Application of the theory on "Internal Elastic Bond" to the equilibrium of arches

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Application of the Theory of „Internal Elastic Bond" to the Equilibrium of Arches.

Anwendung der „Bindungs-Elastizitäts-Theorie" für das Studium der Bogen.

Application de la théorie de „l'élasticité à liaisons internes" à l'équilibre des arcs.

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Deformations in elastic solids may be simply and systematically studied if, instead of relying on special hypotheses as to stresses (Saint-Venant), the study is based on a special hypotheses regarding displacements.

In many cases the displacements of elastic bodies may be supposed to occur very approximately as if there existed special internal hinges. By the introduction of such hinges some degree of freedom is removed from the system and from the analytical point of view a notable simplification results, the problems of elasticity being reduced to ordinary total differential equations (instead of partial differential equations). The greater simplicity of the new method consists in this device, and the author proposes to name it the Method of Internal Elastic Bond. It lends itself very well to the study of deformations in elongated elastic solids, whether their axis be straight or curved.

The hinges thus introduced within the elastic solid are represented by the assumption that transverse sections (that is to say sections at right angles to the axis) remain plane; in other words any plane section undergoes only pure deformation in its own plane, with rigid displacement.

Thus, in the simplest case of all, that of a plane arch, the analytical study of deformations reduces itself to the problem of solving a system of four ordinary total differential equations having four unknowns.

The solution of these equations provides us with the components of the elastic displacement.

Consider the case of a beam of constant thickness with a straight axis. Here the differential equations can be integrated at once, and the shear force and bending moment will at once be obtained. Now let us suppose that the beam, while still of constant cross section, has a curved axis. It may then be shown that the displacements can be calculated by the addition of correcting terms to the displacement obtained in the first case, these correcting terms being made to take

account of the curvature of the arch. By the introduction of new correcting factors it is also possible to extend the solution to the case where the thickness varies.

It is impossible to explain the details of the calculation in a few words but these may be found in the notes already published by the present author in the „Rendiconti dell'Accademia dei Lincei" of Rome and in the Comptes Rendus de l'Académie des Sciences, Paris.[1]

The different types of arches which are important in engineering — namely those having circular, elliptical, parabolic, etc., axes — can all be related to the theory briefly summarised above.

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[1] See *E. Volterra:* 1° Elasticità vincolata e sua schematizzazione matematica. Rend. Acc. R. dei Lincei vol. XVI — serie 6°—2° semestre fascicolo 5 e 6 — settembre 1932. — 2° Questioni di elasticità vincolata: 1° Componenti di deformazione e potenziale elastico in coordinate qualsivogliono id., id., vol. XX id., id. fascicolo 11 dicembre 1934. — 3° Id., id., id.; II° Forma appropriata del ds^2 e conseguenze del vincolo geometrico. Id., id., id., vol. XX id., id., fascicolo 12 dicembre 1934. — 4° Id., id., id.: III° Espressione della ϑ e della ψ nel caso generale. Le equazioni dell' elasticità vincolata pei solidi la cui fibra baricentrica è piana. Id., id., id. vol. XXI, id., id., fascicolo 1° — gennaio 1935. — 5° Id., id., id.: IV° Significato del vincolo geometrico. Id., id., id. — Seduta del 1° marzo 1936. — 6° Sugli archi elastici piani: 1. Le equazioni differenziali delle deformazioni. Id., id., id. Seduta del 15 marzo 1936. — 7° Id., id., id.: 2. Direttrice rettilinea Id., id., id. Seduta del aprile 1936. — 8° Id., id., id.: 3. Direttrice, qualsiasi. Id., id., id. Seduta del 19 aprile 1936. — 9° Sur la déformation des arcs élastiques. Comptes rendus des séances de l'Académie des Sciences, t. 202, p. 1558.

See also *E. Volterra:* 10° Elasticità libera ed elasticità vincolata. Applicazioni del concetto di elasticità vincolata. "Atti del Congresso Internazionale di Matematica" di Zurigo, settembre 1932. — 11° Ricerche sugli archi elastici: 1° Metodo generale ed applicazione alle travi ad asse rettilineo. "Annali dei Lavori Pubblici" Anno 1936 — XIV°.