forward, the three conditions numbered (1), (2) and (3) above are combined as follows:

$$\sigma_{(n_g \cdot g)} + \sigma_{(n_p \cdot p)} \leqq n_B \cdot \sigma_B = \sigma'_B \qquad \text{(I)}$$

where n_g is the factor of safety for dead load, and

n_p is the factor of safety for live load, while

n_B, which is less than unity, is the factor of safety of the material as such.

If, now, the coefficients n_p and n_g are so chosen that the ratio n_p/n_g is sufficiently great — for instance, 1.5 — then safety against overturning (the problem of stability) is automatically assured and no additional requirements need be stipulated. σ_B is the breaking stress or yield point as determined by experiment, as, for instance, the compressive strength of concrete at 28 days. The lower value $\sigma'_B = n_B \cdot \sigma_B$ is *defined* as the nominal breaking stress; it may, therefore be used as a definite basis for subsequent calculations.

Similar proposals have already been put forward by *Gerber* and others, but have never been fully worked out.

The nominal breaking load is definitely given by $n_p \cdot p + n_g \cdot g$ etc. and the nominal conditions for the breakdown of a structure are determined from the nominal breaking stresses and nominal breaking loads. If *Hooke*'s Law is to be abandoned as a basis for design — as has already been done in many respects for reinforced concrete — it must be replaced by other working principles, and since it is known that the properties of materials as determined experimentally cannot be directly applied to materials as used in actual structures it is better to distinguish certain safe "nominal" properties which can be attributed to the materials, so as to serve as a consistent and logical basis for design, rather than to rely on an arbitrary factor of safety.

When a number of external loads are present, such as, for instance, a vertical live load, a wind load, and additional loads due to shrinkage, temperature, settlement of the supports, etc., probable combinations can be allowed for in the following way:

$$\sigma_{(n'_g \cdot g)} + \sigma_{(n'_p \cdot p)} + \sigma_{(n_w \cdot w)} + \sigma_{(n_x \cdot x)} = n_B \, \sigma_B \qquad \text{(II)}$$

wherein n'_g and n'_p are given lower values than n_g and n_p in Equation (I).

This principle can, of course, be carried further, but for practical purposes it is sufficient to lay down conditions (I) and (II). Additional stresses resulting from statical indeterminacy are less dangerous, from the point of view of breakage, than stresses due to loads, and generally speaking these are smaller than as calculated by *Hooke*'s Law because in the constructional materials adopted the line of stress is bent towards the axis of deformation, and moreover the additional stresses become smaller when the deformation is permanent. Thus n_x may be given a lower value than n_p and n'_p.

Where one particular moving load predominates over the others, as for instance, where the horizontal live load is much greater than the wind and braking loads, it is sufficient to satisfy one condition of form (II), and this is in fact the general case.

The present practice of requiring two separate conditions to be satisfied — one with and one without the additional loads — is inconsistent. In the case of statically indeterminate structures the usual requirement that $\sigma_g + \sigma_p \leqq \sigma_{zul}$ is apt to be applied in conjunction with certain assumed additional loads, instead of $\sigma_g + \sigma_p + \sigma_{addnl} \leqq \sigma'_z$ (wherein σ'_z denotes an increased permissible stress) being taken as the criterion which governs the dimensions, and as a result the degree of safety possessed by a statically indeterminate structure is often made to appear smaller than that of a structure which is determinate.

It is preferable, as here proposed, to adopt a lower factor of safety in respect of the additional loads than in respect of principal loads, seeing that the former cannot by themselves cause breakage, and the probability of maximum additional loads occurring simultaneously with maximum live load is smaller than that of the occurrence of maximum live loads by themselves.

Two different groups of factors of safety may be used according as the calculations are required to be more or less accurate: for instance $n_{g,1}$ — $n_{p,1}$ — $n_{x,1}$ and $n_{B,1}$ will give a higher degree of accuracy than $n_{g,2}$ — $n_{p,2}$ and $n_{B,2}$.

Considerations of this kind can be practically applied in structural designing. There is good justification for equating certain stresses, such as the erection stresses in the completed structure, to the "own weight" stresses, and in many cases if this is done the calculations are still further simplified — as for instance in the case of *Melan* structures where it is required to take account of the pre-imposed stresses in the rigid reinforcement. The general effect of the conditions of safety here proposed is to make it possible to take special account of special stresses without complicating the calculations. This fact is very important, for with the old method of calculation there was no way of making allowance for differences in liability to increase as between the different kinds of stress.

A further peculiarity of the *Melan* system of construction will now be mentioned. If, for instance, the pre-existing stress in the rigid steel reinforcement amounts to two-thirds of the permissible stress, then, according to the usual method of calculation, the total cross section may only be stressed up to $\frac{\sigma_{j,\,zul}}{3 \cdot 15} \cdot (F_b + 15\,F_j)$ — but this limitation is unjustified, for it would imply that if the pre-stress has been equal to $\sigma_{j,\,zul}$ the total cross section (concrete + rigid reinforcement + round reinforcement) is unable to carry any further load at all.

Using the old method of calculation, very arbitrary distortions have to be introduced in order to avoid an increase in the pre-imposed stress, and still more exception made from the ordinary rules of design. By the proposed method, however, the calculations are simplified as follows:

$$n_g \cdot \sigma_{j,\,\text{pre-stress}} + n_g \cdot \sigma_{j,\,g,\,\text{completed}} + n_p \cdot \sigma_{j,\,p,\,\text{completed}} \leqq n_B \cdot \sigma_B$$

(and similarly as regards the concrete stress): that is to say the calculations may be based upon the separate loadings which will actually arise, and finally all the stresses may be added together. Care need only be taken not to fix the ratio n_p/n_g too small.

It may happen that the dead load is imposed in the form of a live load, either as regards the proportion it bears to the assumed values, or because it is actually movable. There might, therefore, be a temptation to assume part of the fixed load as being movable, but such a procedure is unpractical because it introduces an unnecessary complication into the calculations by implying that there are two movable loads, differently constituted, instead of one, and because there is a limit to the movability of the "fixed" load. Moreover it is difficult to look upon large cross girders as movable. On the other hand it is easy to visualise a slab of varying thickness, so that the dead load will not be uniformly distributed over its area as assumed.

It is better to allow this freedom of movement of the "fixed load" to be covered by the factor of safety applied to the moving load, but where the fixed loads are very large in proportion to the moving loads such an assumption becomes insufficient. To meet this exceptional case it is both logical and practical to require that the total movable load must be taken as not less than a certain fraction of the total stationary load (for instance 10 %) in each structural member. This question, however, will only arise as regards the principal members of large structures subject to small live loads.

6) *The principal advantages of using the new proposals.*

a) The scope of the new proposals is more general than that of the usual methods of calculation.

b) The two main groups of defects which should be covered by the factor of safety — namely defects in material and defects in load — are each covered by their separate coefficients.

c) Safety as regards stability is automatically assured without the need for stating special requirements.

d) The existence of a large dead weight, which in general is to be looked upon as an advantage (being, for instance a protection against explosion risks, dynamic effects, noise, etc.) will not needlessly be penalised.

e) Where a structure is subsequently found, by accurate investigation, to have been particularly well built, it may without risk be more heavily loaded.

f) Test loadings, involving the imposition of increased live loads at the most dangerous places, may be carried out without undue risk.

g) It ought to be possible to identify the true factor of safety of any given structure with the ratio between the absolute maximum live load that can be brought to bear thereon at the moment of breakage and the live load that has been assumed in the calculations. This definition is not of course an entirely satisfactory one, but the nominal factor of safety against breakdown ought not to be too different from the true value in this sense. Such consistency can be obtained by the method of calculation now proposed, but not by that ordinarily followed.

h) The wide significance here attributed to the factor of safety can, in this way, at least be given a logical basis, and need not merely be regarded as a vague and unpractical symbol, as it must be when the ordinary method, based on permissible stresses, is used.

i) The nominal breaking stresses, the nominal breaking load and therefore the nominal breaking conditions can all be worked out.

k) The deviations from *Hooke's* Law, etc., which are admissible in approximate calculations, can be made subject to definite and consistent rules.

l) Safety against cracking, against repeated loading etc., may be attained by the same means, and more convincingly than by the usual methods.

m) The basis of calculation is rendered more consistent, and the statical calculations themselves are made simpler and more reliable, especially as regards structures in which questions of stability, pre-stressing, etc. are involved. The values finally adopted for the safety coefficients must be consistent with the rules governing both design and erection.