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The Theoretical Maximum Spans of Reinforced Concrete Arch Bridges.

Die theoretisch größtmöglichen Spannweiten von Eisenbetonbogenbrücken.

Les portées théoriquement maxima des ponts en arc de béton armé.

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M. *Boussiron,* in his exhaustive and interesting paper as printed in the Preliminary Report, has included a calculation and diagram of the average concrete section of reinforced concrete arch bridges of different spans in relation to the concrete compressive stress where the ratio of rise to span remains constant at $\frac{f}{l} = \frac{1}{5}$ (Preliminary Report, page 739, Fig. 11). The basis for his calculations is the theoretical principle which he explains $\left(l = \frac{\varepsilon \cdot R^n}{e^\alpha} \ldots\right)$ and the assumption that the arch has to carry a live load of 2 tonnes per linear m (corresponding to $\sim \frac{2}{4} = 0.5$ tonnes per m²) in addition to its own weight and a dead load of 4,6 tonnes per m, representing the roadway, suspension bars, etc. The variation in temperature is assumed at $\pm 25^0$ C.

From the curves given it will be seen that for $\frac{f}{l} = \frac{1}{5}$ and $\sigma_{b perm} = 100$ kg/cm² the maximum possible span is approximately 600 m, or with $\sigma_{b perm} = 150$ kg/cm² approximately 900 m.

In amplification of this work and of the contribution to the discussion made by Professor *K. Gaede,* the present writer proposes to give an account of his own investigations which lead to a generalised determination of the maximum possible spans of reinforced concrete arch bridges.

Basic assumptions and principles.

According to this study, the form of arch which allows the longest span is of the hingeless type, built in at either end and supporting the roadway above.

Using the method of calculation given by *Dr. A. Straßner*,[1] the thicknesses at the crown and springing corresponding to different amounts of rise with $\sigma_{b\,perm} = 100$ and $150\ kg/cm^2$ will be determined on the assumptions stated below:

I. *Nature of arch.*

Fixed hingeless arch of full cross section with the roadway above.

II. *Calculation.*

(The basic idea is that subject to a particular law of change of loading the axis of the arch may be represented as a geometrical function of the line of thrust for dead load, and that the statically unknown values may then be obtained from the equations of elasticity. In the same way, the variation in thickness of the arch may be calculated from a law of change. The notation adopted by *Dr. A. Straßner* is retained here throughout.)

Further: —

1) The planes of action of the forces coincide with the principal longitudinal planes of symmetry.

2) The system of axes in a vertical direction is determined through the choice of values m_a and m_b such that

$$\gamma_a + \varepsilon_a m_a = 0$$
$$\gamma_b + \varepsilon_b m_b = 0$$

In other words, the angle of the abutment at the springing under a loading $H = 1$ and the angle of the built-in cross section are in agreement for equal or opposite loading.

3) The system of axes in a horizontal direction is determined by a suitable choice of values z_a and z_b such that

$$z_a (\alpha_a + \beta + \varepsilon_a) = z_b (\alpha_b + \beta + \varepsilon_b).$$

4) Equilibrium exists between internal and external forces.

5) The modulus of elasticity E remains the same for the whole of the arch.

6 The distribution of stress follows *Navier*'s straight line law.

7) The proportion between stress and strain is constant (Hooke's law) $\sigma = \varepsilon \cdot E$

8) $Z = \int z^2 \cdot \frac{r}{r+z} \cdot dF \approx J$, or accurately: $J = \int z^2 \cdot dF$

$$\frac{r}{r+z} = 1 - \frac{z}{r} + \left(\frac{z}{r}\right)^2 - \left(\frac{z}{r}\right)^3 \ldots\ldots\ldots\ldots$$

In the case of a rectangular cross section this gives

$$Z = J\left[1 + \frac{3}{5}\left(\frac{d}{2r}\right)^2 + \frac{3}{7}\left(\frac{d}{2r}\right)^4 \ldots\ldots\ldots\right]$$

which in turn gives $Z = 1.0015\ J$ when $r = 10\ d$.

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[1] *Dr. A. Straßner:* Neuere Methoden zur Statik der Rahmentragwerke. Berlin 1927.

9) No account is taken of the following:

a) The value $\frac{M}{r}$ in relation to N in the expression $\varepsilon = \frac{1}{E \cdot F} \cdot \left(N + \frac{M}{r}\right)$;

hence $\varepsilon = \frac{N}{E \cdot F}$.

b) The value $\frac{\varepsilon}{r}$ in relation to $\frac{M}{EJ}$ in the expression for $\frac{\Delta d\varphi}{ds} = \frac{M}{EJ} + \frac{\varepsilon}{r}$;

hence $\frac{\Delta d\varphi}{ds} = \frac{M}{EJ}$.

10) The arch is symmetrical and is firmly fixed on each side, so that $z_a = z_b = \frac{l}{z}$

$$m_o = \frac{\int y_o \cdot dw}{\int dw}; \qquad \psi = 0.$$

11) A geometrical law of loading holds good: $g_z = g_s \left(1 + \frac{y'}{f}(m-1)\right)$.

12) The axis of the arch coincides with the line of thrust under its own weight:

$$y' = \frac{f}{m-1}(\mathrm{Cos}\,\zeta k - 1).$$

13) The moment of inertia of any given cross section of concrete varies in accordance with a geometrical law:

$$\frac{Js}{J_z \cos\varphi} = 1 - (1-n) \cdot \zeta^1.$$

14) The calculation of thickness of the arch is governed solely by the compressive stress in the concrete, all tensile stresses being taken up by the steel reinforcement.

III. *Loading.*

1) From own weight:

The change in cross section from the crown to the springing follows in accordance with the above-mentioned law from the equation

$$\frac{Js}{J_z \cos\varphi} = 1 - (1-n)\,\zeta^1, \text{ wherein } n = \frac{Js}{J_k \cos\varphi_k}.$$

2) Due to the weight of the floor construction and superstructures of the bridge.

A load of 2 tonnes per m² will be assumed to include the average weight of the roadway paving, the slab and the longitudinal and cross girders. In addition the following load in tonnes per m² will be allowed for the necessary superstructure:

1.9 tonnes per m² for spans up to . . . . l = 250 m
4.0 tonnes per m² for spans . . . . l = 500 m
8.0 tonnes per m² for spans . . . . l = 750 m

This simplification involves errors, which do not, however, appreciably affect the final result.

3) Due to live load.

Here a uniform moving load equal to p = 1.0 tonnes per m^2 is assumed, corresponding approximately with the loading for an (Austrian) first-class highway (Oenorm B 6201, Case 1). Since we are dealing mainly with spans of over 100 m the value p is made amply high enough to cover the possibility of a future increase in loading. With very large spans the live load has so small an effect that a reduction in p would not alter the final result, consequently the value of p = 1.0 tonnes per m^2 is here retained. For the subsequent calculation of M_p and N_p the ordinates of the influence lines calculated by *Dr. A. Straßner* are utilised.

4) The variation in temperature is taken at $\pm$ 15⁰ C and allowance for shrinkage is made by assuming a drop in temperature of — 15⁰ C. These assumptions appear to be entirely justified on the basis of experiments carried out on the Langwieser viaduct and on the Hundwilertobel bridge in Switzerland. By the adoption of special methods of construction the shrinkage effects can be considerably reduced, but in the present investigation this possibility has been neglected.

5) No account is taken of stresses due to wind loading, braking loads and movement of the supports.

Results.

a) *Investigation with* $\sigma_{b\,perm}$ = *100 kg/cm²*.

Firstly only the arch of 250 m span will be considered. By plotting the thicknesses of the arch calculated in accordance with the various amounts of rise,

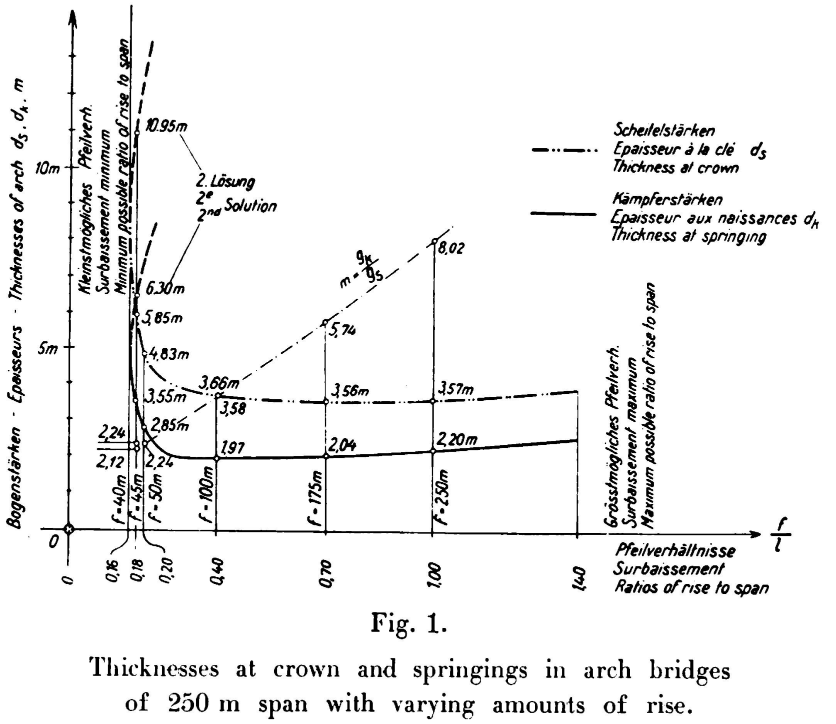

Fig. 1.

Thicknesses at crown and springings in arch bridges of 250 m span with varying amounts of rise.
$\sigma_{b\,perm}$ = 100 kg/cm².

a clear picture of their magnitude to correspond with varying amounts of rise f is obtained (Fig. 1).

In the resulting curves the ordinates for the smallest possible ratio of rise become tangents, but then the curves assume their maximum curvature and as the rise of the arch increases they rapidly flatten out. The thicknesses of arch after touching their minimum value again increase slightly when the amount of rise f continues to increase, but subsequently the curves straighten very rapidly, still with small increments. The curves finally terminate at that ratio of rise which corresponds to the calculated maximum f possible. The broken line represents the relationship

$$m = \frac{g_k}{g_s}.$$

The relation of the crown thickness to changes in the rise alone is further represented in Fig. 2, which will be readily understood from Fig. 1.

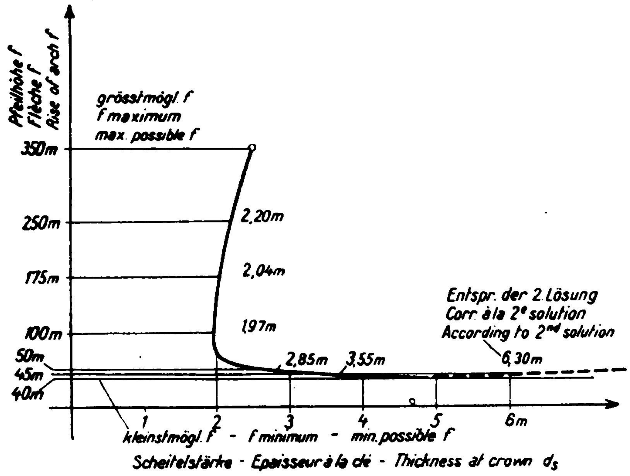

Fig. 2.

Crown thicknesses in arch bridges of 250 m span with varying amounts of rise.

$\sigma_{b\ perm} = 100\ kg/cm^2$.

For the sake of conciseness the further investigations will be discussed only from the point of view of the thickness at the crown. Since the crown and the springings of an arch are subject to the same laws the results obtained in relation to the former will apply also to the latter.

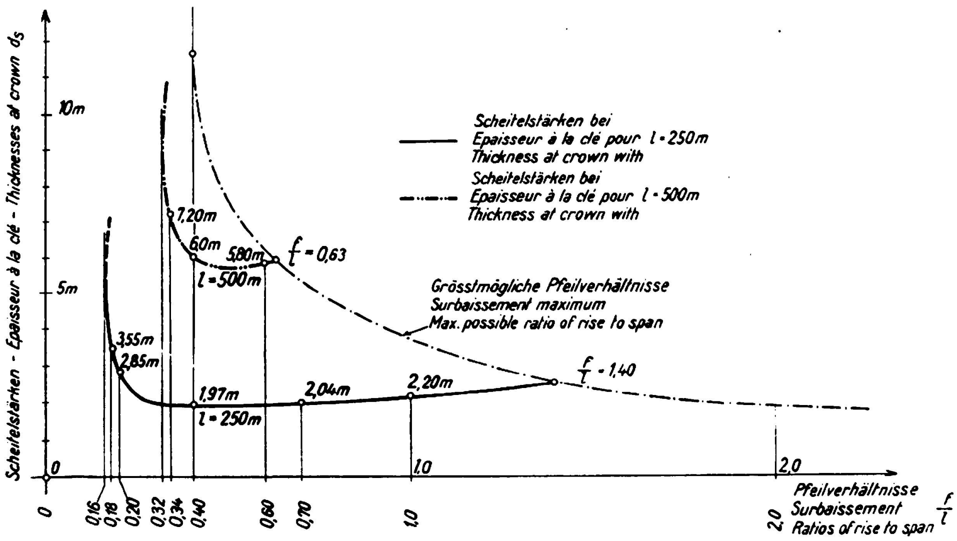

Fig. 3.

Crown thicknesses in arch bridges of varying spans and varying amounts of rise.

$\sigma_{b\ perm} = 100\ kg/cm^2$.

If, now, the calculated crown thicknesses of arches of different spans are plotted as in Fig. 3, it will be seen that the end points of the several curves indicating the maximum possible amounts of rise of arch admit of being con-

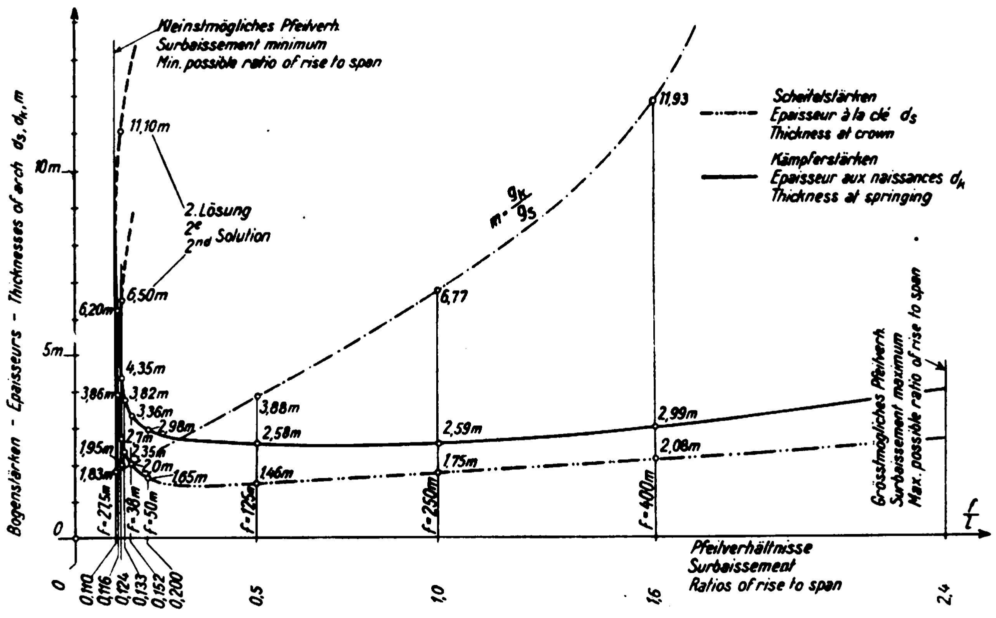

Fig. 4.

Thicknesses of crown and springings in arch bridges of 250 m span with varying amounts of rise.

$\sigma_{b\,perm} = 150$ kg/cm².

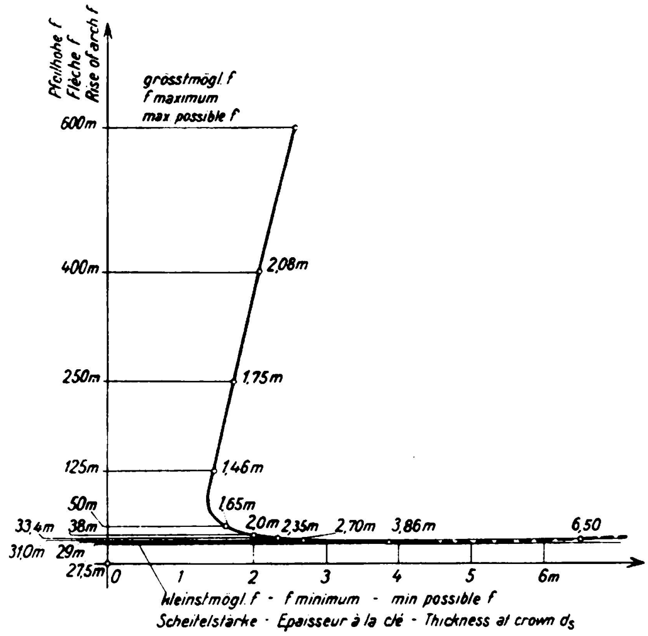

Fig. 5.

Crown thicknesses in arch bridges of 250 m span with varying amounts of rise.

$\sigma_{b\,perm} = 150$ kg/cm².

nected by a smooth curve, which is shown as a broken line in the figure. *If such points for different spans, and also for the ratios corresponding to minimum amounts of rise, are plotted as in Fig. 7, the intersections indicate the maximum obtainable lengths of span.*

The area enclosed between the upper and lower bounding lines covers all the possible arches. It will be seen that as the span increases this area is rapidly reduced until finally it becomes a point. *Thus the maximum possible span, in this case $l = 650$ m, is possible only with one particular ratio of rise, namely* $\left(\frac{f}{l} = 0.40\right)$.

b) *Investigations with* $\sigma_{b perm} = 150 \; kg/cm^2$.

Here again reference will at first be made only to the case of the arch of 250 m span represented in Fig. 4. The dependence of the thickness at the crown on the varying amount of rise is represented in Fig. 5 and the lines so obtained show a similar trend to those found under a). Hence what has been said above is also valid here, but in accordance with the greater permissible stresses for the concrete the limiting values are now different as shown in Figs. 4 to 7. *The maximum span is seen to be 1000 m, which once again is only obtainable with a ratio of rise equal to* $\frac{f}{l} = 0.40$.

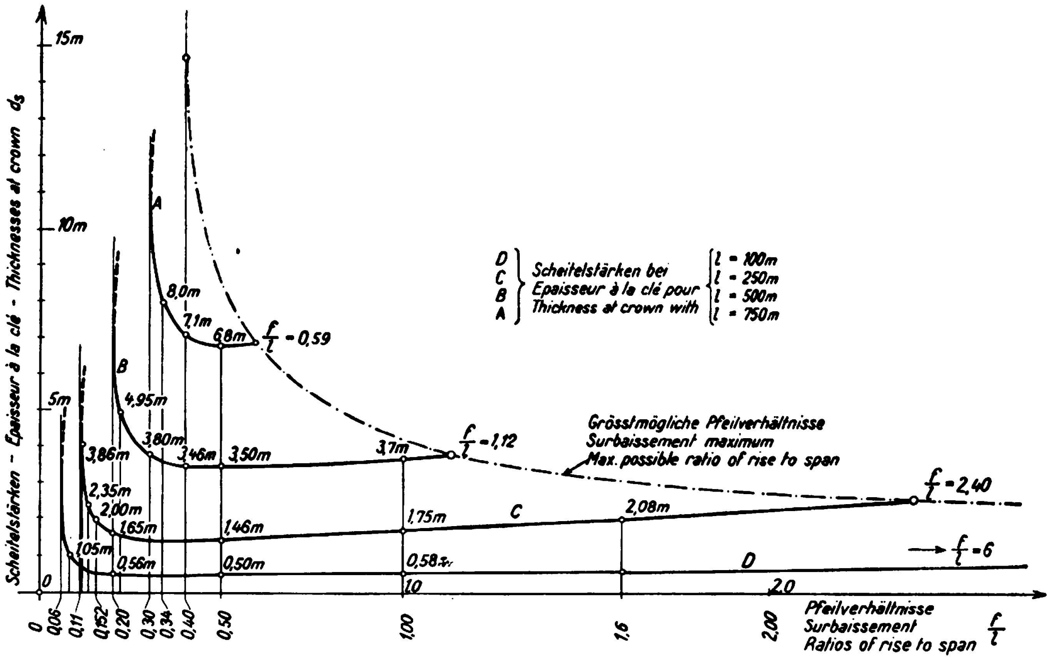

Fig. 6.

Crown thicknesses in arch bridges of varying spans with varying amounts of riso. $\sigma_{b perm} = 150$ kg/cm².

The range of possible ratios of rise $\frac{f}{l}$ for different spans and different permissible stresses in the concrete is shown in Fig. 7.

The possibility of further progress in the construction of reinforced concrete arch bridges rests on the fact that it is being seriously contemplated, even today, that compressive stresses in the concrete of 200 to 300 kg/cm² might be used in exceptional bridges of this type. Preliminary designs have been made for reinforced concrete arches of 400 m free span (*Hawranek*, with $\sigma_{bperm} = 160$ kg/cm^2) and even 1000 m free span (*Freyssinet*, with $\sigma_{bperm} = 280$ kg/cm^2).

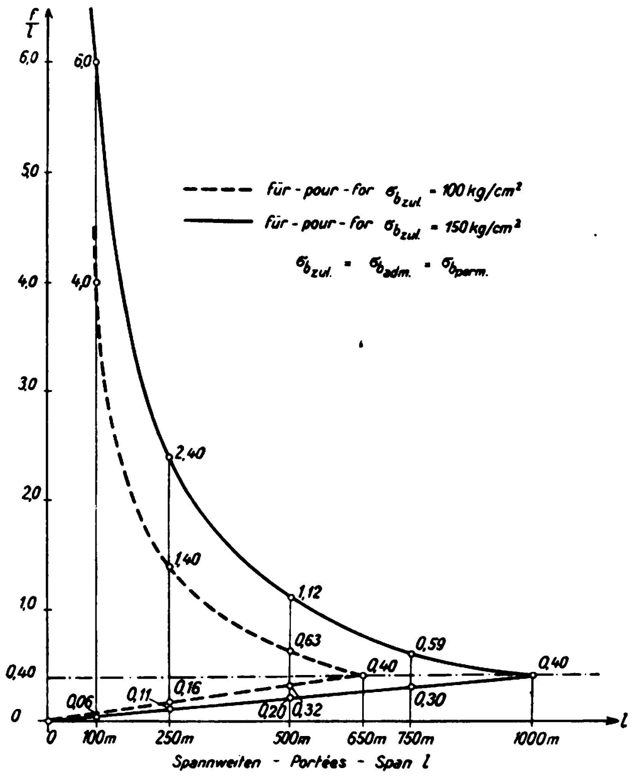

Fig. 7.

Range of possibility of arch bridges, and limiting values of spans, under the assumptions made.

The development of technique in the preparation and placing of concrete, together with the theoretical investigations that are being made into the statical conditions which govern structures of this character, render it likely that such designs may actually be realised.