Long span bridges

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Long Span Bridges.

Weitgespannte massive Brücken.

Ponts de grande portée.

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Doubtless the most satisfactory way of determining the maximum span of bridge capable of being built in a particular material would be actually to build it. This, of course, is not always possible. An alternative is to prepare a design, if possible a design for a project ready to be carried out. Either of these methods is subject to the disadvantage that it holds good only in reference to a particular conjunction of permissible stress, load, ratio of rise to span, length of span, etc., and does not allow conclusions to be directly drawn in reference to other sets of conditions.

For this reason it appeared desirable to develop formulae that would be generally applicable.

1. Reinforced concrete arch bridges.

The author has arrived at definite formulae of this kind for concrete *arch* bridges. In order to do so it was necessary, of course, to simplify by idealising certain of the usual mathematical treatments, and this must be borne in mind when estimating the reliability of the result.

The following assumptions were made, using the notation indicated in Fig. 1.

a) An arch, based on the "pressure line" due to the normal loading condition: i. e. dead load plus half the live load uniformly distributed over the span.

b) Distribution of the total load q over the length of the bridge in accordance with the relationship

$$q = q_S \left[1 + (m - 1)\, y/f\right] \tag{1}$$

$$m = \frac{q_K}{q_S} \tag{2}$$

c) Variation of section of the arch according to

$$F = \frac{F_S}{\cos \varphi} \tag{3}$$

d) Stress at centre of gravity, under normal load condition a), given by

$$\sigma_m = \mu \sigma_{perm} \qquad 0 < \mu < 1 \tag{4}$$

Here μ serves to indicate to what extent under the assumed condition of loading, the permissible stress σ_{perm} is utilised. μ may therefore be designated the coefficient of utilisation. From Fig. 1 we obtain:

$$H = \frac{(Q+G) \cdot \beta l}{f} \tag{5}$$

and from (4) it follows that

$$H = \mu \cdot \sigma_{perm} \cdot F_S \tag{6}$$

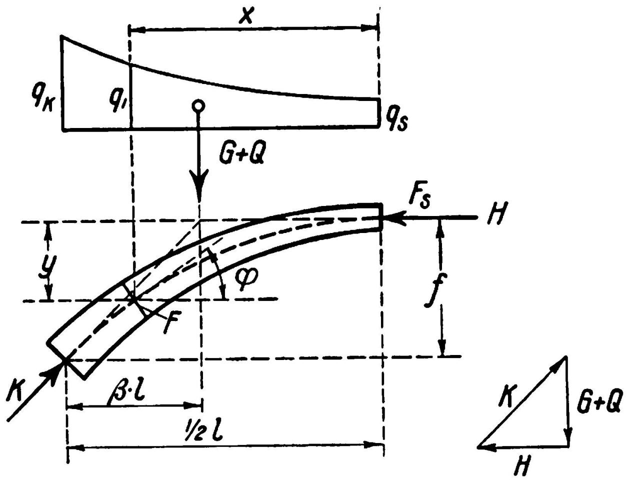

Fig. 1.

From (1) we obtain the distance $\beta \cdot l$ from the springing for the load $G + Q$ (where G represents the weight of the arch itself and Q that of the superstructure and roadway with one half of the live load):

$$\beta = \frac{m-1}{2c\sqrt{m^2-1}} \tag{7}$$

$$c = \operatorname{Ar\,cosh} m = \operatorname{Ar\,cosh} \sqrt{\left(\frac{q_K}{q_S}\right)} \tag{8}$$

Equations (1) and (3) enable the cross section of the arch to be calculated and hence the weight G of one half of the arch.

$$G = \frac{1}{2} \gamma F_S \cdot l \left[1 + \alpha (f/l)^2\right] \tag{9}$$

$$\alpha = \frac{c}{(m-1)^2} (\operatorname{Sin} 2c - 2c) \tag{10}$$

Combining (5), (6) and (9) we obtain

$$\varkappa = \frac{G}{Q} = 1 : \left[\frac{2\mu\sigma \cdot f/l}{\gamma \cdot l \cdot \beta \left[1 + \alpha (f/l)^2\right]} - 1\right]$$

$$\varkappa = 1 : \left[\frac{\mu\sigma}{\gamma \cdot l} \cdot \frac{2n}{\beta (1 + \alpha n^2)} - 1\right] = 1 : \left[\frac{\mu\sigma}{\gamma \cdot l} \cdot \delta - 1\right] \tag{11}$$

wherein, for abbreviation, there is written

$\sigma = \sigma_{perm}$, $n = f/l$, $\gamma = \gamma_B$ = specific weight of the arch by volume. (12)

In view of (7) and (10) the new coefficient δ becomes:

$$\delta = \frac{2n}{\beta (1 + \alpha n^2)} = \frac{4nc \cdot \sqrt{m^2-1} \cdot (m-1)}{(m-1)^2 - c n^2 (\operatorname{Sin} 2c - 2c)} \tag{13}$$

The figure $\varkappa$ obtained from equations 11 and 13 gives the proportion between the weight of the arch itself and that of the super-structure including the roadway and half the live load; in other words, the *relative amount of material* required for the arch. $\varkappa$ provides a convenient means of assessing the result of various influences on the amount of material required and on the possible span. In particular, the condition:

$$2\mu\sigma n = \beta \gamma l (1 + \alpha n^2) \tag{14}$$

applied to the case when $\varkappa = \infty$, determines the theoretical limit of feasibility for the arch. It will be seen that this is governed only by the geometrical conditions and by the mechanical properties of the material, not by the intensity of the loading.

In the case of arch bridges of "open superstructure", wherein suspension bars, spandrel walls, or columns transfer the weight of the roadway onto the arch, the approximate assumption may be made that the loading of the arch under normal conditions of loading is distributed in approximately the same way as its own weight:

$$m = \frac{q_K}{q_S} = \frac{g_K}{g_S} = \frac{1}{\cos^2 \varphi_K} = 1 + \left(\frac{dy}{dx}\right)_K^2 \tag{15}$$

Hence m is determined as a function of the ratio of rise to span n = f/l in accordance with the formula:

$$\frac{1}{2c}\sqrt{\frac{(m-1)^3}{m^2-1}} = \frac{1}{2 \operatorname{Ar\cosh} m}\sqrt{\frac{(m-1)^3}{m^2-1}} = n \tag{16}$$

Here the coefficients α and β determined from equations (7) and (10) as being dependent on m, and also δ according to equation (13), have been reduced to functions of the ratio of rise n alone. In the following table the coefficients m, α, β, δ are given for various ratios of rise to span:

f/l = n	m	α	β	δ
1 : ∞ = 0	1.00	5.33	0.25	0
1 : 10 = 0.1	1.18	5.47	0.242	0.784
1 : 7 = 0.143	1.38	5.60	0.234	1.068
1 : 5 = 0.200	1.83	5.70	0.223	1.460
1 : 3.5 = 0.286	3.00	5.87	0.201	1.920
1 : 2.5 = 0.400	7.50	6.86	0.165	2.310
1 : 1.78 = 0.562	20.00	8.13	0.129	2.450
1 : 1 = 1.000	100.00	10.67	0.095	1.805

The only item now outstanding in order to make use of the fundamental formula (11) is the coefficient of utilisation μ (4). The value of this increases in proportion as the dead load becomes the predominant factor, and it also increases with the permissible stress, so that as a general rule it is higher in the case of long span bridges than in that of smaller spans. To a large extent, however, it can be varied in accordance with the design and method of construction of the bridge. Indeed, it may be stated that the improvement of the coefficient of utilisation constitutes the chief problem before the designer of a long span arch bridge, and it follows from this that no universally valid statement can be made as to the magnitude of μ. It is the business of the designing engineer to regulate this figure according to the conditions of each particular case.

For the purpose of the preliminary calculations, the author has made use of the following relationship based upon considerations which are here omitted for lack of space:

$$\mu = a \cdot \sqrt[4]{\sigma_{perm}} \qquad (\sigma_{perm} \text{ in } t/m^2) \tag{17}$$

Here a is a figure amounting to between 0.1 and 0.12 for the ratio of rise to span liable to arise in practice. The highest values for a are applicable where $f/l = \frac{1}{5}$ to $\frac{1}{4}$, but the value of a decreases in either steeper or flatter arches:

Hence (11) becomes:

$$\varkappa = 1 : \left(\frac{a \cdot \sigma^{5/4} \cdot \delta}{\gamma \cdot l} - 1 \right) \tag{18}$$

With the aid of this equation all the figures and graphs given below have been calculated. These latter are correct only under the special assumptions (15) and (17) while equation (10) is of more general validity. It may be remarked that the curves do, in principle, represent actual conditions.

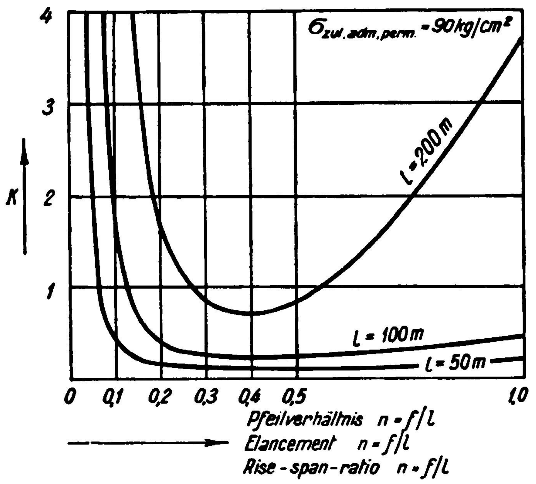

Fig. 2.

Material requirements in relation to rise-span ratio.

$$\varkappa = \frac{\text{weight of arch}}{\text{weight of superstructure} + \frac{1}{2}\text{ live load}}$$

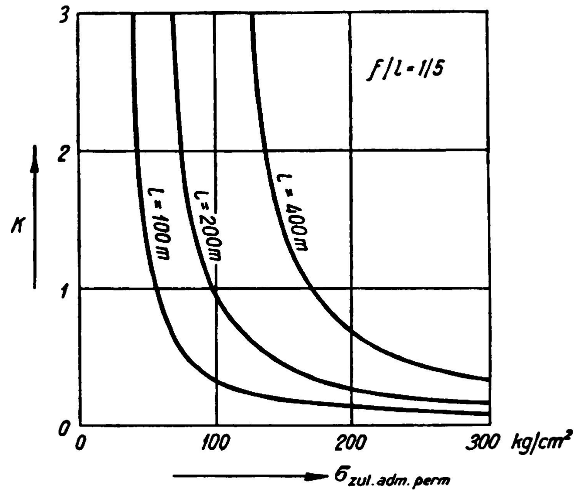

Fig. 3.

Material requirements in relation to the permissible stresses.

In Fig. 2 the requirement of material $\varkappa$ is shown relatively to the ratio of rise to span n. All the curves reach their minimum for a ratio of rise to span of approximately $^1/_3$. In practice a flatter arch is more usual, and the curves rise towards the left, but for moderate spans up to 50 m they to so only slowly, so that the excess quantity of material if a flatter arch is chosen is not great. If, however, the span becomes large the curve of $\varkappa/n$ rises very steeply, and such spans are practicable only if the ratio of rise to span is kept near the minimum indicated by this curve. Actually the optimum value is not exactly as indicated here, but a little further left, somewhere between $^1/_4$ and $^1/_6$, because, among other reasons, the super-structure over a flatter arch is cheaper and easier to build, and a non-uniform distribution of the live load produces in such cases smaller stresses.

Fig. 3 shows the relative requirement of material as a function of the permissible stress. The curves indicate that the saving in material that can be realised by increasing the permissible stress is much smaller in small spans than in large spans. For instance, if the stress is increased from 100 to 150 kg/cm² the reduction in $\varkappa$ for a span of 100 m is from 0.32 to 0.20, equal to 37 %, but with 200 m span it is from 0.93 to 0.43, equal to 54 %. The principal point to be noticed is that there is a lower limit of permissible stress for every span, below which the construction of that span becomes impracticable.

Assuming a particular ratio of rise to span, it is possible to calculate the theoretical maximum span to correspond with any given permissible stress. In practice, of course, such spans are not attained. The question of how far practice falls short of the theoretical limit is not solely a technical one, but is determined by various considerations, especially those of an economic nature. It might be approximately correct to assume that in practice about two thirds of the theoretical maximum span is attainable. Fig. 4 has been calculated on this basis, showing the practically attainable spans for reinforced concrete arch bridges as

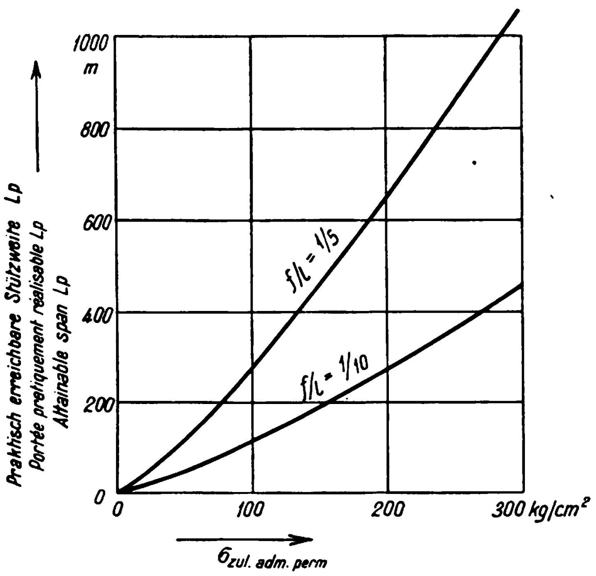

Fig. 4.

Spans attainable in practice with reinforced concrete arch bridges.

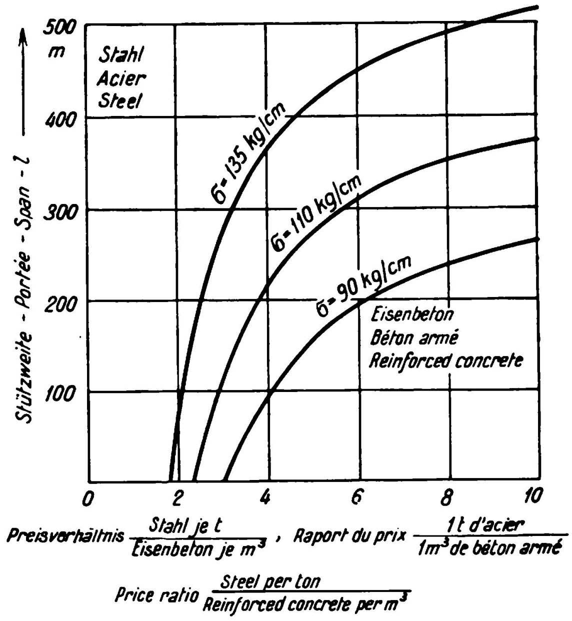

Fig. 5.

Cost comparison between steel and reinforced concrete arch bridges.

functions of the permissible stress σ in the form of two curves plotted to correspond with the ratios of rise to span $^1/_{10}$ and $^1/_5$ respectively. It will be seen that in the latter case approximately twice the span can be obtained as with the flatter arch which has a ratio of $^1/_{10}$. Special attention may here be drawn to a few important points in the $^1/_5$ curve: using a concrete with a permissible stress of 100 kg/cm² an arch of about 270 m span can be built; with 200 kg/cm² the possible span is 600 m, and for 300 kg/cm² it is about 1000 m.

To a certain extent it is possible to exceed these spans, but only at the cost of a greatly increased consumption of material in the arch and to the detriment of the competitive power of reinforced concrete. The most important of its competitors is the steel arch, and since the formulae here developed are applicable also to the latter a comparison of cost between concrete and steel arches can be made. For small spans, as a rule, the reinforced concrete is the more economical; then a certain critical span is reached at which the cost of the two materials is equal, and beyond that point the steel arch is the more economical. The critical span depends on the proportionate cost of the two materials. Fig. 5 shows the limiting spans in relation to the proportionate cost of 1 ton of steel and 1 m^3 of reinforced concrete. The permissible stresses are taken at 2100 kg/cm^2 for the steel and at 90, 110 and 135 kg/cm^2 for the reinforced concrete. The region in which the reinforced concrete is economically more advantageous lies to the right and below the curves, and the corresponding region for steel lies to the left above.

What is particularly notable in this diagram is the large increase in the reinforced concrete zone which accompanies the by no means immoderate increase assumed in its permissible stress. For instance, taking a cost ratio of 4 to 1, the limit of competability of the reinforced concrete with $\sigma_b = 90$ kg/cm^2 is about 100 m span, whereas with 110 kg/cm^2 it becomes 220 m, and with 135 kg/cm^2 as much as 360 m.

It would scarcely be possible to express in a more striking manner the paramount importance of improving the quality of concrete and thereby increasing its permissible stress. In this connection it should be noted that everything which can be done towards reducing those additional extreme-fibre stresses which are a consequence of irregular distribution of the live load, temperature variations, shrinkage, etc. will have the same effect as an increase in the permissible stress. The great efforts that are being made with this object are, therefore, fully justified.

II. Reinforced concrete beam bridges.

Apart from arch bridges, which type alone comes into question for the largest spans, reinforced concrete construction in the form of beams has a large field of application for medium spans. In these structures the distribution of the moments over the supports and within the spans plays a similarly decisive part in determining the amount of material required as is played by the choice of the ratio of rise to span in arch bridges. The following considerations will serve to indicate how far this is true.

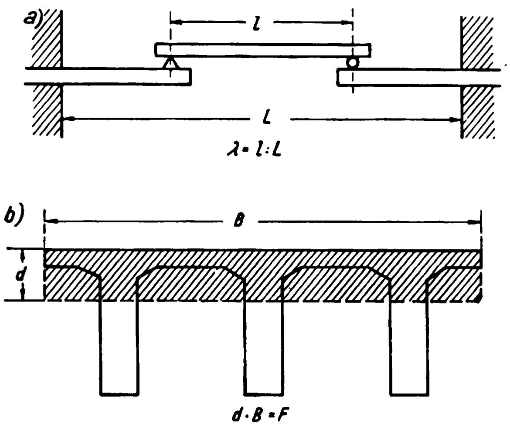

Fig. 6.

An opening of width L may be bridged by a structure consisting of a freely supported girder of span l carried on two symmetrically arranged cantilevers (see Fig. 6a). The proportion between the spans l/L will be denoted by λ, the value of which lies between 0 and 1. $\lambda = 1$ corresponds to the case of a simple beam on two supports and $\lambda = 0$ to that of two cantilevers each of length L/2. Between these limits there may be any intermediate forms similar to that sketched.

It was a matter of difficulty[1] to work out generally valid formulae similar to those for arch bridges. As a beginning, therefore, a series of designs were prepared for road bridge superstructures in reinforced concrete of different spans and with different values of λ and the amount of material required for each of

[1] See *Gaede:* „Balkenträger von gleichem Widerstande gegen Biegung." Die Bautechnik 1937, Heft 10, S. 120/122.

these was expressed in terms of thickness d of a slab of the same volume, and the area of the bridge floor (see Fig. 6b). The calculations were based upon the reinforced concrete stresses of 60 and 70 kg/cm² prescribed in the German regulations, the latter of these values being used for the increased stresses in the region of negative moments.

In Fig. 7 the average quantity of material d, in m³/m² of floor area, is shown as a function of the span ratio λ for several different spans L. According to this, the minimum quantity of material is obtained with $\lambda = 0$ for all spans; that is to say, in a structure consisting of two cantilevers alone. The quantity of material increases as the length of the simply supported girder increases, and reaches its maximum value when $\lambda = 1$ which corresponds to the case of a beam between two supports. The curves become more and more steep as the total span L in-

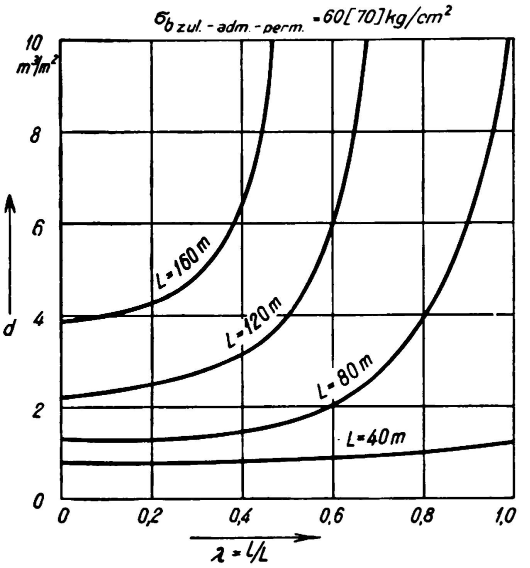

Fig. 7.

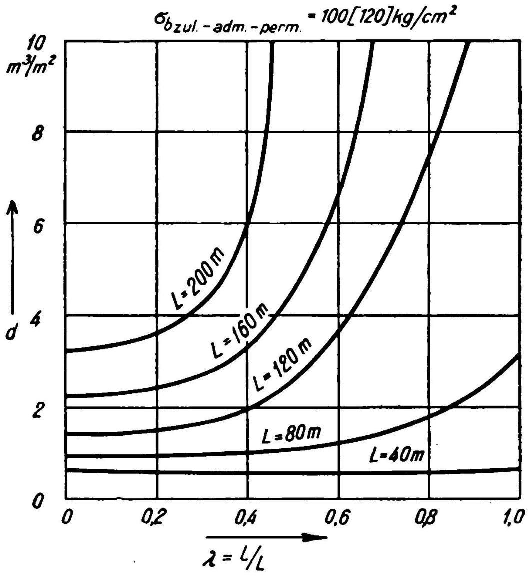

Fig. 8.

Material requirement d for reinforced concrete girder bridges in m³/m².

creases, which shows that for very large spans a simple beam, or even cantilevers with proportionately long beams between them, would be uneconomical or even impossible to build. Large spans can be bridged, with an economically acceptable amount of material, only by the use of cantilever girders acting in the same way as those with small intermediate girders. It is not essential in such a case to introduce hinges, for the same effect may be obtained by means of hingeless continuous girders provided there is a suitable distribution of the moments of inertia and artificial pre-stressing (by dropping or lifting the supports) is applied. If side openings are not otherwise available for the purpose of imposing the heavy restraint moments, it may be expedient to introduce special measures for this purpose, such as counterweight arms, or specially provided openings at the sides, etc.

Fig. 8 shows a corresponding result to that in the preceding graph, assuming a permissible stress of 100 or 120 kg/cm² such as may conceivably be realised in the future.

For practical reasons λ will not be reduced to zero, but the span of the intermediate beam will be made say 0.2 to 0.4 of the total span. The values obtained from Figs. 7 and 8 to correspond with this condition are incorporated in the next diagram, Fig. 9, which shows the quantity of material for suitably arranged cantilever beams in relation to the spans, using either of the stresses already contemplated.

As in the case of the arch bridges, a comparison between the quantities of materials in these cases and those in steel girder bridges allows the limits of competability of reinforced concrete by comparison with steel to be determined. These limits are represented in Fig. 10 once again in relation to the cost ratio between 1 ton of steel and 1 m^3 of reinforced concrete. Here again emphasis should be laid down on the great increase of the region in which reinforced concrete may be enabled to compete by raising the permissible stress. Measures designed to reduce the extreme fibre stress of the concrete, such as those suggested by Professor *Dischinger*, play the same part as a corresponding increase

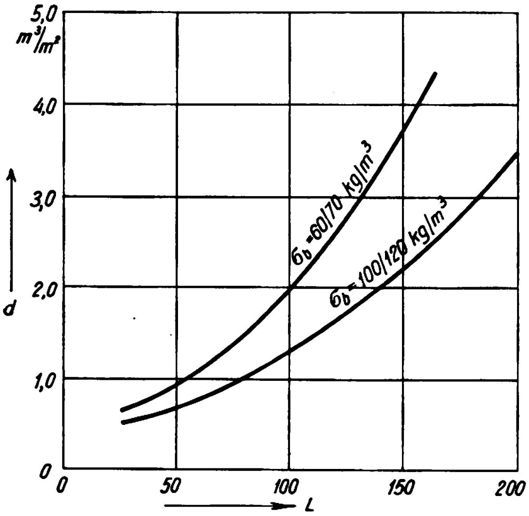

Fig. 9.

Material required for reinforced concrete girder bridges in m^3/m^2.

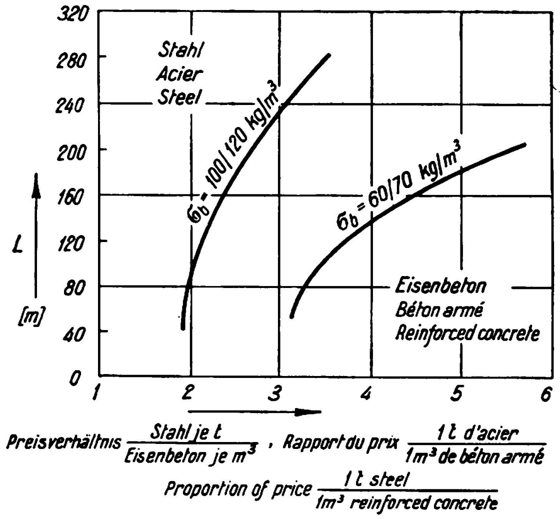

Fig. 10.

Cost comparison between steel and reinforced concrete beam bridges.

in the permissible stress, and offer like the latter, a suitable means of increasing the competability of reinforced concrete in this important field of application.

Finally the author would draw special attention to the fact that these comparisons of cost, whether for arch or beam bridges, must not be regarded as more than rough approximations and in particular that no account has been taken of the greater quantity of material usually necessary in the piers and abutments in consequence of the great weight of the reinforced concrete superstructure, these quantities being so greatly dependent on special local circumstances as not to admit of generalised treatment. It must not be overlooked that this circumstance will tend to swing the balance of the comparison more or less strongly to the disadvantages of reinforced concrete.

An attempt to take into account the supports in a comparison of cost for arch bridges has been made by *Dr. Glaser* in Zeitschrift des Oesterreichischen Ingenieur- und Architektenvereins, 1934, N° 39/40, pages 233 and foll., and similar solutions might possibly be made for beam bridges. This work by *Dr. Glaser* followed upon an earlier one by the present author in Bauingenieur, 1934, N^os^ 13/14 and 17/18.