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Inelastic behaviour and safety of structures

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Limitation of elastic behaviour

Methods applied in the analysis and design of structures are based on the assumption that structural materials are perfectly elastic; if they were, no structure would be safe, even under normal conditions of service. The real strength of a material is not so much in its capacity to resist, as in its ability to « yield ». However, as the performance of a structure in service depends upon its retaining permanently its initial shape, at least within a relatively narrow margin of tolerance, the extent and rate of yielding must be controlled. Thus, it is not only the ability to yield, but to yield by the right amount at the right time, that determines the performance of a structural material.

The observation of a linear relation between stress and strain in the tension test of such a material suggests that results of conventional methods of structural and stress analysis will be fairly accurate if the induced stresses remain within the range of the test. However, for any but the simplest case, the differences between the computed average stresses and the real, localised stresses occuring in a structure are considerable; it is, therefore, the very slight deviation from linearity in the stress-strain diagram that creates the conditions for the application of linear elastic theory in structural analysis, since this deviation is the expression of the property of the material to redistribute, by limited « inelastic » action, the high local stresses towards the level of the computed average stresses.

It is known that the higher the rate at which load or strain is applied, the more likely the occurrence of elastic (brittle) fracture; also, the more often a rapid load cycle is repeated, the more probable is it to produce fracture, if the load intensity exceeds a certain limit. In both cases the truly elastic material, if it existed, would be apt to fracture at a comparatively low load intensity, whatever its intrinsic cohesive strength. A small amount of inelastic action would do more to improve its performance than a substantial increase in the cohesive strength. In this case, however, it is less the extent, than the rate at which the inelastic action takes place, that determines the benefit resulting from it; inelasticity developing over a long period would have no effect. On the other hand temperature gradients produce high residual tensile stresses adjacent to structural welds. Unless partial relaxation of these stresses through inelastic action takes place subsequently, these points of residual stresses are most likely to bring about fracture in service at a relatively low intensity of the load applied. There is, however, no need for the rate of this relaxation to be very high, since considerable time will elapse before the weld is called upon to carry the full stress intensity.

A statically indeterminate structure, if it is perfectly elastic, becomes unsafe if the most highly stressed member or section is loaded to fracture. Yielding of this member or section by an amount of the order of magnitude of elastic deformation is usually sufficient to produce, in the course of loading, such redistribution of stresses within the structure that a load, substantially exceeding that under which fracture of the elastic structure would occur, can be safely supported. Such redistribution, however, does not only require a certain *amount* of inelasticity; it requires that the *rate* of yielding be at least as high as that of loading, since otherwise the structure would not be able to mobilise its inelasticity in time, before being destroyed as a result of its perfect elasticity.

Hence, while the presence of inelastic behaviour in general is essential to ensure the safety of structures designed under the assumption of perfect elasticity, this inelasticity has various aspects; it must be defined with regard to its basic character and to the effects it produces.

Inelasticity

Deformation is the visible effect of the transformation of energy taking place within a structure during application and release of external loads. When work W is done on a structure or part of it by the forces acting during the time interval dt, one part is transformed into kinetic energy W_K producing motion and the rest, by mobilising the internal reaction of the structure, appears as strain-work W_S, producing changes of dimensions and of shape. Under the usual assumption of negligible acceleration forces

$$\frac{dW_k}{dt} = 0 \text{ and } \left(\frac{dW}{dt}\right) = \left(\frac{dW_s}{dt}\right).$$

According to the laws of thermodynamics the total strain-work W_S is transformed partly into free potential energy W_F and partly into bound energy W_D; moreover, every real mechanical process is accompanied by an increase in the amount of bound energy. Hence

$$\frac{dW_s}{dt} = \frac{dW_F}{dt} + \frac{dW_D}{dt} \quad \text{and} \quad \frac{dW_D}{dt} > 0\,;$$

therefore the free energy

$$\frac{dW_F}{dt} = \frac{dW_s}{dt} - \frac{dW_D}{dt}$$

is bound to decrease unless the mechanical change of state is reversible $\left(\frac{dW_D}{dt} = 0\right)$. Equilibrium in the mechanical system is attained if the free energy W_F becomes a minimum, or if $\left(\frac{dW_F}{dt}\right) \rightarrow 0$.

The part of the applied strain-work stored up reversibly as potential energy produces deformation which is fully recoverable on load release; the remaining strain-work applied is dissipated into heat. This irreversible process produces the inelastic phenomena which are (1) viscous flow (creep), (2) elastically restrained inelastic deformation (hysteresis and after-effect) and (3) fragmentation of internal structure of the material (permanent set, work-hardening). Thus the increase in the bound energy of the system by the energy W_D dissipated into heat has three different aspects :

(1) Creep, which is the effect of relaxation on an atomic scale of high internal energies by spontaneous heat-energy fluctuations. It is, therefore, both temperature and time-sensitive and is produced in amorphous materials if the stored-up potential energy is dissipated without any restraint. Under constant load it increases with time, and the amount of inelastic deformation attainable is limited only by instability or acceleration of the process, causing separation.

(2) Elastically restrained inelastic deformation results, if the action of the load is opposed simultaneously by elastic and inelastic elements existing within the material or the structure, or produced by plastic slip in the course of energy application, without causing more than slight, localised changes in the internal structure of the material. The energy dissipation within the inelastic elements is accompanied by a gradual redistribution of the internal reactions from the inelastic into the elastic elements which, upon load release, causes a delayed, partial recovery of the inelastic deformation. The potential energy responsible for delayed recovery is being stored in the course of the redistribution of internal forces; although it remains mechanically recoverable and thus elastic, the recovery is delayed by the resistance of the inelastic elements; the amount of energy reversibly stored up at any moment after release of the applied strain-energy, is proportional to the amount of deformation which remains to be recovered at that moment. The inelastic deformations are necessarily of the order of magnitude of the elastic deformations. It is within this range that they increase with time at a decreasing rate, if the load is kept constant.

(3) Fragmentation of the internal structure is produced, if the material is unable to support the strain-energy applied in a state of equilibrium, unless it succeeds in mobilising the necessary additional reactions by changes in its crystaline structure (crystal-fragmentation and rotation);

after the limit of the essentially elastic reaction has been exceeded, substantial inelastic deformation takes place. In this process the applied strain-energy is expended irreversibly in the work of producing the changes in the internal structure of the material; when, upon load release, the material attempts to recover its initial shape, this tendency is, however, opposed by the newly formed internal structure, which in the process retains a certain part (according to observations about 15 to 30 percent) of the energy expended in the fragmentation, to form a potential « opposing » that of the applied loads. In metals this « work-hardening potential » is stored up as elastic, but mechanically irrecoverable, « latent » energy; it « blocks » any inelastic action under subsequent load-cycles within the range of the previously applied level of strain-energy. It is, however, not stable, and gradual relaxation through spontaneous heat-energy fluctuations takes place over relatively long periods, as the « work-hardened » system tends towards a more stable, lower energy level; increasing external temperatures, by intensifying the internal heat-energy fluctuations, are bound to speed up the process of relaxation, while a lowering of temperatures produces a retarding effect. In non-metallic materials, fragmentation of the internal structure does not produce an appreciable, « latent » potential, since it is accompanied by permanent local destruction of the cohesion of the material which, upon load release, may produce further fragmentation.

The « adaptation » or « self-strengthening » under load of a statically indeterminate structure produced by redistribution of forces or moments resulting from inelastic deformation within the redundant members or sections, is essentially a phenomenon belonging to the second group; because of the existence of a definite yield-limit in plasticised metal members or sections, the inelastic deformation is not fully recoverable even after long delay. Part of the applied strain-energy is therefore stored up in the form of a latent « work-hardening » potential of the structure. However, mechanically irrecoverable changes in the structure by *fragmentation* of the material can only be expected to occur, if the inelastic deformation produced by loading cycles gradually increases beyond the range of elastic deformations and reaches the true work-hardening range of the material. Creep and phenomena of hysteresis and after-effect are strongly dependent on time; fragmentation is practically time-insensitive. It is this difference which makes it possible to split the observed total inelastic deformation into its different constituents.

Significance of inelastic behaviour

The performance of a structure, expressed by its reaction under the action of external loads applied, can only be observed in terms of deformations, unless it has been damaged to such an extent that signs of fracture are apparent. Since this stage is too advanced in the direction of exhausting the ultimate resistance, as to be considered in an analysis of « performance », it is the relation between deformation, particularly inelastic deformation, and performance, which has to be relied upon as a key to the problem of structural performance, by providing an indication for what is usually termed « structural damage ».

« Structural damage » is a rather loosely applied term, which covers different aspects relating to the ability of a structure to perform its function. As this ability, considered over the expected period of service, can be defined in the two different terms of « purpose » and of « safety », these two aspects should be considered with regard to the correlation between inelastic deformation and structural damage : (1) inelastic deformation as a measure of damage to the *purpose* of the structure, to be termed « functional damage »; and (2) inelastic deformation as an indication of a reduction in the carrying capacity or the *safety*, representing the real « structural damage ».

Functional damage of a structure is usually defined directly in terms of either elastic or inelastic deformation, by setting tolerances of deflections, both transient and permanent, which are derived from the conditions the structure has to meet in service. Undesirable elastic deformations are reduced by increasing dimensions, although this simple method is not necessarily effective with regard to inelastic deformations. The relations between functional damage and inelastic deformation are self-evident; from this point of view the most undesirable type of inelastic deformation is creep since, as a result of the comparatively long periods of service of engineering structures, even extremely small creep-rates are bound to produce appreciable irrecoverable deflections, particularly under conditions of high dead-load stresses.

In analysing the relations between real structural damage and inelastic deformation, both the damage to stability and the damage to strength must be considered separately.

Since the stability of essentially compressed members or structures is a direct function of their rigidity, any type of inelastic deformation, producing an apparent reduction of the elastic modulus of rigidity, is bound to have a damaging effect. This is apparent from the results of the usual analysis of buckling in the inelastic range. Because of the lack of restraint, creep is again the most dangerous type of inelastic deformation since, under sustained loading, it produces a gradual reduction of stability with time. Increase of the deflection with time is, therefore, an indication of a decrease of stability. On the other hand if the rate of energy application by loading is considerably higher than the rate of dissipation, the inelastic deformation vanishes, and the stability is bound to increase towards Euler's limit. The stability under transient and impact loads is, therefore, higher than under sustained or slowly applied loads of the same intensity.

The strength or ultimate resistance of a structure can be expressed in terms of a limiting energy potential that can be stored up prior to fracture. Therefore, the rate $\left(\frac{dW_F}{dt}\right)$ at which this limit is approached, determines the rate at which resistance is being exhausted. Since

$$\frac{dW_F}{dt} = \frac{dW_s}{dt}\left(1 - \frac{\frac{dW_D}{dt}}{\frac{dW_s}{dt}}\right),$$

this rate is reduced with increasing rate of energy dissipation, that is with increasing inelastic deformation. The higher the proportion of applied energy dissipated, the smaller the proportion available to build up the free energy towards its ultimate limit.

It is, however, necessary to differentiate between the three *types* of energy dissipation, and to consider that, while viscous flow constitutes an energy-release by thermal fluctuations that does not affect the strength at all, and while the effect of inelasticity, restricted to the order of magnitude of elastic deformation causes beneficial redistribution of internal forces, it is the energy expended in fragmentation of the internal structure that either produces immediate local damage, as in non-metals or, as in metals, facilitates it, by blocking beneficial inelastic deformation through the formation of a « strain-hardening » potential. Thus, only this latter part of the dissipated energy and the inelastic deformation produced by it can be assumed to be a direct indication of structural damage. If this type of inelastic deformation appears and progressively increases under the loads applied, the resistance of the structure is being gradually exhausted; if, after it has appeared, its progress is checked by the reformation, due to work-hardening, of a continuous elastic network within the material or the structure, this is an indication that the progress of damage has been stopped. The different significance of inelasticity due to creep and due to plastic slip in design is the result of the fact that only inelastic deformation due to slip produces relief of peak stresses and stress-concentrations; creep does not affect the elastic stress distribution.

Analysis of performance

The analysis of performance and safety of structures by observation of their deformation requires the observation of behaviour both during the loading and unloading. In the process of loading, elastic and inelastic deformations appear superimposed; elasticity can only be defined and observed by unloading, which produces a purely elastic response of the structure, unless the period of unloading is long enough for after-effects to be noticeable.

The inadequacy of the conventional interpretation of structural performance in terms of inelastic deformation is well illustrated by the general specification-requirements for the load-test of reinforced concrete structures. These specifications generally distinguish between a total deflection, a recovery upon unloading and a permanent deflection, which is the difference of both; they usually stipulate definite figures for total deflection under a load sustained for a certain period, and for recovery after a certain time. Since concrete manifests both creep and delayed recovery, both of which produce permanent deflections, but have no injurious effect on the strength of the material, it is evident that the deflection cannot be an indication of the extent of damage produced within the structure. This damage can be expressed only in terms of that part of the permanent deflection which is being caused by local fragmentation of the internal structure, and which is the amount of permanent deflection remaining after the effects of creep and delayed recovery have been split-off. Moreover, it is not the absolute amount of this deflection itself, but its progressive change under consecutive load cycles which indicates, whether the strength and safety of the structure is expected to decrease. Thus it is essentially the *fatigue strength* of the structure which determines its performance and safety and which is indicated by the injurious part of the inelastic deformation.

This part of the deformation can be computed from observations of the total deflection and of the recovery of the structure at two time-intervals, if the rate of creep of the concrete has been determined by separate tests. By expressing, for each time of observation, the observed deflection and recovery in terms of the constituent deformations [Δ (observed) = Δ (creep) + Δ (after-effect) + Δ (fragmentation) + Δ (elastic)], a sufficient number of linear equations for the computation of the unknown constituents is obtained.

Résumé

Calcul de constructions et d'éléments de construction en se basant sur le comportement parfaitement élastique du matériau. La seule garantie efficace pour une sécurité suffisante est toutefois son comportement dans le domaine plastique qui assure une réserve d'adaptation pour des sollicitations exagérées.

Le comportement plastique des matériaux de construction et des constructions elles-mêmes peut s'exprimer par une répartition de l'énergie de déformation inélastique et présente divers aspects. Chaque type de déformation inélastique présente, suivant le procédé de construction et suivant la sécurité exigée, une signification différente, car chaque type présente une forme de dissipation de l'énergie essentiellement propre à son système. L'auteur examine pour terminer la relation entre les déformations plastiques et les dégâts occasionnés aux constructions.

Zusammenfassung

Konstruktion und Konstruktionsteile werden berechnet und deren Spannungen bestimmt unter der Annahme eines vollkommen elastischen Verhaltens des Materials. Der einzig wirkliche Schutz für eine genügende Sicherheit im Gebrauch ist jedoch ihr unelastisches Verhalten, welcher die nötigen Reserven für die Beanspruchung und Anpassungsfähigkeit sichert.

Das unelastische Verhalten des Konstruktionsmaterials und der Konstruktionen drückt sich aus in der Art der Verteilung der enthaltenen Formänderungsenergie und hat verschiedene Erscheinungsformen. Jede Art von unelastischem Verhalten hat, je nach bautechnischer Anwendung und verlangter Sicherheit, eine verschiedene Bedeutung, weil dies zu einem andern Streuungsprozess der Formänderungsenergie in Wärmeenergie führt; die Wechselwirkung zwischen unelastischer Deformation und Bauschäden wird untersucht.

Summary

Structures and structural parts are designed and their stresses determined on the assumption of perfect elastic behaviour. However, the only real safeguard of their satisfactory performance in service is in their inelasticity, which provides the necessary reserves of strength and of adaptability.

The inelasticity of engineering materials and structures is the expression of the mode of dissipation of the input strain-energy and has various aspects. Each type of inelasticity has a different significance as far as structural performance and safety is concerned, since it pertains to a different process of dissipation of strain-energy into thermal energy; the correlation of inelastic deformation and structural damage is analysed.