The phenomena of buckling

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The Phenomena of Buckling.

Über Kipperscheinungen.

Sur les phénomènes de déversement.

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The problem of stability against buckling and bulging has now been the subject of extensive researches, but the theory of collapse by torsion (tilting) has received little attention. In design practice, up to the present, the lack of a simple formula for calculation to be applied in determining stability of members subject to bending has usually led to the compressive flange being considered as separated from the remaining portion of the girder. In what follows below the relationship between this view of the matter and the complete solution of the problem of the collapse (tilting) of girders of **I** section[1] will be pointed out. For this purpose we shall take as an example the simplest case of all, where the bending moment is constant and the girder so loaded is of constant **I** cross section.

If, now, the bending of the flange is neglected, that is to say the critical moment is determined as for a beam of rectangular cross section, we obtain the value:

$$M_{o,\,kr} = \pi \cdot \frac{\sqrt{B_2 \cdot C}}{l}, \qquad (1)$$

wherein $B_2 = E.J_y$ represents the lateral resistance to bending and $C = G.J_d$ represents the resistance to torsion. The effect of vertical bending, which as a rule is small, is here neglected.

If now we consider the compressive flange as detached, then the product of *Euler*'s buckling load P_E multiplied by the distance h of the flanges determines the critical moment of the girder in the usual way:

$$M_{Fl,\,kr} = P_E \cdot h = \frac{\pi^2 \cdot B_2 \cdot h}{a\, l^2}, \qquad (2)$$

since the resistance of a flange to bending may be equated to one half of the resistance of the girder to lateral bending.

[1] *S. Timoshenko:* Sur la stabilité des systèmes élastiques. Annales des Ponts et Chaussées, 1913. — *S. Timoshenko:* Stability of plate girders subjected to bending. Preliminary Report, I.A.B.S.E., Paris, Congress 1932. — *F. Stüssi:* Stability of a girder subject to bending. Publications, I.A.B.S.E. Vol. 3, 1935. — *F. Stüssi:* Exzentrisches Kippen. Schweizerische Bauzeitung. Vol. 105, 1935.

According to *Timoshenko*, the critical moment of the I beam is

$$M_{kr} = \pi \frac{\sqrt{B_2 \cdot C}}{l} \cdot \sqrt{1 + \frac{\pi^2}{a^2}} \tag{3}$$

wherein a^2 is a simplification for the expression

$$a^2 = \frac{4\,Cl^2}{B_2\,h^2}$$

If, now, the value obtained by this simplification is introduced into Equation (3), a right-angled triangle as in Fig. 1 will serve to indicate the simple connection that exists between the three values of the critical moment which are under consideration:

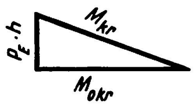

Fig. 1.

$$\underline{M_{kr} = \sqrt{M^2_{o,\,kr} + (P_E \cdot h)^2}.} \tag{4}$$

In order to picture the numerical significance of the two components the critical extreme fibres stresses for an I NP 16 beam are represented in Fig. 2. Throughout the elastic region M_{kr} is only slightly larger than $M_{o,\,kr}$; hence the buckling load on the compression flange plays only a very subordinate part in the determination of the critical moment. The assumption usually made in design that the carrying capacity of the girder is governed only by the buckling load on the compression flange is unsatisfactory, attributing, as it does, a decisive influence to what is only a subordinate partial influence. The correctness of the complete solution of the problem should be evident from the experimental points plotted in

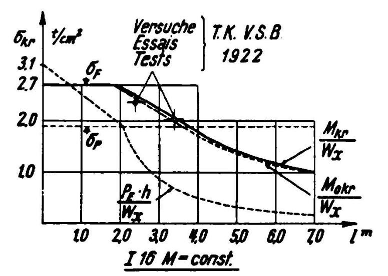

Fig. 2.

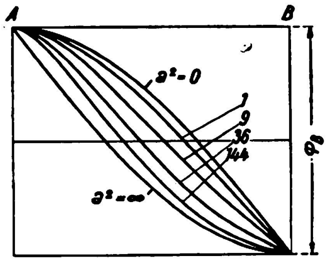

Fig. 3.

Fig. 2 which correspond to the condition of loading here considered, and which are reproduced from an unpublished report of the Technische Kommission des Vereines Schweizerischer Brückenbau und Eisenhochbau Fabriken.[2]

The simple connection between the critical moment of the rectangular beam and the buckling load of the compression flange, as represented in the right-angled triangle, is as consequence of the fact that with the loading here under consideration the deformation curves for buckling and for tilting are both sine curves, and therefore are both of the same form. The author has investigated,

[2] T.K.V.S.B.: Bending tests at the Swiss Federal Testing Institute, Zürich, May 1932. Lateral buckling of the compression flange of an I-beam. Experiments Nos. 1 and 5.

as an example, the case where this agreement in shape cannot be obtained, namely the case of a cantilever loaded with a constant bending moment. The two deformations which correspond to the beginning of instability due to torsion and to buckling, react upon one another, so that the curve of distortion φ changes its form according to the dimensions. The shape of this curve is represented in Fig. 3 for different values of the abbreviation a^2, wherein $a^2 = 0$ corresponds to the buckling problem (resistance to torsion $C = 0$) and $a^2 = \infty$ corresponds to resistance to torsion (stiffness of flange = 0). Since the mutual effect of the two limiting curves is the same for every case, assuming an arbitrary amount of fixation, and is the same, therefore, if the stiffness is increased, the critical moment of the **I** girder must be greater here than is measured by the hypotenuse of the triangle which has for its other two sides

$$M_{o,\,kr} = \frac{\pi}{2} \cdot \frac{\sqrt{B_2 \cdot C}}{l}$$

and

$$P_E \cdot h = \frac{\pi^2}{4} \cdot \frac{B_2 \cdot h}{2 \cdot l^2} = \frac{\pi^2}{4\,a} \cdot \frac{\sqrt{B_2 \cdot C}}{l}.$$

Here again, however, the effect of the buckling load for the compression flange is only of subordinate importance under practical conditions (Fig. 4).

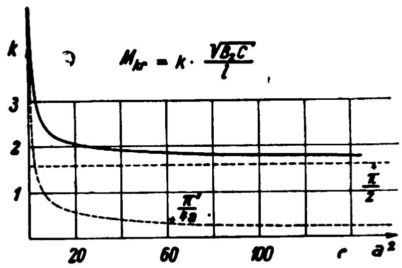

Fig. 4.

The accuracy of the investigations on buckling of beams, now available in a simple form, should make it possible to pay attention to this important group of problems of stability in the structural practice.