Zeitschrift: IABSE congress report = Rapport du congrès AIPC = IVBH Kongressbericht

Band: 2 (1936)

Artikel: Elastically built-in arch dams

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DOI: https://doi.org/10.5169/seals-3339

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Elastically Built-in Arch Dams.

Elastisch eingespanntes Talsperrengewölbe.

L'arc de barrage élastiquement encastré.

Dr. sc. techn. K. Hofacker,
Zürich.

The expression "arch dam" will be used to indicate an arch having its axis bent to a circular curve and having constant thickness which may be large in proportion to the span. By contrast with the "bridge arch", which may be calculated accurately enough on the basis of *Navier's* theory of bending, the arch dam requires to be analysed by reference to the mathematical theory of elasticity if an accurate picture of the real conditions of stress is to be obtained.

If the water pressure acting on an arch dam is devided up in the customary way between a combined system of horizontal arches and vertical cantilevers, then any desired loading diagram may be drawn for each of the separate elements. The method of calculating the stresses in the vertical slab-shaped elements of a beam in accordance with the actual conditions of stress and strain has been known for a long time, even experimental investigations in this direction have frequently been carried ont. The calculation of stresses in the horizontal curved elements of an arch has hitherto been worked out only for the special case of a rigidly fixed arch; neither does the author know of any exact measurements of the stresses or strains effected in the laboratory on models of such arch dams. It was, therefore, deemed to be of special interest to carry out a theoretical and experimental investigation into the general question of an elastically built-in arch loaded with any desired water pressure.[1]

We will assume a slice of annular shape subjected to the uniplanar conditions of stress shown in Fig. 1.

Any given loading diagram may be represented with the aid of a Fourier mathematical series:

$$\sigma'_r = A'_0 + \sum_{n=1}^{\infty} A'_n \cdot \cos n\varphi + \sum_{n=1}^{\infty} B'_n \cdot \sin n\varphi \tag{1}$$

In Fig. 2 we see the stresses operating on an element dF at the point 0 and may write down the condition of equilibrium. In view of the relation between stresses and elongations, that is to say the differences in displacements u and v

[1] *K. Hofacker:* Das Talsperrengewölbe, 1936. Gebr. Leemann & Co., Zürich.

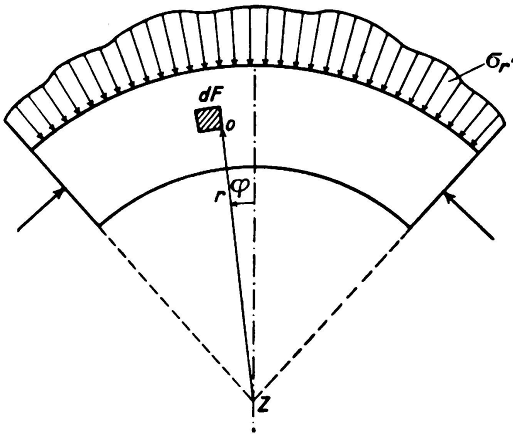


Fig. 1.

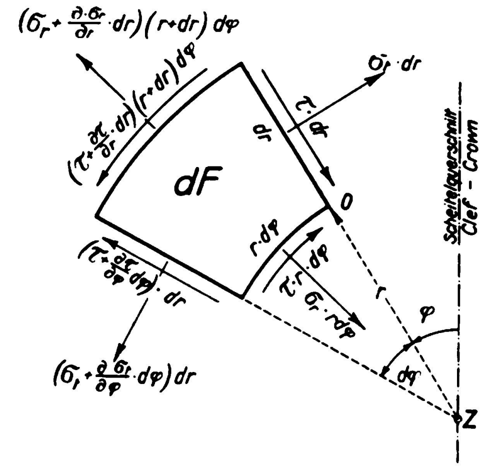


Fig. 2.

measured in a radial and a tangential direction respectively, we obtain the two differential equations:

$$\frac{1}{r^2}\frac{\partial^2 u}{\partial\varphi^2}+\frac{2m}{m-1}\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}\right)$$
$$+\frac{1}{r}\frac{\partial^2 v}{\partial r\cdot\partial\varphi}-\frac{1}{r^2}\frac{\partial v}{\partial\varphi}+\frac{2m}{m-1}\left(\frac{1}{mr}\frac{\partial^2 v}{\partial r\cdot\partial\varphi}-\frac{1}{r^2}\frac{\partial v}{\partial\varphi}\right)=0 \tag{2}$$

$$\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}-\frac{v}{r^2}+\frac{2m}{m-1}\frac{1}{r^2}\frac{\partial^2 v}{\partial\varphi^2}$$
$$+\frac{\partial^2 u}{\partial r\cdot\partial\varphi}\cdot\frac{1}{r}\frac{m+1}{m-1}+\frac{\partial u}{\partial\varphi}\cdot\frac{1}{r^2}\cdot\frac{3m-1}{m-1}=0 \tag{3}$$

The general solutions for u and v read as follows:

Radial displacements

$$E\cdot u=-\frac{m+1}{m}\cdot\frac{a_0}{r}+\left[\frac{2(m-1)}{m}\cdot b_0-\frac{m+1}{m}c_0\right]\cdot r+\frac{2(m-1)}{m}\cdot c_0\cdot r\lg r$$
$$+\left(\frac{m-1}{2m}\cdot a_1+2\beta_1\right)\varphi\cdot\sin\varphi-\left(\frac{m-1}{2m}\cdot c_1+2\delta_1\right)\varphi\cdot\cos\varphi$$
$$+\left[\left(a_1+\frac{m-1}{m}\cdot\beta_1\right)\lg r+\frac{m-3}{m}\cdot b_1 r^2+\frac{m+1}{m}\frac{\alpha_1}{r^2}\right]\cos\varphi$$
$$+\left[\left(c_1+\frac{m-1}{m}\cdot\delta_1\right)\lg r+\frac{m-3}{m}\cdot d_1 r^2+\frac{m+1}{m}\cdot\frac{\gamma_1}{r^2}\right]\sin\varphi \tag{4}$$
$$+\sum_{n=2}^{\infty}\left[-\frac{m+1}{m}\cdot n\cdot a_n\cdot r^{n-1}-\left(\frac{2n}{m}+(n-2)\frac{m-1}{m}\right)b_n\cdot r^{n+1}\right.$$
$$\left.+\frac{m+1}{m}\cdot n\cdot\alpha_n\cdot r^{-n-1}+\left(\frac{2n}{m}+(n+2)\frac{m-1}{m}\right)\cdot\beta_n\cdot r^{-n+1}\right]\cos n\varphi$$
$$+\sum_{n=2}^{\infty}\left[-\frac{m+1}{m}\cdot n\cdot c_n\cdot r^{n-1}-\left(\frac{2n}{m}+(n-2)\frac{m-1}{m}\right)\cdot d_n\cdot r^{n+1}\right.$$
$$\left.+\frac{m+1}{m}\cdot n\cdot\gamma_n\cdot r^{-n-1}+\left(\frac{2n}{m}+(n+2)\frac{m-1}{m}\right)\cdot\delta_n\cdot r^{-n+1}\right]\sin n\varphi$$

Tangential displacements

$$E \cdot v = -\frac{m+1}{m} \cdot \frac{\alpha_0}{r} + 4 c_0 r \cdot \varphi + \left(\frac{m-1}{2m} \cdot a_1 + 2 \beta_1\right) \varphi \cdot \cos \varphi$$
$$+ \left(\frac{m-1}{2m} \cdot c_1 + 2 \delta_1\right) \cdot \varphi \cdot \sin \varphi$$
$$+ \left[-\left(a_1 + \frac{m-1}{m} \cdot \beta_1\right) \lg r - \frac{m+1}{2m} \cdot a_1 + \frac{5m+1}{m} b_1 r^2 + \frac{m+1}{m} \cdot \frac{\alpha_1}{r^2} - \frac{m+1}{m} \cdot \beta_1\right] \sin \varphi$$
$$+ \left[\left(c_1 + \frac{m-1}{m} \cdot \delta_1\right) \lg r + \frac{m+1}{2m} \cdot c_1 - \frac{5m+1}{m} \cdot d_1 r^2 - \frac{m+1}{m} \cdot \frac{\gamma_1}{r^2} + \frac{m+1}{m} \cdot \delta_1\right] \cos \varphi \qquad (5)$$
$$+ \sum_{n=2}^{\infty} \left[\frac{m+1}{m} \cdot n \cdot a_n \cdot r^{n-1} + \left(n \frac{m+1}{m} + 4\right) b_n \cdot r^{n+1} + \frac{m+1}{m} \cdot n \cdot \alpha_n \cdot r^{-n-1} + \left(n \frac{m+1}{m} - 4\right) \cdot \beta_n \cdot r^{-n+1}\right] \sin n \varphi$$
$$+ \sum_{n=2}^{\infty} \left[-\frac{m+1}{m} \cdot n \cdot c_n \cdot r^{n-1} - \left(n \frac{m+1}{m} + 4\right) \cdot d_n \cdot r^{n+1} - \frac{m+1}{m} \cdot n \cdot \gamma_n \cdot r^{-n-1} - \left(n \frac{m+1}{m} - 4\right) \cdot \delta_n \cdot r^{-n+1}\right] \cos n \varphi$$

From these displacements the stresses may be calculated as follows:

Radial stress

$$\sigma_r = \frac{a_0}{r^2} + 2 b_0 + c_0 (2 \lg r + 1) + \left(\frac{a_1 + \beta_1}{r} + 2 b_1 r - \frac{2 \alpha_1}{r^3}\right) \cos \varphi + \left(\frac{c_1 + \delta_1}{r} + 2 d_1 r - \frac{2 \gamma_1}{r^3}\right) \sin \varphi$$
$$+ \sum_{n=2}^{\infty} [n (1-n) \cdot a_n \cdot r^{n-2} + (n - n^2 + 2) b_n r^n - n (n+1) \cdot \alpha_n \cdot r^{-n-2} - (n^2 + n - 2) \beta_n \cdot r^{-n}] \cos n \varphi \qquad (6)$$
$$+ \sum_{n=2}^{\infty} [n (1-n) \cdot c_n \cdot r^{n-2} + (n - n^2 + 2) \cdot d_n r^n - n (n+1) \cdot \gamma_n \cdot r^{-n-2} - (n^2 + n - 2) \cdot \delta_n \cdot r^{-n}] \sin n \varphi$$

Tangential stress

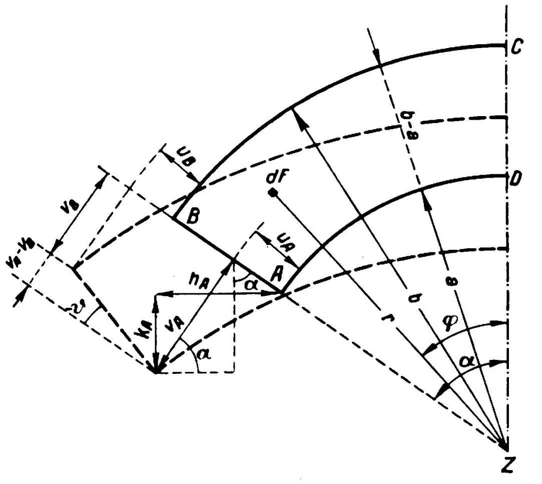
$$\sigma_t = -\frac{a_0}{r^2} + 2 b_0 + c_0 (2 \lg r + 3) + \left(6 b_1 r + \frac{2 \alpha_1}{r^3} + \frac{\beta_1}{r}\right) \cos \varphi + \left(6 d_1 r + \frac{2 \gamma_1}{r^3} + \frac{\delta_1}{r}\right) \sin \varphi$$
$$+ \sum_{n=2}^{\infty} [n (n-1) \cdot a_n \cdot r^{n-2} + (n+1)(n+2) \cdot b_n \cdot r^n + n (n+1) \cdot \alpha_n \cdot r^{-n-2} + (n-2)(n-1) \cdot \beta_n \cdot r^{-n}] \cos n \varphi \qquad (7)$$
$$+ \sum_{n=2}^{\infty} [n (n-1) \cdot c_n \cdot r^{n-2} + (n+1)(n+2) \cdot d_n \cdot r^n + n (n+1) \cdot \gamma_n \cdot r^{-n-2} + (n-2)(n-1) \cdot \delta_n \cdot r^{-n}] \sin n \varphi$$

Shear stress

$$
\begin{aligned}
\tau = \frac{\alpha_0}{r^2} &+ \left(2 b_1 r - \frac{2\alpha_1}{r^3} + \frac{\beta_1}{r}\right)\sin\varphi \\
&- \left(2 d_1 r - \frac{2\gamma_1}{r^3} + \frac{\delta_1}{r}\right)\cos\varphi \\
&+ \sum_{n=2}^{\infty} \left[n(n-1)\cdot a_n \cdot r^{n-2} + n(n+1)\cdot b_n \cdot r^n \right. \\
&\qquad \left. - n(n+1)\cdot \alpha_n \cdot r^{-n-2} - n(n-1)\cdot \beta_n \cdot r^{-n}\right]\sin n\varphi \\
&- \sum_{n=2}^{\infty} \left[n(n-1)\cdot c_n \cdot r^{n-2} + n(n+1)\, d_n \cdot r^n \right. \\
&\qquad \left. - n(n+1)\cdot \gamma_n \cdot r^{-n-2} - n(n-1)\cdot \delta_n \cdot r^{-n}\right]\cos n\varphi
\end{aligned} \tag{8}
$$

Knowing the general laws governing stresses and displacements, the question becomes one of determining the constants with the aid of the marginal conditions, by equating the corresponding values of σ_r from Equation (6) and σ'_r from Equation (1), that is to say, by identifying the trigonometrical terms which correspond to these coefficients.

As regards the radial edges, only conditions governing the displacement of the extreme fibres can be laid down. When the arch is rigidly fixed the condition obtains that the extreme fibres undergo no displacement. With elastic fixation the displacements of the extreme fibres of the arch must have the same values as the corresponding points in the abutment, which is loaded by normal and shear stresses in the cross section where the arch is built-in. This question has been considered in greater detail in the publication by the writer already cited. With a view to simplifying the method of calculation the radial and tangential



$$h_A = u_A \cdot \sin\alpha + v_A \cdot \cos\alpha$$
$$k_A = v_A \cdot \sin\alpha - u_A \cdot \cos\alpha$$
$$\vartheta = \frac{v_B - v_A}{b - a}$$

Fig. 3.

displacements for the corner points A and B of the arch, the calculated increase in length h_A of the inner chord of the arch, and the rotation ϑ suffered by the cross section at the springing, are all shown in Fig. 3.

The theoretical studies were checked by measurements on celluloid models.

Fig. 4 shows the appearance of an elastically built-in arch dam which is loaded by radial pressures on the outside face.

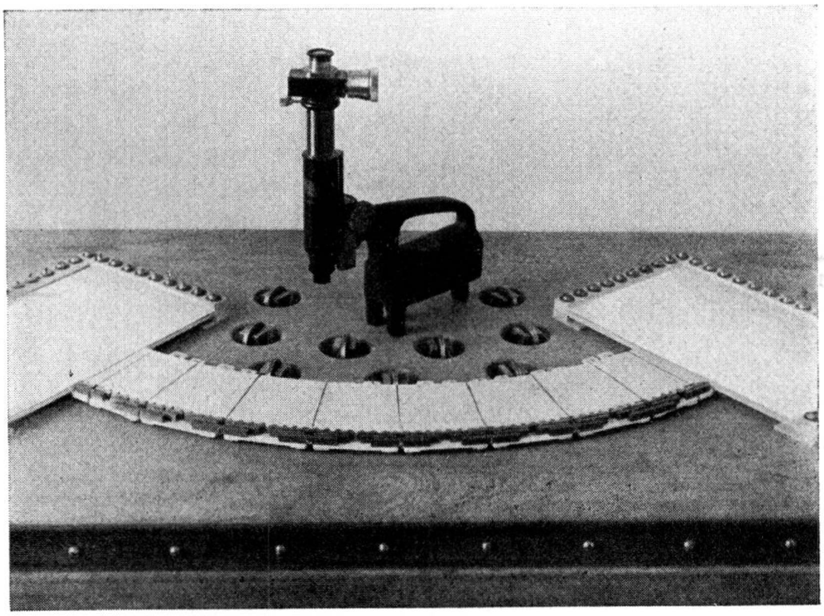


Fig. 4.

In Fig. 5 are shown the displacements of the periphery of the circular arc as measured with the aid of a microscope, and also the amounts of the dis-

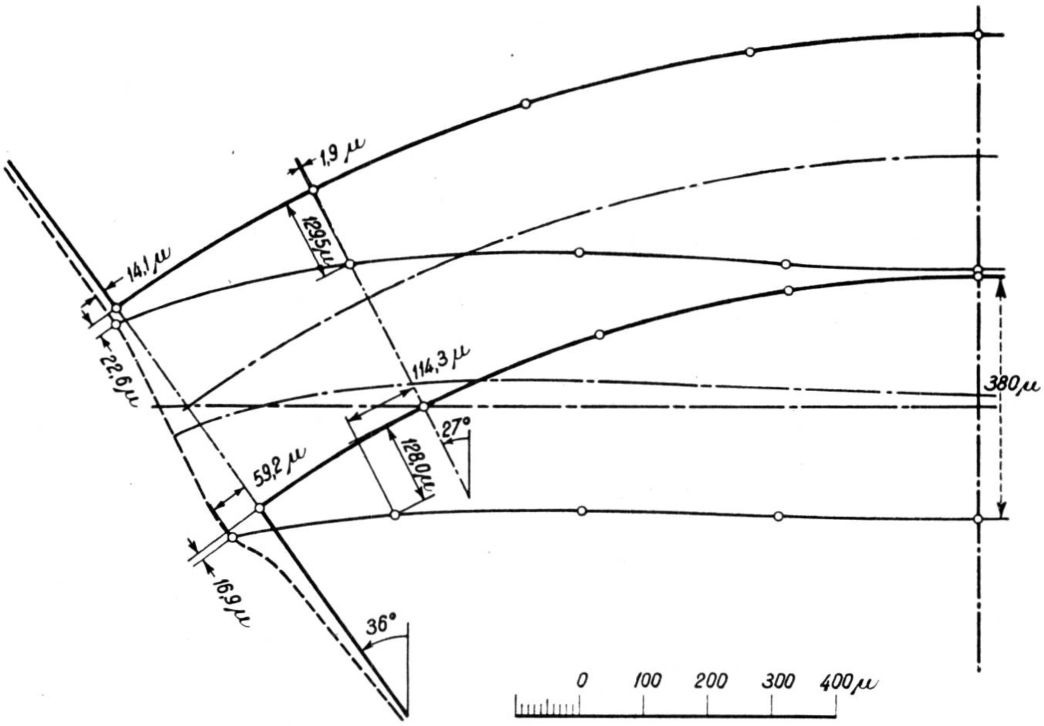


Fig. 5.

placements measured in the two sections $\varphi = 36^0$ and $\varphi = 27^0$. If the drop in the crown at the inner edge be calculated, as for instance by regarding the displacements of the extreme fibre of the section $\varphi = 27^0$ as abutment displacements in respect of the elastically stressed portion of the arch at this

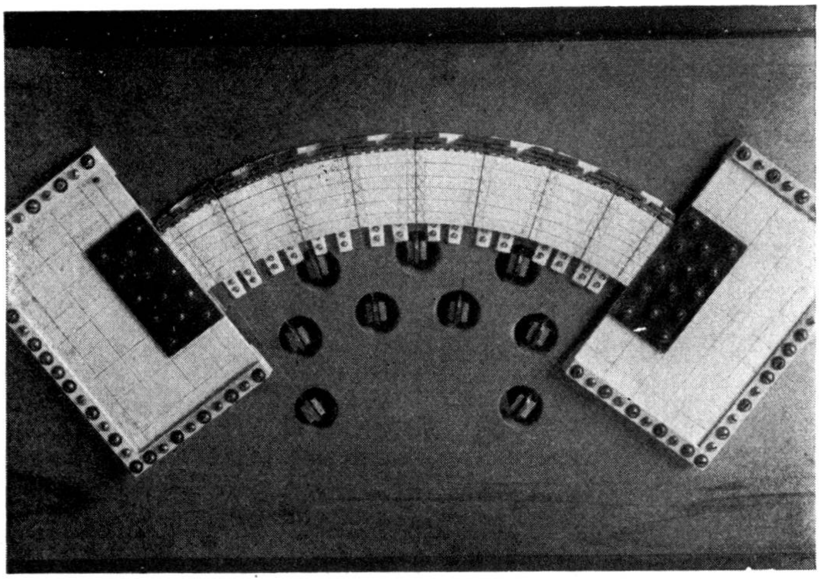


Fig. 6.

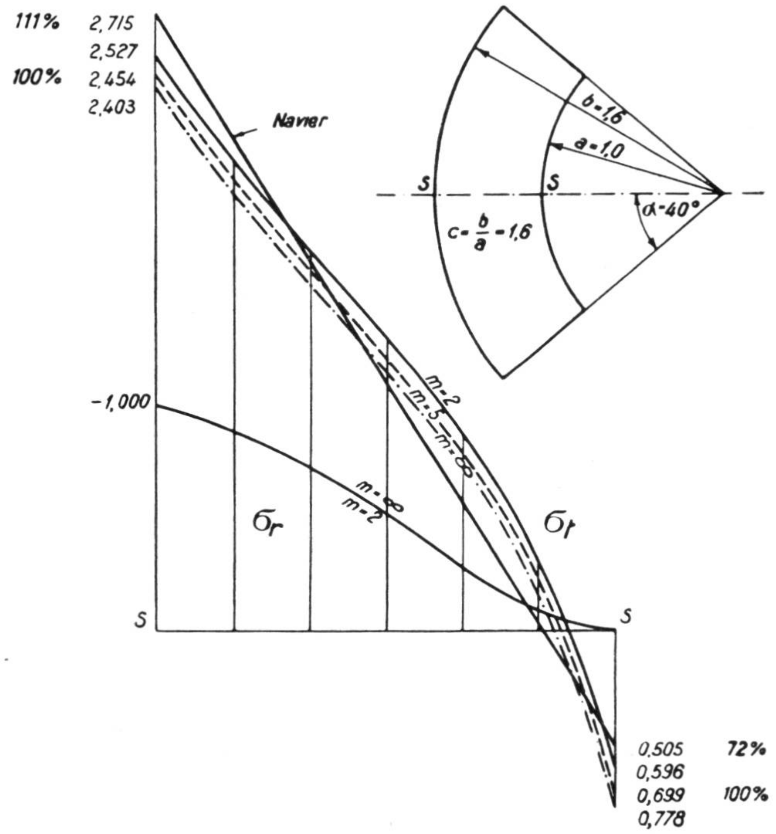


Fig. 7.

radial section; it will be seen that the measured values are only about one-third of one per cent. greater than those calculated; the agreement, therefore, is close enough to allow of recognising the theoretical bases of the problem.

If the drop at the crown on the inner edge is calculated by reference to the measured amount of displacements at the corner points of the abutment section, the result so obtained differs by approximately 4 % from the measured amount. Fig. 7 shows the model of the rigidly fixed arch. If it is assumed in this case that the law of stress and strain holds good as far as the portion which is built in, then the drop at the crown, as calculated, works out some 15 % lower than as measured. The greater amounts of deformation which in fact arise in the neighbourhood of the built-in cross section are the result of the concentration of stress which exists on the side exposed to the air. The investigations [2] in this direction which have hitherto been made are based on the assumption of rigid restraints at the ends.

By reference to an example of an arch dam subject to water pressure, the stress diagram according to the accurate theory will be compared with that obtained by means of the *Navier* approximation, which has hitherto been the only method in use for examining elastically built-in aches. In Fig. 7 there may also be recognised the influence of the *Poisson* number m for the transverse contraction of the stress values. The approximate solution gives values which are approximately 28 % too low for the tensile stresses at the crown assuming a *Poisson* ratio of $m = 5$ in concrete.

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[2] *M. Caquot:* Annales des Ponts et Chaussées, 1926, IV, July-August, p. 21. *R. Chambaud:* Génie Civil, 1926 (vols. 99 and 100).