Lateral stability of I-beams

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Stabilité latérale des poutres à âme pleine
(Méthode par superposition)

Kipperscheinungen von I-Trägern
(Eine Superpositionsmethode)

Lateral stability of I-beams
(Method of super-position)

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The problem I am going to tackle is not a new one in technical literature, as it dates back to 1899, when it was first treated simultaneously by A. G. Michell and Ludwig Prandtl.

Further development rests mainly upon works of S. Timoshenko, presenting the special features of the rolled steel joist, and of H. Wagner, who described the phenomenon, which he called "Drillknickung", i.e., buckling by twist.

The problem is to-day fully solved mathematically, but the exact solution is normally so involved that it has no practical bearing — which brings us to the point of this discussion, the approximate methods.

Our joist with I Section has the following leading features :

Maximum flexural rigidity	$A = EI_1$
Minimum flexural rigidity	$B = EI_2$
Torsional rigidity	$C = GI_t$
Flexural rigidity of one flange	$D \sim \frac{1}{2} B$.

Its deflections will be measured from a system of co-ordinate axes x, y, u ; x along the centre line of the undeflected beam; y at right angles to x in the web; and u at right angles to both x and y. The deflected beam will at the point (x, y, u) be twisted an angle β.

If vertical moments (i.e., acting in the plane of the web) M and horizontal moments M_h are applied to the centre line of the beam and if they are just sufficient to make the beam buckle, then the leading equations will be:

$$-A\frac{d^2y}{dx^2} = M + \frac{A-B}{B}\beta M_h \,, \tag{1}$$

$$-B\frac{d^2u}{dx^2} = M_h + \frac{A-B}{A}\beta M \,, \tag{2}$$

$$MM_h + (A-B)\left(\frac{M^2}{A} - \frac{M_h^2}{B}\right) + \frac{AB}{A-B}\frac{d}{dx}\left[C\frac{d\beta}{dx} - \frac{d}{dx}\left(\frac{1}{2}Dh^2\frac{d^2\beta}{dx^2}\right)\right] \,. \tag{3}$$

If the maximum value of M be M_1, we will write our solution as:

$$M_1 = \frac{k}{l}\frac{A}{A-B}\sqrt{BC}$$

k is a constant characteristic of the case of loading, and l is the span.

We will here confine ourselves to an I-cross-section, symmetrical about both its axis constant along its length, and free to rotate around a vertical axis over the supports. Our first case shall consist of vertical moments only.

Introducing $m = \dfrac{M}{M_1}$ and $\alpha^2 = \dfrac{\frac{1}{2}Dh^2}{Cl^2}$ and $l\dfrac{d\beta}{dx} = \beta'$.

Equation (3) becomes:

$$\beta m^2 k^2 + \beta'' - \alpha^2\beta'''' = 0 \,. \tag{4}$$

An exact solution is arrived at by a "trial and error" method: guessing β in $\beta m^2 k^2$ and integrating this expression.

A satisfactory approximation, however, is found by inserting the average β, $\beta_{av} \sim 0.8\,\beta_{max}$ and solving the following two equations:

$$0.8\,\beta_{max}\, m^2 k_{M_1}^2 = -\beta''$$

$$0.8\beta_{max}\frac{m^2 k_{M_2}^2}{\alpha^2} = \beta''''$$

and then using

$$k = \sqrt{k_{M_1}^2 + k_{M_2}^2}$$

$\dfrac{1}{k_{M_1}^2}$ is found as the maximum bending moment in a simple supported beam with the distributed load 0.8 m^2 and $\dfrac{\alpha^2}{k_{M_2}^2}$ will be the maximum deflection of the same beam (EI = 1, length = 1).

If the distributed load p and the single forces P of our rolled steel joist do not act at the centre line of the beam but at a distance a above the centre line, then we should load our simply supported beam once more — this time with $\dfrac{\beta}{\beta_{max}} \cdot (pl^2 + Pl)\dfrac{a}{C}$. The maximum moment should be 1 (producing k_{P1}) and the maximum deflection should be α^2 (producing k_{P2}).

The final factor k, including for all the effects we have now dealt with, is found as the solution to the equation :

$$\frac{1}{\left(\frac{k}{k_{M_1}}\right)^2+\frac{k}{k_{P_1}}}+\frac{1}{\left(\frac{k}{k_{M_2}}\right)^2+\frac{k}{k_{P_2}}}=1 \, .$$

We have not yet taken any longitudinal forces in the beam into account. If we disregard them, I have just shewn how to find k, and this value we will call k_M. If, on the other hand, we only let the longitudinal force N (which we will write as $N=\frac{\varkappa B}{l^2}$) act, we will then get from the normal Euler formula $\varkappa_E=\pi^2$.

If we finally include for both longitudinal force and bending moments, our result will be found from the following equation :

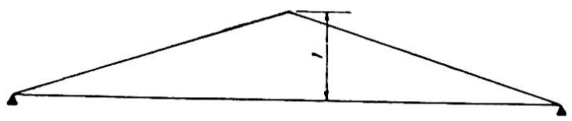
$$\frac{\varkappa}{\varkappa_E}+\left(\frac{k}{k_M}\right)^2=1 \, .$$

Time does not permit the full proof of the various formulae I have given. It must be sufficient to mention that they are based upon the shape of the deflection being the same type for all the parts into which we split the load and the beam. If the types differ, the result will be too small values of $\varkappa$ and k, which only means that we are on the safe side. It is possible, however, in some of these special cases to give special formulae taking this into account.

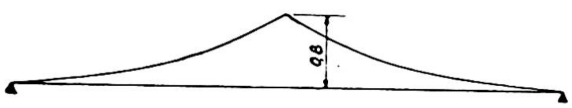
Finally I will show you an example of how to use our formulae.

I choose a rolled steel joist subject to a longitudinal force $N=\frac{\varkappa B}{l^2}$ and a single force P acting at the centre of the beam on top of the upper flange.

m is then simply a triangle :



$0.8\ m^2$ will consist of two 2nd degree parabolas :



This is used as load on a simply supported beam.

$$\text{Maximum moment}=\frac{1}{k_{M_1}{}^2}=\frac{1}{20} \, ; \qquad k_{M_1}=4.5 \, .$$

$$\text{Maximum deflection}=\frac{\alpha^2}{k_{M_2}{}^2}=\frac{7}{1440} \, ; \qquad k_{M_2}=14.3\,\alpha \, .$$

This beam should next be loaded with —

$$\frac{\beta}{\beta_{max}}(pl^2+Pl)\frac{a}{C}=Pl\frac{a}{C}=4M_1\frac{h}{2C}=2\frac{k}{l}\frac{A}{A-B}\sqrt{BC}\frac{h}{C}=4k\frac{A}{A-B}\alpha$$

i.e., a single force $\frac{4\,kA\alpha}{A-B}$ acting at the centre.

$$\text{Maximum moment} = 1 = \frac{1}{4} \times \frac{4\,k\mathrm{A}\alpha}{\mathrm{A}-\mathrm{B}}\,; \qquad k_{\mathrm{P}_1} = \frac{\mathrm{A}-\mathrm{B}}{\mathrm{A}\alpha}\,.$$

$$\text{Maximum deflection} = \alpha^2 = \frac{1}{12} \times \frac{k\mathrm{A}\alpha}{\mathrm{A}-\mathrm{B}}\,; \qquad k_{\mathrm{P}_2} = \frac{12\,(\mathrm{A}-\mathrm{B})\,\alpha}{\mathrm{A}}\,.$$

The result of P only is then —

$$\frac{1}{\left(\frac{k_{\mathrm{M}}}{4.5}\right)^2 + \frac{k_{\mathrm{M}}}{\frac{\mathrm{A}-\mathrm{B}}{\mathrm{A}\alpha}}} + \frac{1}{\left(\frac{k_{\mathrm{M}}}{14.3\,\alpha}\right)^2 + \frac{k_{\mathrm{M}}}{\frac{12\,(\mathrm{A}-\mathrm{B})\,\alpha}{\mathrm{A}}}} = 1$$

and P *and* N together gives

$$\frac{\varkappa}{\pi^2} + \left(\frac{k}{k_{\mathrm{M}}}\right)^2 = 1\,.$$

Assuming $\frac{\mathrm{A}-\mathrm{B}}{\mathrm{A}} \sim 1$, $\alpha \sim \frac{1}{2}$ (as a special case) one gets

$$\frac{1}{\left(\frac{k_{\mathrm{M}}}{4.5}\right)^2 + \frac{k_{\mathrm{M}}}{2}} + \frac{1}{\left(\frac{k_{\mathrm{M}}}{7.15}\right)^2 + \frac{k_{\mathrm{M}}}{6}} = 1 \qquad k_{\mathrm{M}} = 5.1$$

$$\frac{\varkappa}{\pi^2} + \left(\frac{k}{5.1}\right)^2 = 1$$

$\varkappa =$	$-\pi^2$	-5	0	$+5$	$+\pi^2$
$k =$	7.2	6.3	5.1	3.6	0

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Résumé

L'auteur de ce mémoire établit les équations fondamentales de la stabilité au renversement latéral. Il établit des solutions approchées pour différents cas simples, par exemple : charges verticales ou charges normales; l'effet de la poutre elle-même (considérée comme une poutre rectangulaire de faible épaisseur) ou l'effet des semelles; et même l'influence des charges appliquées au-dessus de la ligne des centres. Finalement, l'auteur indique des formules applicables au cas où plusieurs influences agissent simultanément.

Zusammenfassung

Es werden die Hauptgleichungen des Kipproblems dargestellt. Eine Näherungslösung wird gefunden, indem eine Reihe von Spezialfällen untersucht wird, wie z.B.: lotrechte Belastung oder Normalkräfte; die Wirkung des Balkens an sich selber (idealisiert als schmaler Rechteckquerschnitt), contra die Wirkung der Flansche und ferner der Einfluss von Kräften, die oberhalb der Verbindungslinie der Schwerpunkte des Trägers angreifen. Schliesslich werden Formeln angegeben für den Fall, dass mehrere oder alle der Einzelfälle gleichzeitig wirken.

Summary

The leading differential equations governing the lateral stability of beams are put forward. An approximate solution is found by dividing the process into a series of " plain cases " such as vertical loading or longitudinal force; further the effect of the beam itself (corresponding to that of a narrow rectangular beam) against the special effect of the flanges; also the effect produced by the loading acting above the centreline of the web. Finally, formulae are produced giving the joint effect of several or all " plain cases " acting simultaneously.

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