Design of concrete mixes for bridge and other constructions

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Design of concrete mixes for bridge and other constructions

Etude des mélanges de béton pour ponts et autres ouvrages

Bestimmung der Betonmischung für Brücken und andere Bauwerke

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INTRODUCTION

The lack of a concrete technology based upon recognised scientific principles is a serious deterrent to advances in the economic design of structures and the release of some other structural materials which may be in short supply. Furthermore, there is a grave lack of basic knowledge of concrete as a material and it must be supposed therefore that many of the assumptions made for purposes of design fail to accord with the true characteristics of the material, demonstrable both under carefully controlled conditions and in the field. It is noteworthy that while much attention is given to the design of structures, stress analysis and even the metallurgy of steel in the university training of civil engineers, little more than an hour or so is devoted to the subject of concrete during the whole of the student's academic career. Nevertheless, on a basis of what can be but the most superficial knowledge of the behaviour and characteristics of concrete this same student will be expected to produce a plain or reinforced-concrete structure in which the design loading may be partially or wholly borne by the concrete, as well as any incidental loads due to local conditions and uncalculated characteristics of the material itself.

The advent of prestressed concrete demands the development of a much more complete and realistic concrete technology based on a properly co-ordinated and planned research. Much of the research that has taken place over the last forty years is worthless, either because sufficient and close control of the experimental conditions was not maintained or because the various pieces of research work, while good in themselves, could not be co-related owing to the lack of a common basis of comparison.

The comparison on a common basis of one concrete with another is extremely difficult. To appreciate this consider one aspect only, the degree of compaction. Obviously it is desirable that in comparing concretes generally, the degree of compaction to which the concretes are brought should be the same and, for preference, should be that of full compaction. The accurate determination of the fully compacted state has presented some difficulty, particularly in regard to hand-placed concrete, but the introduction of mechanical placing as represented by vibration has simplified this problem. The method and apparatus employed to this end are referred to later.

It is well known that the strength of concrete is greatly affected by the presence of voids. This source of variation may be eliminated from experimental results by always working with concrete specimens that are fully compacted. If, however, only fully compacted concretes are to be used, then it is clear that the workability of the concrete becomes an important factor, since the amount of work required to produce full compaction in one mix may be quite inadequate in bringing about the same state in another. However, the same apparatus used in determining the state of full compaction will at the same time measure the workability of the concrete in terms of the time taken to reach that state.

A considerable research programme has been carried out at London University and at Durham. This work has made it evident that where a relatively large volume of concrete can be compacted as a whole by vibration, as distinct from the very localised and indefinite effect normal in hand ramming, a mix may be designed on a mathematical basis instead of relying upon the wasteful and arbitrary proportions so commonly employed. The theory and practice of this mix design was worked out in the laboratory and then applied on a considerable range of jobs with excellent results. The only assumption made in postulating the theory is that with suitable and adequate vibration any correctly designed concrete mix may be fully compacted.

EFFECTS OF VIBRATION

It has been apparent to many engineers that the use of intense vibration will bring about segregation in a concrete if the treatment is continued for a sufficient length of time. Because of this reaction of concretes to vibration many have assumed that vibration is either destructive of good concrete or if it is applied at all it should be for very short periods only. These observations are perfectly correct but the deductions made from them are unsound. The action of the vibrators is to reduce the space between all solid particles in the concrete, in fact to reduce the volume occupied by the mix when first introduced into the mould. The closing of the voids forces out those materials in excess of requirements for the formation of a fully consolidated concrete. Clearly, those materials of the lowest viscosity will be the most easily displaced and as a rule sand, cement and water in the form of mortar come to the surface of the mix together with entrapped air. The term consolidated has been used here intentionally and not as an alternative to compaction. The sense in which it is used is the same as in soil mechanics, namely, to indicate a state of density attained by the changed proportions of the ingredients of a soil system subjected to a compacting force or pressure.

Mortar is not the only ingredient of the concrete which may be in excess. It is quite possible to have an excess of coarse material present which will separate out under the action of vibration. The accompanying diagram (fig. 1) indicates the three states of mix proportions which can arise.

The aim of vibration is, in fact, to produce a controlled segregation of air and surplus water as a secondary effect resulting from the primary object of bringing about the consolidation of the concrete. If this object is to be achieved without the undesirable features associated with the existence of a disproportionate amount of one or other material in the concrete, it is evident that the mix must be designed to meet specific requirements. These requirements will be influenced by four main considerations:

(1) The placing conditions.
(2) The type, grading, shape and surface texture of the aggregates.
(3) The method of compaction.
(4) The particular characteristics required of the concrete.

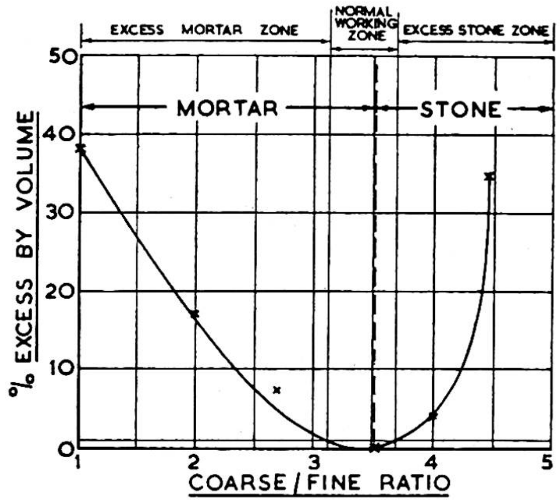

Fig. 1

PLACING CONDITIONS

The type of concrete, its workability, degree of coarseness and richness, and hence the mix design, must be greatly influenced by the volume to be placed and the general configuration of the formwork. That is to say, whether it encloses large free areas as in mass work or relatively complicated, narrow, and densely-reinforced members. The depth of section may have an important bearing upon the workability of the concrete, particularly if the steel density in the lower part of the member is great because, when it is impossible for the placing gang to see the flow of the material about and between the bars, special precautions must be taken to increase the workability at these points so as to ensure the complete filling of the whole of the enclosed volume. In circumstances such as these the mix must be specifically designed by the engineer to meet these requirements. Adjustments to the workability, as may be necessary, should not be left to the mixer driver, as is at present common practice, because this reduces the level of control and cannot be considered good engineering.

In mass concrete special conditions arise which permit the use of a large mean size of aggregate, as well as appreciably leaner mixes of lower workability for any specified strength. This is because in large volumes the aggregates are little restricted in their action under the influence of the vibrators, and are able to re-orient themselves more easily and completely in relation to their neighbours than they can in more restricted placing conditions. As a consequence the density of packing is brought closer to the upper limit, with the result that the void ratio is greatly reduced and with it the quantity of cement paste required. This permits the use of less water or cement, or both, depending upon the ultimate characteristics demanded of the concrete and economic considerations generally.

SIZE OF AGGREGATES

The degree of restriction within the formwork must decide the choice of maximum as well as mean particle size. Two main considerations govern this choice. In the first place, the distance between shutters must be so related to the maximum size as to ensure freedom for the vibrator to bring about redistribution and re-orientation of the coarse aggregate. In the second place, the mean size of the coarse aggregate must be such as to have a free passage between bars and through the cover-space between the formwork and the bars. The reinforced-concrete designer should remember that his design and steel layout, not forgetting enforced overlaps and intersections, will have a very profound influence upon the required workability of the concrete, for the reasons stated above, and upon the free use of internal vibrators in the process of placing.

TYPE OF AGGREGATE

The important bearing which the type of aggregate, its grading, shape and surface texture has upon the behaviour of freshly mixed concrete, as well as upon its post-curing qualities, is not generally understood. It is therefore hardly surprising that the science of mix design is seldom applied even where the bulk of concrete being placed would fully justify its employment in the interests of economy and quality of products.

Considered from the aspect of workability alone, since this is dependent upon the internal friction of the mix, it is obvious that the surface area and surface texture of the aggregates must play an important part. The shape and surface texture both influence the angle of repose of a particular aggregate and this again affects the workability of the concrete made from it. The size of aggregate is important because of the relationship between surface area to mass and its effect upon the number and size of the individual voids in a given volume of the material. The surface texture of most crushed-rock aggregates is rough and therefore able to carry a larger weight of water per square centimetre of projected area than is carried by a crushed-flint aggregate, the surface of this material usually being smooth. Although the weight of water present on the former is larger than on the latter the effective amount of water available for lubrication is probably no greater, and may be less in the case of a granite or whinstone than for a crushed flint. The workability of any concrete for a given water content increases as the effective surface area of the mix is decreased, therefore it should be the object of the designer to obtain a smooth, rounded aggregate, having as large a mean size as possible, so as to reduce to the minimum the lubrication requirements of the mix and hence the value of the water–cement ratio. In this way for a given aggregate–cement ratio, a concrete can be produced which will possess one of the three following alternative properties:

(i) a high compacting factor or workability,
(ii) a high compressive strength,
(iii) a low cement content for a particular strength

or a compromise may be made in which the speed of placing is somewhat reduced and a satisfactory compressive strength is attained accompanied by an appreciable saving in the use of cement.

Compressive strength is, however, but one of the qualities of concrete which interest the designer. It is one which gives perhaps the least trouble to attain, as, except for certain prestressed work, the highest minimum values demanded in general civil and structural works are easily within the capacity of concretes much leaner than those generally specified. In practice the designer's choice of aggregate characteristics must be guided by such considerations as availability and the principal function that the finished concrete has to perform. For instance, whether a high modulus of rupture is desirable as in road and runway pavements, or it may be that the concrete is to form an impervious skin to protect steelwork. He must consider the effect of thermal variations in relation to the development of internal stresses in the concrete itself, or of external forces applied by it to another part of the structure. Again some aggregates possess a high absorption and their use so reduces the workability of the concrete that insufficient compaction is attained, giving rise to a reduced compressive strength and leaving the concrete, where exposed, vulnerable to frost attack.

In designing a concrete mix the bulk density of the coarse aggregate is of very great importance as also is the grading. Although the bulk density cannot be determined from the grading, the designer must know the distribution of the particle sizes in the mix, since the size of the voids depends upon the size of the particles enclosing them. For instance, the bulk density of a cubic foot of $1\frac{1}{2}$ in. to $\frac{3}{4}$ in. aggregate may well be the same as that of an equal volume of $\frac{3}{4}$ in. to $\frac{3}{8}$ in. aggregate derived from the same rock and having the same general shape; but whereas the number of voids in the former might be of the order of 700, those in the latter will not be less than 4,000. It is therefore illogical to use the same grading of filler for these two aggregates, since, while that which would satisfactorily fill the $\frac{3}{4}$ in. to $\frac{3}{8}$ in. aggregate could be used also to fill the larger voids, the total surface area of this small-sized filler would materially reduce the workability of the resulting concrete to below that which could have been attained by the use of another filler having a more suitable size relationship; that is to say, a coarser sand.

Thus in designing a mix it is evident that the grading and mean particle size of the sand must receive the engineer's close attention. It is also evident that if the sand is to act as a filler, then all its particles must be able to pass into and through the interstices of the coarse aggregate; otherwise, during the process of compacting, the sand particles of excessive size will wedge the large stones apart, upsetting the initial bulk density of this material and rendering the design data invalid. In most present-day specifications it is the practice to call for what is termed a good concreting sand. Such a sand is described in the British Standard 882/1944 as a Class A concreting sand. On inspection it will be found that the grading ranges from 95% passing $\frac{3}{16}$ in. sieve to 10% passing sieve No. 100. Such a sand cannot make a satisfactory filler for coarser aggregates if their mean size is less than $1\frac{1}{2}$ in., that is to say, an aggregate graded between $2\frac{1}{2}$ in. and 1 in. Both experience and calculation have shown that for the range of aggregates in more general use the more important sand sizes are those lying between No. 25 and No. 100 B.S. sieves.

METHOD OF COMPACTION

The method of compacting the concrete has a very important bearing upon the design of the mix and particularly upon the ratio of sand to stone. The two principal methods in general use are hand ramming and vibration. Concretes which are to be hand rammed must possess a high workability in order that the formwork may be adequately filled and the reinforcement thoroughly embedded. The degree of compaction normally attained by manual placing is seldom if ever as high as that reached when vibration is employed. In consequence the weight of coarse aggregate per unit volume of concrete will be lower in hand-rammed concrete and hence the quantity of filler required in the form of mortar will be greater. This mortar may be made either rich or lean depending upon the workability and strength required of the concrete. Because of the higher mortar content mixes suitable for or designed for hand placing are intrinsically less workable than mixes designed for vibration, because of the larger volume of sand required to fill the less densely packed coarse aggregates; the resulting increased surface demands added water for a given workability, but as hand ramming is also less efficient in compacting the concrete than vibration a further increase in workability must be obtained by the addition of still more water so as to lubricate the appreciably larger surface area of the combined aggregates caused by the increase in sand content.

The object of compaction is to convert the freshly placed concrete into a dense homogeneous mass. Such a mass forms a completely related system of particles having a more or less uniform distribution of particle sizes. It is clear, therefore that whatever process of compacting is employed the best results will be attained by treating the mass as a whole, or where subdivision of volume is essential then these volumes should be of the largest size practicable. Hand ramming is a process which is very localised in its action and effect, and since the system of particles as a whole becomes more and more continuous as its density increases, the local groupings will be badly upset by the insertion of the tamping bar, and the accompanying regrouping which takes place at the point of insertion is bound to influence the orientation and distribution of particles in the immediate neighbourhood. While the same criticism may be made of the internal vibrator, the degree of local disturbance at the point of entry or withdrawal is small compared with the very much larger mass of concrete treated at one time by a vibrator, and consequently the disintegrating effect of the vibrator's insertion or withdrawal is virtually negligible compared with that caused by a tamping bar. Furthermore, by withdrawing the vibrator slowly a redistribution and re-alignment of particles takes place. Such an adjustment cannot take place on the withdrawal of a tamping bar except in very wet mixes.

The mechanism of vibration as applied to concrete has been dealt with in some detail by the author in his book *The Design and Placing of High Quality Concrete.* It is shown that the effect of vibration is to reduce the fraction between particles and, by gently disturbing the mix, to break down arching so that there is a general collapse of the particles. The work done in this process is proportional to the change of centre of gravity from the freshly placed condition to the compacted condition. As compaction proceeds the coarse aggregates close up more and more, forcing out from between them the more fluid components of the mix. The same phenomenon of concentration takes place in the mortar which, in turn, expels surplus water and some cement particles where these are in excess. Thus as compaction proceeds the amount of lubricating water available increases with a consequent increase in the rate of compaction.

Advantage may well be taken of the segregation of the mixing water under the influence of vibration, particularly in difficult placing conditions where a high workability is specially desired, this being attained by the addition of an increased quantity of water. This additional water, provided it is not excessive, will be brought to the surface of the concrete where it cannot produce undesirable void space. Because of the capacity of the vibrators to force surplus water out of a mix it is usual to find that the strength and other qualities of the concrete in the job are better than those in the test cubes where these are hand rammed.

PERIOD OF TREATMENT

It is essential that the vibrational treatment given to the concrete should be sufficient to overcome the internal friction of the concrete in changing from the uncompacted to the compacted state and the resistance offered to its flow by the shuttering and reinforcement. The period of treatment must depend upon the useful work done on the concrete in unit time by the vibrator, the number of vibrators employed, the volume of concrete treated and the workability of the mix. The configuration of the formwork, its overall dimensions and the density and disposition of reinforcement also influence the time required to bring about full compaction. It is undesirable to prolong the vibration beyond the time required to reach the compact state, not because the concrete will be harmed by over-vibration, for this is unlikely to happen in normal practice with a correctly designed mix, but because the continuation of vibration after compaction has been attained is uneconomical. The very short periods of vibrational treatment given to concrete in general practice are quite inadequate to the end in view, but are dictated by the fear that so-called "over-vibration" will cause segregation and loss of grout through shutter leakage. While these fears are justified to some extent, since segregation and grout leakage do take place in many cases, the cause is not over-vibration but the use of unsuitable mix proportions and poorly constructed shuttering. It is most unfortunate that in designing shuttering, shutter layouts, and working sequences, the fluidity produced in the concrete by vibration is frequently overlooked. In passing, it is interesting to note that shutters which do not leak quite liquid concrete when it is manually placed, fail to contain stiff mixes under the influence of vibration, a point which is not appreciated by those responsible for the manufacture and erection of formwork.

CHARACTERISTICS OF SET CONCRETE

The characteristics required of the set concrete vary with the purpose for which the concrete is intended. In many instances the predominant requirement is compressive strength developed in bending or as a result of direct loading, as in the case of some columns. The use of concrete in pavements demands two additional features besides compressive strength and resistance to impact: these are a low shrinkage coefficient and a high flexural strength. In prestressed work the concrete to be associated with the very high tensile steels must have all the previously mentioned qualities as well as a low coefficient of creep.

In concrete the modulus of elasticity under load increases with an increase in compressive strength. This means that for a fully compacted concrete the value of E increases with a reduction in the water–cement ratio. However, a low water–cement ratio may be obtained in a concrete having a very high cement content permitting, in turn, a proportionately large water content so that the mix is in fact quite wet. Although such a concrete may be easy to place it will, on hardening, develop appreciable shrinkage, and under the influence of a sustained load the creep will be found to be large. Both shrinkage and creep are closely associated with the total paste content of a unit volume of concrete; it is therefore very desirable that prestressed concrete should contain the largest weight of combined aggregates and as little cement and water as possible commensurate with obtaining adequate workability and strength.

The term "consolidated" has been used in describing the compactness of a concrete in which the quantity of mortar, including the water, is just sufficient to fill the voids in the coarse aggregate when the particles of this material are in intimate contact with each other. Such a compactness of the coarse aggregates can be effected by the application of vibration. Concrete of this type contains 10% to 15% less mortar, and hence cement paste, for a given strength than does that in more general use. It will be obvious that since the shrinkage and creep take place largely in the mortar, which contains the bulk of the paste, a reduction in the mortar present in the mix must materially reduce these movements. Since the coarse aggregates are in general contact, by virtue of the consolidated state of the concrete, the creep must be more closely related to the behaviour of the aggregates themselves under the influence of the load than to the mortar in these specially designed mixes.

SUMMARY OF MIX REQUIREMENTS

To sum up before embarking upon the method of design, the principal characteristics which the mix must possess are:

(1) The mean size of the coarse aggregate should be such as to permit the particles to pass between the shuttering and any immediately adjacent bars, and also between the bars.
(2) The mean size of the combined aggregates should be as large as possible, taking into account the condition imposed by (1) above, while at the same time ensuring the effectiveness of the sand as a filler for the coarse material; that is to say, the whole of the fine aggregate must be able to pass into and through the coarse fraction in its compacted state.
(3) The aggregate–cement ratio should be as large as possible while ensuring adequate workability to allow the attainment of the fully compacted state of the concrete, in the particular placing condition, with the water–cement ratio dictated by strength requirements.

MIX DESIGN

In designing a concrete mix, as in any other problem of design, assumptions, which may not be strictly true because they do not envisage some of the number of variable factors, have to be made for purposes of simplification. Yet in practice these assumptions approximate so closely to the truth on account of other compensating errors or conditions that calculations based upon them give reasonably workable and satisfactory results.

The particles of both coarse and fine aggregates as well as those of the cement vary enormously in shape and in the relationship of volume to surface area, so that an accurate mathematical approach to mean particle size, individual void volume, and surface area is out of the question. However, by the assumptions that the particles are spherical in form and that the mean diameter of particle in any one size group, i.e. those lying between two adjacent sieves such as $\frac{3}{4}$ in. and $\frac{3}{8}$ in., is equal to half the sum of the upper and lower limits, a system of mix design can be evolved which is applicable to the general run of aggregates, whether natural or crushed, provided these are of good shape as normally judged.

Working from first principles the proportions of a mix may be determined in the following way.

A cubic foot of fully compacted concrete will contain:

1 cubic foot of combined aggregate, and
P cubic feet of cement paste.

Now the amount of paste present will depend upon the aggregate–cement ratio and the water–cement ratio, which in turn depend upon the ultimate characteristics required of the concrete. If the aggregate–cement ratio and the water–cement ratio have been fixed by consideration of workability and strength, the specific bulk-density required of the combined aggregates may be calculated from the formula:

$$\text{Specific bulk-density} = \frac{y}{\frac{1}{d_c} + \frac{y}{d_a} + x} = Z^*$$

where
y=aggregate–cement ratio
x=water–cement ratio
d_c=specific gravity of cement
d_a=mean specific gravity of the combined aggregates

Now if K is the specific bulk-density of the coarse aggregate,

$$K = \frac{\text{Weight per cubic foot}}{62{\cdot}5},$$

then $(Z-K)/Z \times 100$ will give the weight of sand required expressed as a percentage of the combined aggregates.

It is important in making this calculation to choose a coarse aggregate of suitable bulk-density, otherwise the proportion of sand will be either too small or too great to produce a sound concrete. The value of the bulk-density required is that which will be attained in the coarse aggregate by the method of compaction to be adopted in placing the concrete. The value of the bulk-density will depend also upon the nature of the grading. It is evident that a coarse aggregate so graded as to have a very high bulk-density will demand a very small sand content, particularly if the mix is rich, i.e. a large volume of cement paste is present.

For instance, consider the case where the calculated specific bulk-density of the combined aggregate is 1·95 and the coarse aggregates graded from $\frac{3}{4}$ in. to $\frac{3}{16}$ in. have a bulk-density of 103 lb./ft.,^{3} that is, a specific bulk-density of 1·65; the percentage of sand required will be:

$$\frac{1{\cdot}95-1{\cdot}65}{1{\cdot}95} = 15{\cdot}4\%$$

It would not be impossible to make a sound concrete with this very low sand content, but because of the small mean size of the coarse aggregate and hence the very much smaller size of the individual voids in it, the sand would have to be extremely fine to pass freely through them, with the result that its surface area would be very large; as a consequence the weight of water required to make a workable concrete might well be much in excess of the permitted quantity based on the chosen water–cement ratio.

It is therefore most desirable to keep the size of the coarse aggregate as large as possible and reduce the range of grading so as to maintain sufficiently large individual voids to be able to accommodate sands ranging from B.S. sieve No. 14 to No. 100.

In practice it is best to use a coarse aggregate having a bulk-density of between 94 and 96 lb./ft.^{3} when suitably compacted, the equivalent specific bulk-densities being 1·5 to 1·54. These values will give sand contents ranging from 24% to 27% of the weight of combined aggregates for aggregate–cement ratios lying between 6 : 1 and 8 : 1. The coarseness of the sand will then depend upon the mean void size in the

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* For proof of this formula see *Design and Placing of High Quality Concrete*, by the author.

coarse material, and this can be determined most easily and with greater practical effectiveness by a special sieving operation carried out as follows.

Two B.S. $\frac{3}{16}$ in. sieves are selected having diameters which are large in relation to the maximum size of the coarse aggregate. One of the sieves is overloaded with the coarse material so that the stones obtrude above the upper edge. The other sieve is then placed on top of the first so that its lower edge is just entered into the upper rim of the loaded sieve. The two sieves are then forced together while being shaken vigorously until the stones between the upper and lower meshes are unable to shift. A known weight of sand, considered suitable for the mix, is then placed in the upper sieve and the sieves are then shaken and twisted as in the ordinary operation of making a mechanical analysis. If the materials are properly dried a proportion of the sand, depending upon its coarseness, will be found to pass right through the two sieves and the aggregate and that part which is of too large a particle size will be discovered lodged in the upper surface of the coarse material when the sieves are separated. An analysis of the sand passing through the sieves will give the maximum particle size which may be expected to act as an efficient filler without causing the coarse material to bulk. As a rule the diameter of the largest filler or sand particle will be found to be about 0·125*d*, where *d* is the size of a square opening through which the mean size of coarse aggregate particle would pass. Thus if the mean size of coarse aggregate lies between $\frac{3}{4}$ in. and $\frac{3}{8}$ in., the sand to be used in making the concrete should pass a sieve four sizes smaller; in this case No. 14.

The mean particle size will be given by:

$$\frac{\Sigma \text{ diameters of particles in sample}}{\text{Number of particles in sample}}$$

Consider a continuous grading from $1\frac{1}{2}$ in. to $\frac{3}{16}$ in., and let r_0 be the radius of the mean particle size in the first size group $1\frac{1}{2}$ in. to $\frac{3}{4}$ in. (group 0). Let N_0 be the number of particles in group 0, N_1 the number in group 1 (size $\frac{3}{4}$ in. to $\frac{3}{8}$ in.) and so on, and N_n the number in group *n*.

Let W=weight of sample; $p_0, p_1 \dots p_n$=the percentage by weight in the respective size groups.

Then the total number of particles is:

$$N=\frac{12^3}{62\cdot5}\times\frac{3W}{4\pi r_0^3 d_a}(p_0 8^0+p_1 8^1+p_2 8^2 \dots p_n 8^n)$$

the sum of the diameters of all particles is D_s

$$D_s=2r_0N_0+\frac{2r_0N_1}{2}+\frac{2r_0N_2}{4}+\dots \text{etc.}$$

$$=2r_0\left(\frac{N_0}{2^0}+\frac{N_1}{2^1}+\frac{N_2}{2^2}+\dots\frac{N_n}{2^n}\right)$$

The number of particles in any group is given by:

$$N_n=\frac{12^3}{62\cdot5}\cdot\frac{3}{4}\cdot\frac{\pi r_0^3 d_a}{W}\cdot p_n 8^n$$

D_s can now be written:

$$D_s=\frac{2r_0}{2^0}\times\frac{3Wp_0}{4\pi r_0^3}\times\frac{12^3\times 8^0}{62\cdot5d_a}+\frac{2r_0}{2^1}\cdot\frac{3Wp_1}{4\pi r_0^3}\times\frac{12^3\times 8^1}{62\cdot5d_a}+\dots\text{etc.}$$

$$D_s=\frac{13\cdot19W}{d_a r_0^2}(p_0 4^0+p_1 4^1+p_2 4^2+\dots p_n 4^n).$$

The mean particle diameter D_m is given by the equation:

$$D_m = 2r_0\left(\frac{p_0 4^0 + p_1 4^1 + p_2 4^2 + \;.\;.\;.\; p_n 4^n}{p_0 8^0 + p_1 8^1 + p_2 8^2 + \;.\;.\;.\; p_n 8^n}\right)$$

As an example take the following gradings of coarse aggregate:

Size group	$1\frac{1}{2}$ to $\frac{3}{4}$ in.	$\frac{3}{4}$ to $\frac{3}{8}$ in.	$\frac{3}{8}$ to $\frac{3}{16}$ in.
A % retained .	20	60	20
B % ,, . .	0	90	10
Value of N . .	0	1	2

For grading A:

$$D_m = 1{\cdot}125\left(\frac{20\times 4^0 + 60\times 4^1 + 20\times 4^2}{20\times 8^0 + 60\times 8^1 + 20\times 8^2}\right)$$

Thus the value of D_m is 0·37 in., and for grading B the value of D_m is 0·43 in.

Calculations of this kind may be criticised on the ground that the answers obtained may co-relate poorly with an exact determination of the mean aggregate size. But it is better to make an estimate based on sound mathematical principles rather than to make no estimate at all or alternatively guess at a value. The information gained from this investigation of the gradings has indicated two points of interest. Firstly, that the probable workability of grading A, given equal conditions, is less than that of grading B, in spite of having a larger maximum aggregate size. Secondly, if bar cover is restricted or the space between bars is one of the major considerations in determining the mean aggregate size, then in spite of the fact that 20% of the total weight in the sample is retained on the $\frac{3}{4}$ in. screen, grading A will give the best overall distribution of material in the process of placing. However, to complete the investigation the two gradings A and B should be plotted in the usual way. Having plotted these, the mean value should be drawn, when it will be seen at once on which side the larger weight of material lies.

It is necessary to introduce a note of caution here so that in examining the grading curves the beginner will not be misled by the apparently high percentage of material coarser than the mean. The important point to be considered is the actual number of particles above and below the mean size.

In the gradings under consideration the following conditions arise:

Grading	No. of particles		
	% above	% below	Mean
A	29·3	70·7	0·37
B	53	47·0	0·43

Thus it can be seen that the risk of a stoppage occurring during placing in using apparently coarser grading is very much less than the normal grading curve would suggest.

GAP-GRADING

The use of gap-grading in mixes designed for high quality or for greater economy in the use of cement has two main advantages. The first of these is directly related to the design aspect of the matter, while the second relates particularly to the quality control of the concrete.

From the design point of view the advantage of a gap-grading is that a coarse aggregate may be built up which has not only the requisite void ratio but also individual voids in it capable of absorbing the available sand grading. As has been indicated, a low void ratio, i.e. a high specific bulk-density, often requires such a small percentage of sand that except under special conditions the mix cannot be compacted. The gaps formed in the grading by the omission of certain sizes can be used to open up the coarse material so that it can accept a more workable volume of sand as well as sand of a coarser grading. The removal of particular sizes reduces the number of voids in the coarse aggregate and so increases their size for a given overall value. A typical gap-grading is shown in fig. 2.

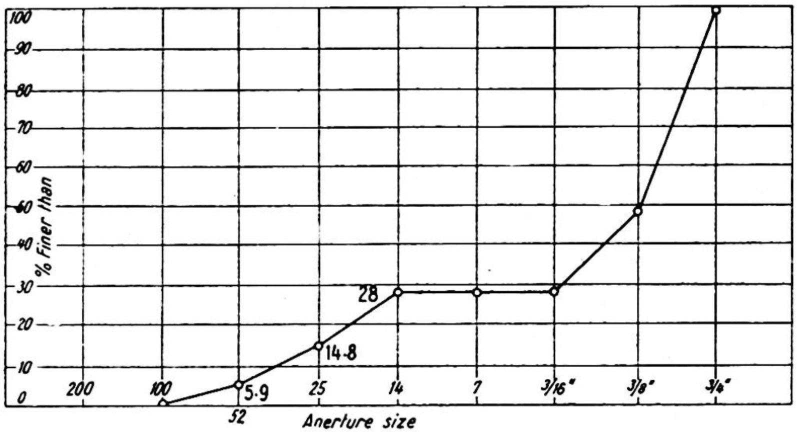


Fig. 2

As a rule the gap in the grading is obtained by the omission of one or more standard sizes or by introducing additional screens where the size of the contract warrants such a step. Usually very good results may be obtained by using one or at most two "single-sized" aggregates such as $1\frac{1}{2}$ in. to $\frac{3}{4}$ in. plus $\frac{3}{8}$ in. to $\frac{3}{16}$ in. if a fine sand is available graded B.S. No. 14 to 100, but where the sand is coarsely graded, $\frac{3}{16}$ in. to B.S. No. 100, then, the $1\frac{1}{2}$ in. to $\frac{3}{4}$ in. alone would be required. This use of single-sized aggregates materially reduces the possible variation in the grading of the coarse aggregate and hence in its bulk-density, and therefore very considerably assists in controlling the quality of the concrete and reducing the coefficient of variation without undue supervision.

The foregoing discussion on mix design will no doubt have given the impression that what has always been regarded as a very simple matter has been converted into an involved process without any easily apparent and worthwhile reward by way of structural economy in time or material. This is, in fact, not true, there being three main advantages to be derived from the engineering standpoint, each of which may be turned to economic gain.

(1) The use of vibration together with the rational design of the concrete mix makes it possible to obtain relatively high strength characteristics from the concrete while using a mix which is leaner by an appreciable amount than that normally employed. For instance, the British Code of Practice for Reinforced Concrete requires a mix of the proportions of approximately 5 : 1 by weight or 1 : $1\frac{1}{2}$: 3 by volume if a design stress of 1,250 lb./in.^{2} is to be used, and the concrete test cube must give a

strength of not less than 3,750 lb./in.^{2} at 28 days. If, however, a designed mix is used and placed by vibration, the same minimum strength can be achieved with an 8 : 1 by weight concrete. This produces a saving of 235 lb. of cement per cubic yard and at the same time increases the density from 146 to 151 lb./ft.^{3}, thus providing a more impervious concrete.

(2) A mix which has been designed upon the lines indicated in the earlier section of this paper will usually employ single-sized coarse aggregates, and in the majority of jobs only one of these. Thus, as has been pointed out, the grading control is greatly improved and simplified. The simplification of the weigh-batching of mixes appreciably reduces the variation in concrete quality that can arise at this point in the process, and when this is taken in conjunction with the more uniform and complete compaction obtained with vibration the overall variation is markedly less than that usually encountered in normal practice; probably the greatest source of variation will be that due to fluctuation in cement quality, particularly if this material is derived from more than one works. Where control at the site is that which might be expected on a contract involving £100,000, the coefficient of variation on the cube tests will be of the order of 10%. Thus it is reasonable to use a rather lower factor of safety in this type of concrete or the employment of a higher working stress for the same minimum cube strength.

(3) A higher quality of product of greater durability can be obtained at a lower initial cost.

The three main advantages are therefore a reduction in material costs due to the use of leaner mixes and higher working stresses in the concrete; often accompanied by a reduction in the volume of steel required in tension, compression and shear, and a reduction in costs of batching, mixing, and placing, since less material is required and the rate of compaction is about twice that in normal working per man employed. Finally a reduction may be expected in maintenance costs, an item which has become alarmingly high in reinforced-concrete structures of many types, some of which are not more than fifteen years old.

CONCRETE COMPACTION AND THE DESIGN OF MEMBERS

In order that the fullest advantage may be taken of the compacting effects of vibrators and the use of coarse low-workability mixes advocated by the author rather more than usual attention must be given to the design of members and the layout of the steel in them. The satisfactory transfer of the load from the concrete to the steel is dependent upon the bond between these two elements of the member. It is therefore most important to ensure that the concrete can be fully compacted round the bars and that air and water pockets on the underside of bars are eliminated. This compact condition can be assured only if the vibrator can be brought within effective range of the particular section of concrete under consideration. Fig. 3 shows two methods of steel distribution for identical load conditions. The bottom method will be recognised as that adopted in current practice, while the uppermost is an arrangement which lends itself to ease of assembly, placing of the concrete and manipulation of internal vibrators. The triangular cross-sectional arrangement (shown in fig. 4) of the bars and their sizes make it possible to maintain a more uniform stress distribution in the steel irrespective of the position of bars in relationship to the centroid. The maintenance of this steel cross-sectional area to centroid relationship when the bars are taken to the upper part of the beam where it is about to pass over supports does produce an added complication but this is a small inconvenience in view of the ease of placing offered by this type of layout. The spacing of the bars in the triangular arrangement is based upon the value of the mean size of coarse aggregate being not more than 0·45 in., a condition that can be obtained with single-sized 1½ in. to ¾ in. material having the grading tolerance allowed by B.S. 882/1944 below the ¾ in. sieve.

Figs. 3 and 4 show the effect upon the economics of beam design where full advantage is taken of mix design and vibration. The design was taken out upon a simply supported beam having a 20-ton load at the centre of its 60-ft. span. The data given cover design stresses in steel and concrete, bending and resistance moments, steel areas and concrete mixes. The design of the beam which takes advantage of the scientific proportioning of mix and the placing of that mix by vibration has been called "rational design," for want of a better description, while the other is referred to as "normal design." The savings made by the "rational" method are as follows: reduction in dead load 22·2%, giving rise to a reduction of volume of steel of 8·5%, and a saving of just over 36% in cement consumption due to a smaller quantity of leaner concrete being used to obtain a 9% increase in design stress above that used in the normal procedure.

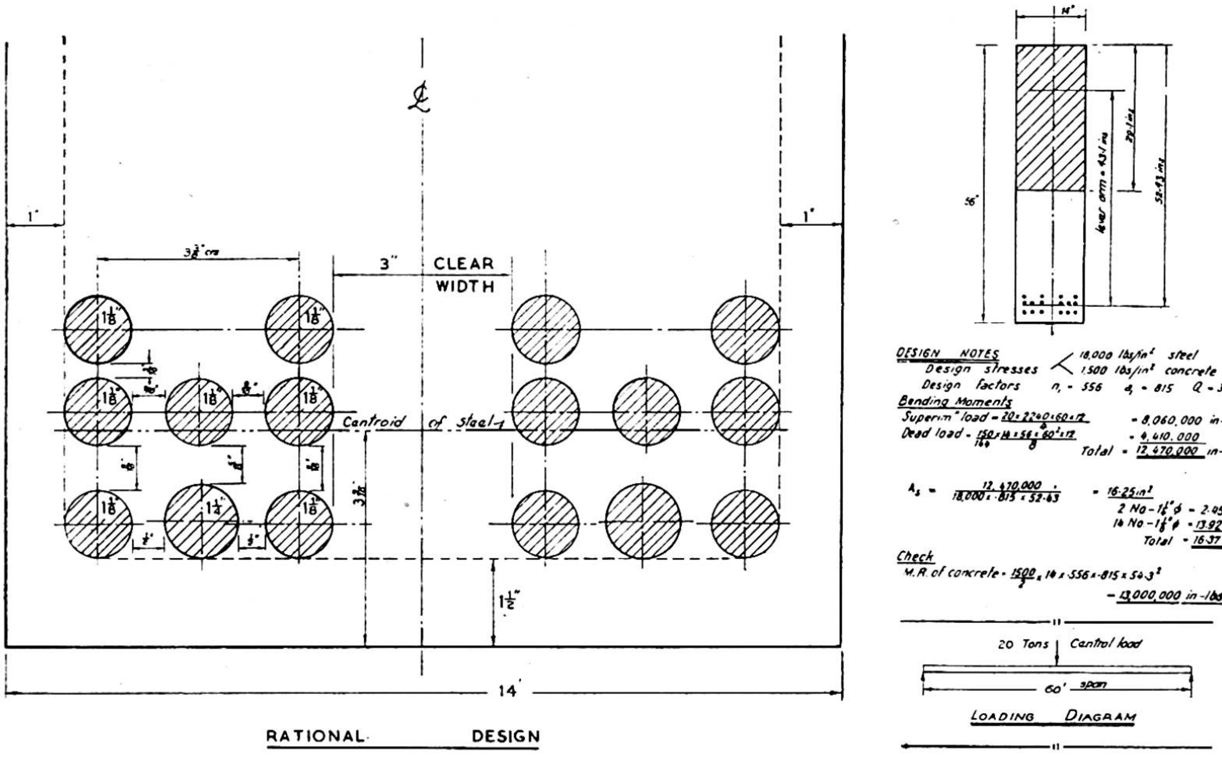


Fig. 3(*a*). The comparison of beam design methods

Advantages of rational design:

For a constant superimposed central load of 20 tons the following savings in material are made possible:

(1) Steel	8·5%
(2) Dead load	22·2%
(3) Cement	36·2%

Mix details: *

Aggregate-cement ratio	1 : 8 (by weight)
Water-cement ratio	0·5 to 0·525
Maximum sized aggregate	1½ in.

* See written instructions for complete details of gradings, etc.

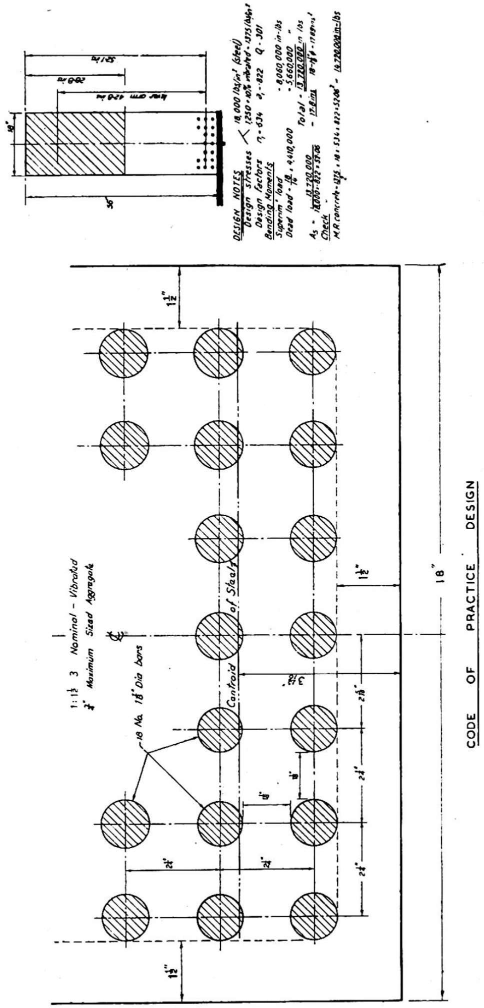


Fig. 3(*b*). The comparison of beam design methods

DESIGN DATA

Normal

Design stresses:

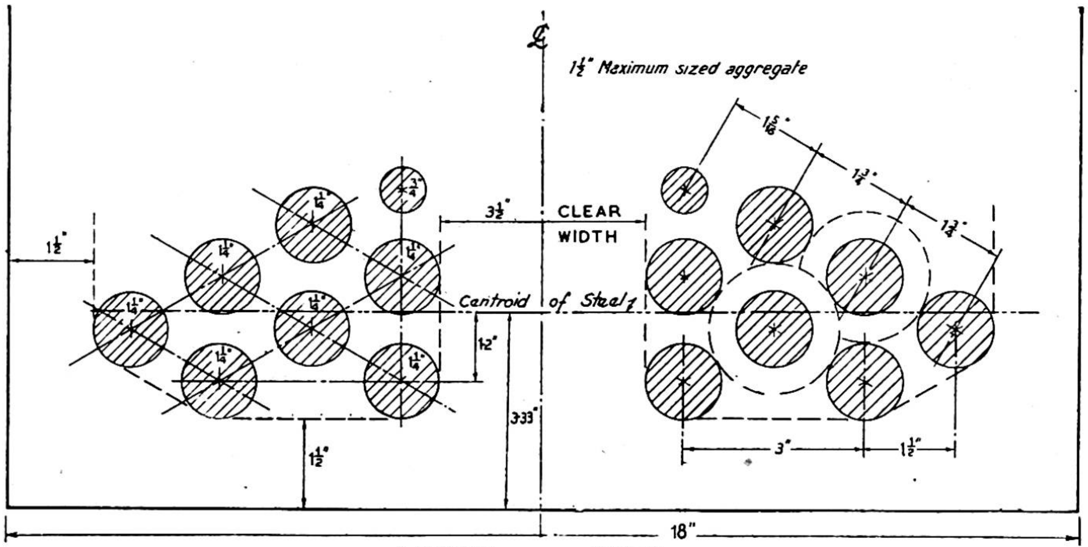
Steel	18,000 lb./in.^{2}
Concrete	1,250+10% vibrated=1,375 lb./in.^{2}
Lever arm factor	0·822 $\Big\}$ Q=301
Neutral arm factor . . .	0·534

Bending moments:

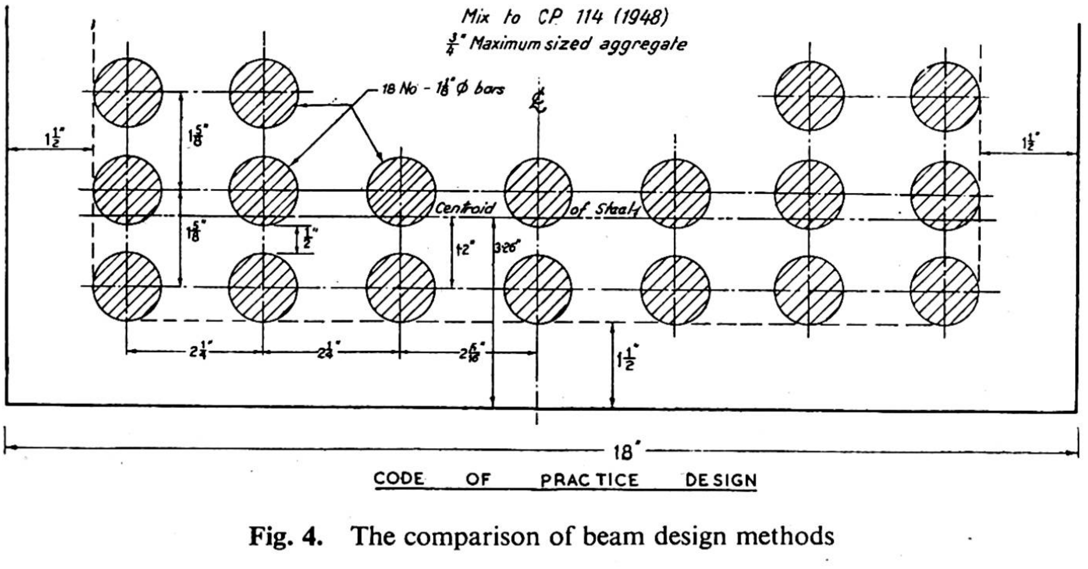
Superimposed load		8,060,000 in.-lb.
Dead load	$\frac{150}{144}\times\frac{18\times56\times60^2\times12}{8}=$	5,660,000 in.-lb.
	Total	13,720,000 in.-lb.

$$A_s=\frac{13{,}720{,}000}{18{,}000\times0{\cdot}822\times52{\cdot}06}=17{\cdot}8\text{ in.}^2\ (18-1\tfrac{1}{8}\phi=17{\cdot}89\text{ in.}^2).$$

Check: M.R. concrete$=301\times18\times52{\cdot}06^2=14{,}720{,}000$ in.-lb.



RATIONAL DESIGN



CODE OF PRACTICE DESIGN

Fig. 4. The comparison of beam design methods

Notes.—The beam section given in the upper detail contains the same area of steel as the lower but has been arranged so as to facilitate the concrete placing by vibration. The centroid of the steel has been maintained at the same level.

It is practical to use an aggregate having a suitable grading of 1½ in. maximum size for the "Rationally Designed" beam, whereas the maximum allowed for in the Code of Practice beam is ¾ in.

Rational

Design stresses:

Steel	18,000 lb./in.^{2}	
Concrete	1,500 lb./ in.^{2}	
Lever arm factor	0·815	$Q=340$
Neutral arm factor	0·556	

Bending moments:

Superimposed load		8,060,000 in.-lb.
Dead load	$\frac{14}{18}\times 5{,}660{,}000=$	4,410,000 in.-lb.
	Total	12,470,000 in.-lb.

$$A_s=\frac{12{,}470{,}000}{18{,}000\times 0{\cdot}815\times 52{\cdot}43}=16{\cdot}25\text{ in.}^2$$

2 No.—$1\frac{1}{4}$ in. $\phi=$ 2·45 in.^{2}
14 No.—$1\frac{1}{8}$ in. $\phi=$13·92 in.^{2}

Total 16·37 in.^{2}

Check: M.R. concrete$=340\times 14\times 54{\cdot}3^2=13{,}000{,}000$ in.-lb.

It is the author's hope that this paper will stimulate sufficient interest in the very real problems of proportioning, mixing and placing concrete to bring about the development of a modern concrete technology.

Summary

The paper points out the need for a more enlightened concrete technology and goes on to analyse the factors influencing the characteristics of concrete, such as: type of aggregate, the effect of the maximum size of aggregate, and the percentage of sand present in the mix. Placing conditions and methods are examined in relation to mix proportions, workability and aggregates and it is shown that mixes can be designed on a mathematical basis, particularly where vibration is used. The effects of vibration are examined in relation to the characteristics of the newly placed and set concrete and special attention is called to the economies that can be effected by mix design, vibration compact and the correct distribution of the reinforcement in members. In one example it is shown that the following savings can be made in a beam of 60-ft. span, carrying a 20-ton central load:

Reduction in dead weight	22%
Reduction in volume of steel	8·5%
Reduction in weight of cement	36%

The savings quoted are made on the quantities that would have been used if standardised design procedure had been adopted and it is shown that the method advocated by the author would tend to give a better quality product than would be expected normally.

The paper is based on the author's research and his work on a number of large contracts carried out in the last five years.

Résumé

L'auteur montre la nécessité d'étendre la technologie du béton. Il passe ensuite à l'analyse des facteurs qui exercent une influence sur les caractéristiques du béton après solidification, telles que la nature de l'agrégat, l'influence de la dimension maximum de ses éléments et le pourcentage de sable dans le mélange.

Les conditions et procédés de mise en œuvre sont examinés corrélativement aux proportions du mélange, à sa maniabilité et aux agrégats eux-mêmes. L'auteur montre que l'on peut étudier les mélanges sur des bases mathématiques, tout particulièrement lorsque l'on a recours à la vibration. Il étudie l'influence de la vibration du point de vue des caractéristiques du béton fraîchement mis en œuvre et du béton après solidification et attire particulièrement l'attention sur les économies que l'on peut réaliser en étudiant avec soin le mélange, la compaction par vibration et la judicieuse répartition des armatures dans les membrures de l'ouvrage. Un exemple montre que sur une poutre admettant une portée de 18 mètres et portant une charge centrale de 20 tonnes, il est possible de réaliser les économies suivantes:

Poids mort	22%
Acier	8,5%
Ciment	36%

Ces économies sont basées sur les quantités de matériaux que l'on emploierait en adoptant des conceptions normalisées et l'auteur montre que la méthode qu'il préconise permettrait d'obtenir un béton de qualité meilleure que celui que l'on prévoit normalement.

Ce rapport est basé sur les recherches effectuées par l'auteur et sur les travaux qu'il a exécutés dans le cadre d'un grand nombre de contrats traités au cours des cinq dernières années.

Zusammenfassung

Der Verfasser verweist auf die Notwendigkeit, die Technologie des Betons weiter abzuklären. Er untersucht die Faktoren, die einen Einfluss auf die Eigenschaften des fertigen Betons ausüben, wie die Art der Zuschlagstoffe, die maximale Korngrösse und der Anteil an Sand in der Mischung.

Das Einbringen des Betons wird im Zusammenhang mit dem Mischungsverhältnis, der Verarbeitbarkeit und der Zuschlagstoffe untersucht. Es wird gezeigt, dass das Mischungsverhältnis auf mathematischer Grundlage bestimmt werden kann, besonders wenn die Vibration benützt wird. Der Einfluss der Vibration wird in Bezug auf den frischen und den fertigen Beton untersucht und besondere Aufmerksamkeit wird auf die Ersparnisse gerichtet, welche durch richtige Mischung, Verdichtung durch Vibration und genaue Verteilung der Armierung erzielt werden können. An einem Beispiel werden folgende Einsparungen an einem 18 Meter langen, von einer Einzellast von 20 Tonnen beanspruchten Balken nachgewiesen:

Verminderung an totem Gewicht	22%
Verminderung an Armierung	8,5%
Verminderung an Zementgewicht	36%

Diese Einsparungen beziehen sich auf die Baustoffmengen, welche für eine standardisierte Ausführung nötig gewesen wären. Der Verfasser zeigt, dass bei Anwendung der angegebenen Methoden ein besseres Qualitätsprodukt erzielt wird, als man normalerweise annehmen kann.

Die Abhandlung beruht auf Forschungen des Verfassers und auf Erfahrungen aus einer groosen Zahl von Arbeiten, die er in den letzten fünf Jahren ausgeführt hat.