Non-uniform shrinkage of concrete due to segregation of coarse aggregate

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Non-uniform shrinkage of concrete due to segregation of coarse aggregate

Hétérogénéité du etraitr du béton due à la ségrégation des gros agregats

Ungleichförmiges Schwinden des Betons als Folge der Absonderung von Grobkorn

TECHN. DR. HENRIK NYLANDER
Professor of Building Statics and Structural Engineering at the
Royal Institute of Technology, Stockholm

PRELIMINARY TESTS

In order to afford an experimental basis for the discussion of the counteractive effect produced by the reinforcing bars on the deformations due to shrinkage in reinforced concrete structures, the author made a series of tests on several reinforced concrete beams in 1948.

The loading devices and the measuring equipment used in the main tests are shown in fig. 1. In order that the influence of the weight of the beam on the deformations should be eliminated as far as possible, the beam was from the beginning subjected to concentrated loads P_1, P_2 and P_3 acting upwards and calculated so that the bending moments due to these loads and to the weight of the beam should be as small as possible. The value of K_{b_H} corresponding to the maximum moment, 5·9 kg.-m. was 1·2 kg./cm.^{2}

The reinforcement used in the test beams is shown in fig. 2. One of the beams was provided with three reinforcing bars, 8 mm. in diameter, at the bottom. The other beam was equipped with the same reinforcement at the bottom and, moreover, with five bars, 8 mm. in diameter, at the top extending over a length of 91 cm. in the central part of the beam.

To measure the effect of shrinkage, a force P acting in an upward direction was applied at the centre A of the beam. This force was controlled in the course of the test so as to obtain a constant zero deflection at the centre. By measuring this load P, it was possible to determine the transfer of moments due to the shrinkage of the beam which was carried on three fixed supports.

Check tests on non-reinforced and reinforced-concrete prisms, 15×15 cm. in

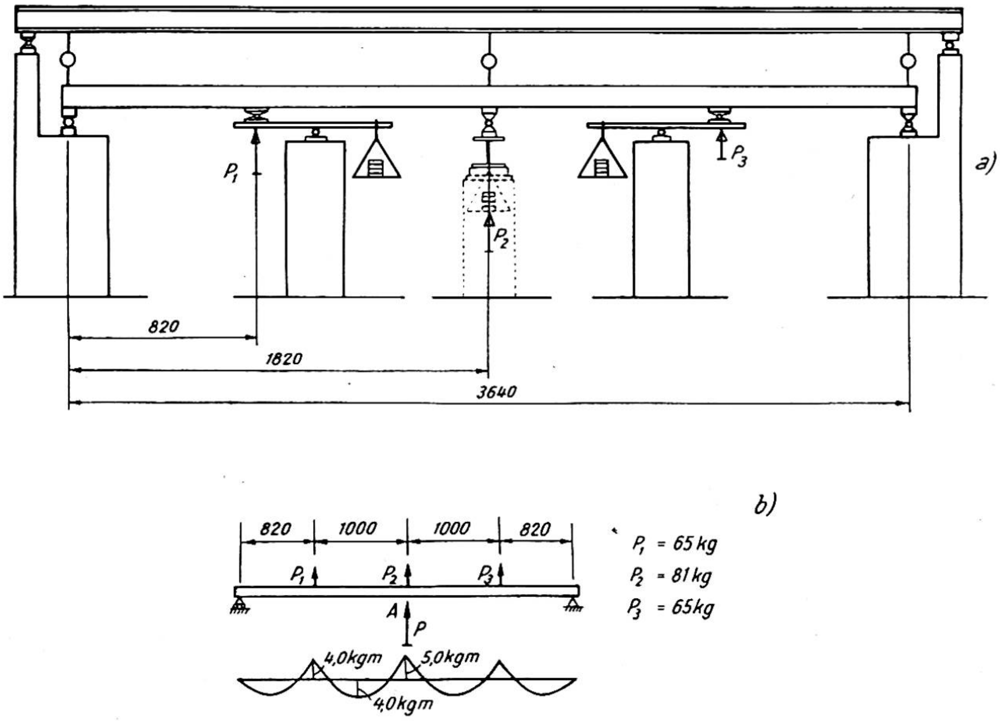

Fig. 1. Loading devices and measuring equipment for the shrinkage tests on concrete beams

cross-section and 90 cm. in length, were made at the same time as the main tests. The total number of prismatic test-specimens was eight viz. four non-reinforced prisms used for determining the coefficient of shrinkage, two prisms reinforced with four bars, 8 mm. in diameter, and two prisms reinforced with eight bars, 8 mm. in diameter, used for determining the coefficient of shrinkage under the counteractive effect of the reinforcement. In order to avoid disturbances in the state of stress at the ends of the concrete prisms, the reinforcing bars were welded to end plates, 30 mm. in thickness. Furthermore, each end plate was provided with twelve threaded round iron bars, 3 mm. in diameter and 75 mm. in length, with a view to ensuring the bond between the end plates and the concrete prisms.

The concrete mix used for the main tests and the check tests had the proportions cement: fine aggregate: coarse aggregate (maximum particle size 32 mm.) 1:4·1:6·1 by

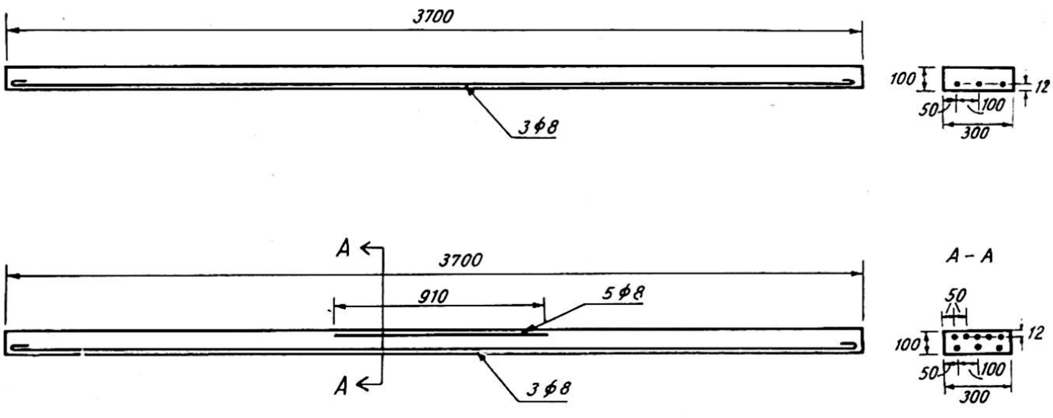

Fig. 2. Dimensions and reinforcement of the beam specimens used in the shrinkage tests

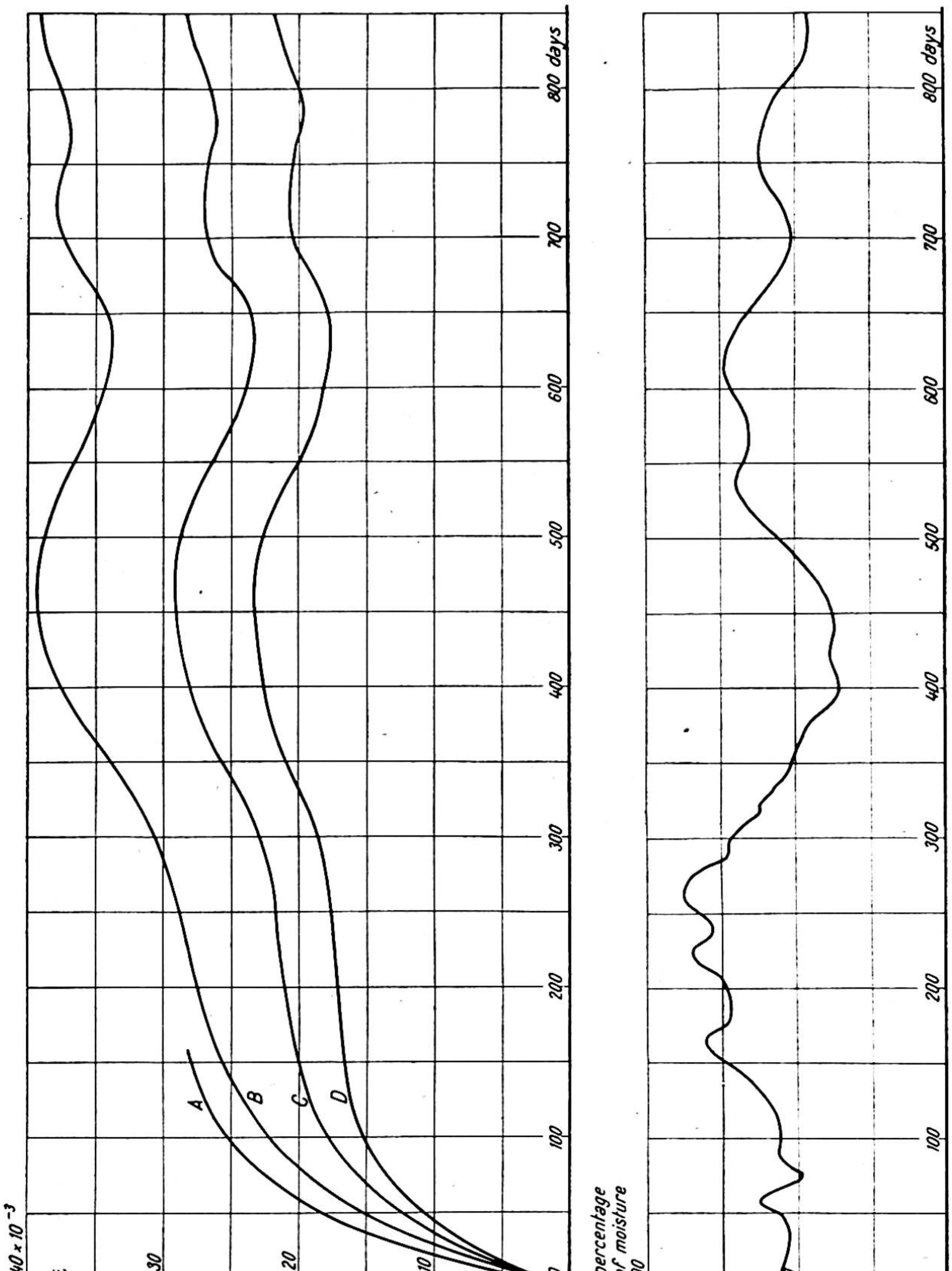

Fig. 3. Shrinkage of non-reinforced and reinforced-concrete prisms plotted as a function of time:

A. ϵ for a prism subjected to a compressive load of 5 kg./cm.^{2} (test results not utilised)
B. ϵ_0 for a non-reinforced prism
C. ϵ_{tot} for a prism reinforced with four bars 8 mm. in diameter
D. ϵ_{tot} for a prism reinforced with eight bars 8 mm. in diameter

weight. The water–cement ratio was 0·85. The workability was 3° to 4° VB. The cube strength determined after 28 days was 265 kg./cm.^{2}

The results of the check tests are reproduced in fig. 3.

In comparing the results and in applying them to other structures, it is convenient to use a simplified method of representation. The author has chosen to use the following simplified assumptions:*

(1) The shrinkage of the concrete is assumed to be independent of the stresses in the concrete.

* The basic phenomena of shrinkage and creep are discussed in a paper by Seed, H. B., "Creep and Shrinkage in Reinforced Structures," *Reinforced Concrete Review*, Jan. 7, 1948. Cf. also Nylander, H. "Korsarmerade betongplattor" (Concrete Slabs Reinforced in Two Directions), *Betong*, No. 1, p. 78, 1950.

(2) The plastic deformations are taken into account by assuming that the value of the modulus of elasticity of the concrete E_c varies with the time.

The author is of the opinion that this method of representation affords a possibility of comparing similar structures (but naturally not of describing the reality). The values to be determined experimentally are ϵ and E_c.

For the discussion of the results obtained from the tests on reinforced-concrete prisms, the following notations are used:

ϵ_{tot} the shrinkage of the reinforced-concrete prism
ϵ_0 the shrinkage of the non-reinforced-concrete prism
A_s the cross-sectional area of the reinforcement
A_c the cross-sectional area of the concrete
E_s the modulus of elasticity of the reinforcement
E_c the "effective modulus of elasticity" of the concrete

The condition that the tensile force in the concrete shall be equal to the compressive force in the reinforcement bars yields the equation:

$$(\epsilon_0-\epsilon_{tot})A_cE_c=\epsilon_{tot}E_sA_s \quad (1)$$

Hence

$$E_c=\frac{\frac{A_s}{A_c}E_s}{\frac{\epsilon_0}{\epsilon_{tot}}-1} \quad (2)$$

The values obtained from the curves shown in fig. 3 give the values of E_c reproduced in Table I as a function of time.

TABLE I

Shrinkage coefficients ϵ_0 and ϵ_{tot} obtained from check tests and values of E_c calculated from these coefficients

Time: days	4 bars, 8 mm. diameter			8 bars, 8 mm. diameter		
	ϵ_0 per thousand	ϵ_{tot} per thousand	E_c kg./cm.^{2}	ϵ_0 per thousand	ϵ_{tot} per thousand	E_c kg./cm.^{2}
30	0·105	0·089	105,000	0·105	0·078	110,000
60	0·170	0·138	81,000	0·170	0·122	82,000
90	0·212	0·171	78,000	0·212	0·148	85,000
120	0·238	0·188	72,000	0·238	0·158	75,000
150	0·255	0·199	67,000	0·255	0·165	69,000
180	0·269	0·206	63,000	0·269	0·169	64,000
210	0·280	0·210	57,000	0·280	0·173	60,000
240	0·285	0·212	55,000	0·285	0·175	59,000
300	0·306	0·228	55,000	0·306	0·184	55,000
400	0·365	0·281	62,000	0·365	0·223	58,000

The following notations are introduced for dealing with the results of the main tests, which were made on concrete beams reinforced at the bottom only:

E_s the modulus of elasticity of the reinforcement
E_c the "effective modulus of elasticity" of the concrete
e the distance from the centre of reinforcement to the centre line of the cross-section

H the total depth of the beam
A_s the cross-sectional area of the reinforcement
A_c the cross-sectional area of the concrete
ϵ_0 the shrinkage coefficient of non-reinforced concrete

If the method of representation outlined in the above is applied to the main tests, the moments M_0 at the ends of the beam are:

$$M_0=\frac{e\,E_s\,A_s}{1+\frac{E_s}{E_c}\frac{A_s}{A_c}\left(1+12\frac{e^2}{H^2}\right)}\epsilon_0 \qquad (3)$$

Hence the deflection at the centre of the beam is:

$$\delta_1=\frac{M_0\,.\,l^2}{8E_cI} \qquad (4)$$

The concentrated load P acting in an upward direction causes the upward deflection:

$$\delta_2=\frac{P\,.\,l^3}{48\,E_cI} \qquad (5)$$

Since the total deflection at the centre of the beam will be equal to zero, from eqns. (4) and (5) it follows that:

$$P=\frac{6}{l}M_0 \qquad (6)$$

where M_0 is determined from eqn. (3).

After the value of M_0 has been obtained from eqn. (3), P can be calculated from eqn. (6). The calculated values of P corresponding to different values of ϵ_0 and E_c are given in Table II.

TABLE II

Calculated values of P, in kg., for different values of ϵ_0 and E_c. Beam provided with simple reinforcement

E_c, kg./cm.^{2}	ϵ_0, per thousand			
	0·1	0·2	0·3	0·5
0·5 . 10^5	15·5	31·0	46·5	77·5
1·0 . 10^5	17·4	37·8	52·2	87·0
1·5 . 10^5	18·2	36·4	54·6	91·0
2·0 . 10^5	18·5	37·0	55·0	92·5

One of the beams used in the main tests was provided with reinforcement both at the bottom and at the top. For this beam, an analogous calculation yields $P=0{\cdot}27$ times the values reproduced in Table II.

Fig. 4 shows observed and calculated values of P.

It will be seen from fig. 4 that the value of P for the beam reinforced at the top and at the bottom is as high as that obtained for the beam reinforced at the bottom only (up to 100 days). This alone is sufficient to indicate that the counteractive effect of the reinforcement on shrinkage has not been the predominant factor. Furthermore, fig. 4 shows curves representing P calculated from the results of the check tests (the curve C refers to the beam reinforced at the bottom only, while the curve D refers to the beam reinforced both at the bottom and at the top). These curves were computed from eqns. (3) and (6), using those values of ϵ_0 and E_c which were obtained from the check tests after the corresponding number of days.

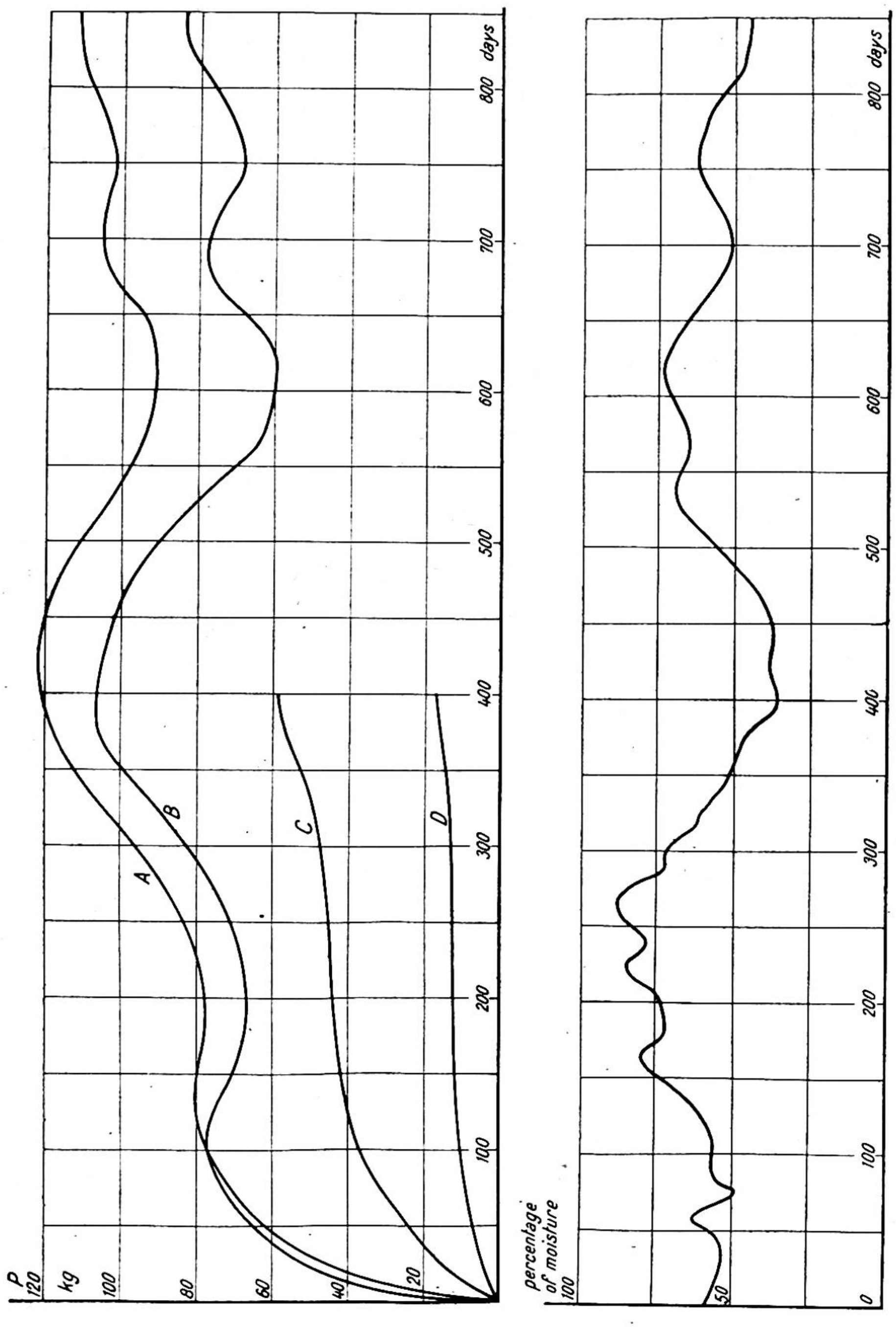

Fig. 4. Observed and calculated values of the concentrated load P (kg.) at the centre of the beam (A in Fig. 1) required for the deflection at the centre of the beam to be equal to zero.

A. Value of P observed in the tests on the beams provided with reinforcement at the bottom only
B. Value of P observed in the tests on the beams provided with reinforcement at the top and at the bottom
C. Value of P calculated from the check tests for beams provided with reinforcement at the bottom only
D. Value of P calculated from the check tests for beams provided with reinforcement at the top and at the bottom

The position of the directly obtained experimental curves A and B in relation to the curves calculated on the basis of the data from the check tests and taking into account only the counteractive effect of the reinforcement on shrinkage affords further evidence for the conclusion that the influence of the counteractive effect produced by the reinforcement on shrinkage has not been predominant. This deviation can probably be attributed to the fact that the shrinkage was not uniformly distributed because the coarse aggregate was not evenly spread over the cross-section. The beams were cast in a horizontal position, and were consolidated by vibration. It is known that vibration tends to further the subsidence of coarse aggregate, at least if the time of vibration is too long. In the upper part of the beam, which is richer in cement, the shrinkage ought therefore to be greater than in the lower part of the beam, where shrinkage is prevented by a higher percentage of coarse aggregate.

If it is supposed that the reaction P at the central support of the beam was dependent on this effect alone, it is possible to calculate the difference $\Delta\epsilon$ between the amounts of shrinkage in the upper and the lower part of the beam by using E_c obtained from the check tests for the experimental curves A and B in fig. 4. As a result, it is found that $\Delta\epsilon$ increases from 0·1% after 30 days to 0·4% after 300 days. The calculated ratio of $\Delta\epsilon$ to the value of the mean shrinkage coefficient ϵ_{mean} obtained from the check tests varies from 1·1 to 1·2.

The preliminary tests described above, which were originally made in order to study the counteractive effect of the reinforcement on shrinkage, showed that this effect in the case under consideration was of minor importance in comparison with the effect produced by non-uniformly distributed shrinkage due to the segregation of the coarse aggregate.

For this reason, the author deemed it desirable to make further tests to determine the order of magnitude of non-uniformly distributed shrinkage in concrete mixes which can be expected to be used in practice.

Type tests for study of non-uniformly distributed shrinkage

The beam specimens used for the tests were cast in a horizontal position. Data on consistency, vibration, etc., are given in Table III.

TABLE III

Vibrated concrete beams

Marking	Number of beams	Concrete mix*—cement: fine aggregate: coarse aggregate	Consistency		Period of vibration per insertion of internal vibrator (seconds)	Compressive cube strength after 28 days
			Slump, cm.	Degrees, VB		
A	1	1:5·1:5·1	0·2	11	20	258, 249, 258 Mean 255
B	1	1:5·1:5·1	0·2	11	120	
C	1	1:4·4:4·4	2·3	5·2	5	266, 272, 258 Mean 265
D	2	1:4·4:4·4	2·3	5·2	120	
F	2	1:4·4:4·4	2·3	5·2	120	
G	1	1:4·4:4·4	2·3	5·2	Tamped	260, 264, 290 Mean 271
H	1	1:3·8:3·8	9·0	0·15	2·5	
I	1	1:3·8:3·8	9·0	0·15	120	
J	1	1:3·8:3·8	9·0	0·15	Tamped	

* The water–cement ratio was 0·75 for all beams.

All beams, except F, were 15 cm. × 15 cm. in cross-section and 85 cm. in length. The beams F were 15 cm. in width, 30 cm. in depth, and 85 cm. in length.

The concrete was consolidated by means of an internal vibrator, 55 mm. in diameter, 500 mm. in length, performing transverse vibrations at a rate of 13,000 per minute.

The internal vibrator was inserted at three points in the sequence indicated in fig. 5. The vibrator was tilted at an angle of 50°, except for the vibration of the beams F, in which the angle of tilt was 70°.

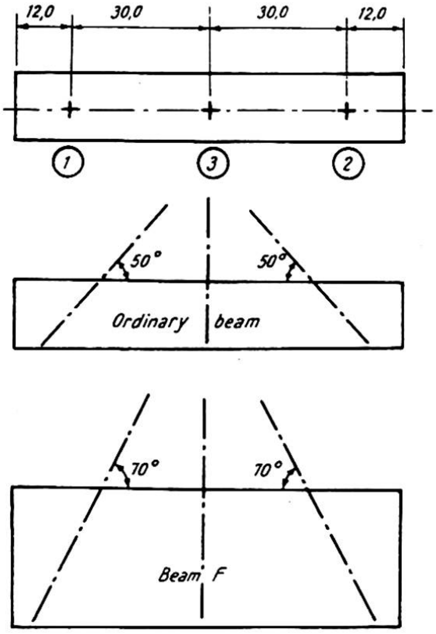

Fig. 5. Position of the internal vibrator (vibrating spade) during vibration of the concrete

After casting, the beams were subjected to moist curing, and were then placed in the testing room, where they were stored in a vertical position and provided with the measuring devices shown in fig. 6.

Channel bars were screwed to the end plates made of steel. The change in length on both sides of the test beam was measured between these channel bars by means of dial gauges, which were attached to the gauge rods A. The gauge rods were articulatedat the top end and guided at the bottom end. In order to ensure that the cross-section of the concrete beam should remain plane at the ends, nine round iron bars, 3 mm. in diameter, were embedded in the concrete and fastened to the end plates.

The results of the tests are reproduced in figs. 7(*a*) to 7(*c*). The curves in these diagrams represent the mean amount of shrinkage ϵ_{mean} and the difference in shrinkage $\Delta\epsilon$ between the top and bottom surfaces of the test beams. The ratio $x=\Delta\epsilon/\epsilon_{mean}$ is of special interest.

The values of x observed after different numbers of days, up to 250 days, are given in Table IV.

It will be seen from Table IV that the values of x are approximately equal for those beams which were vibrated only just so long as was required in order to cause the concrete to fill the form, viz. the beams A, C and H. The non-uniform shrinkage of these beams is about 22% of the mean shrinkage after one month, and about 18% after two months. In the tests on those beams which have been vibrated for two minutes, the beam made of the concrete having the most fluid consistency (beam I) exhibits the greatest value of x; next follow the beams D (consistency 5·2° VB); and last comes the beam B (consistency 11° VB). Thus, the drawbacks of excessive vibration have proved to increase as the consistency becomes more fluid. If we

TABLE IV

Values of $x = \Delta\epsilon/\epsilon_{mean}$ at various instants. The columns for 210 days and 250 days also include the values of ϵ_{mean} in per thousand

Beam	30 days	60 days	90 days	140 days	210 days		250 days	
					ϵ_{mean}	x	ϵ_{mean}	x
A	0·25	0·16	0·14	0·15	0·35	0·17	0·37	0·16
B	0·37	0·35	0·40	0·37	0·33	0·36	0·35	0·34
C	0·22	0·21	0·22	0·23	0·35	0·23	0·36	0·25
D_1	0·81	0·73	0·75	0·56	0·43	0·60	0·44	0·63
D_2	0·84	0·74	0·71	0·65	0·33	0·67	0·36	0·61
G	0	0	0	0·05	0·39	0·08	0·41	0·10
H	0·19	0·16	0·14	0·15	0·35	0·20	0·37	0·19
I	1·10	1·0	0·90	0·84	0·48	0·92	0·51	0·90
J	0·24	0·27	0·25	0·28	0·35	0·31	0·37	0·30
F_1	0·42	0·40	0·33	0·27	0·36	0·33	0·38	0·34
F_2	0·50	0·48	0·49	0·44	0·33	0·42	0·35	0·40

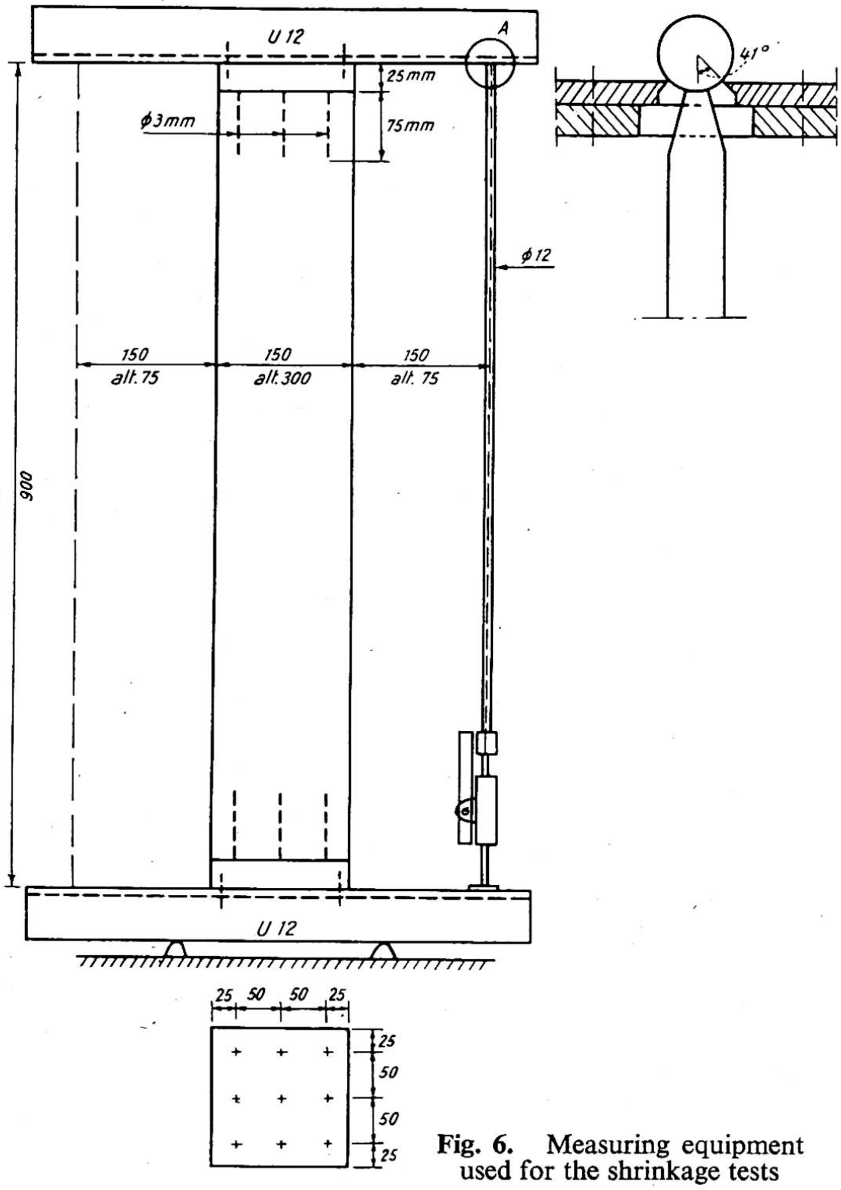

Fig. 6. Measuring equipment used for the shrinkage tests

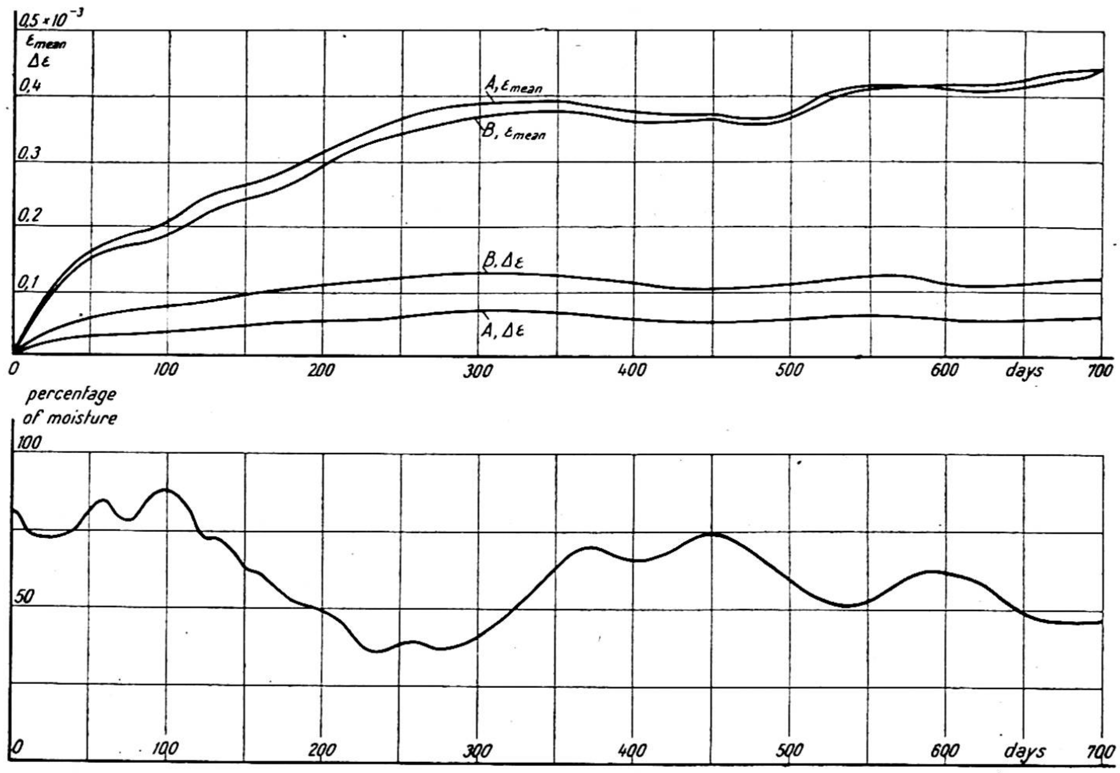

Fig. 7(*a*). Average shrinkage and difference in shrinkage $\Delta\epsilon$ between the top and bottom surfaces observed in the shrinkage tests on beams made in accordance with Table III. Consistency of concrete: 11 degrees VB

examine the beams made of tamped concrete, we find that the beam G (consistency 5·2° VB) is practically free from non-uniform shrinkage, whereas the shrinkage of the beam J (slump 9·0 cm.) is of the same order of magnitude as that of the beams subjected to vibration for a short time. The value of x for the beams 30 cm. in depth (F_1 and F_2) is equal to 55%–60% of the corresponding value for the comparable beams 15 cm. in depth (D_1 and D_2).

The number of tests was relatively small. They have, however, shown that non-

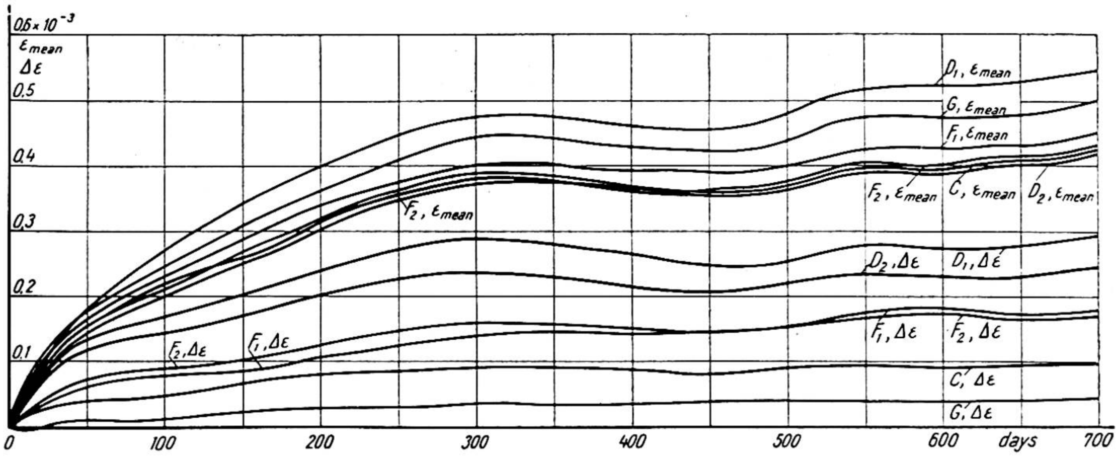

Fig. 7(*b*). Average shrinkage and difference in shrinkage $\Delta\epsilon$ between the top and bottom surfaces observed in the shrinkage tests on beams made in accordance with Table III. Consistency of concrete: 5·2 degrees VB. Slump test: 2·3 cm.

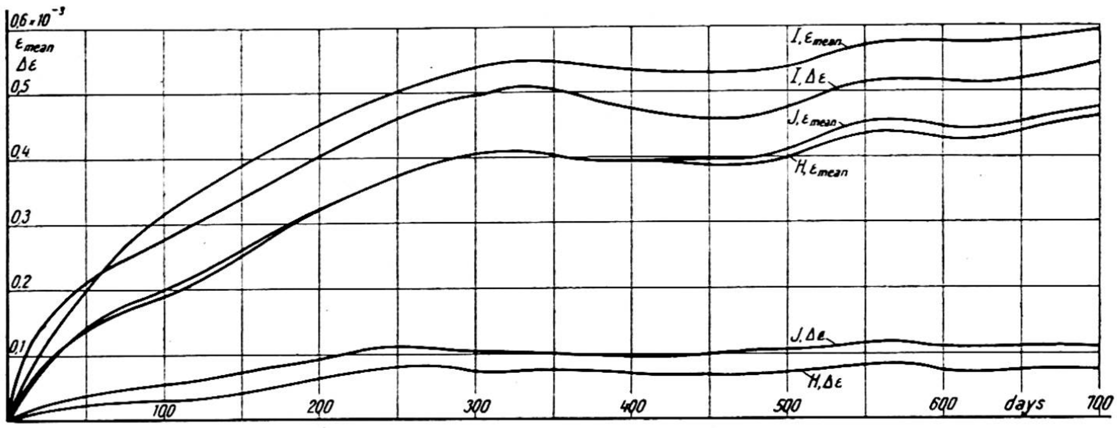

Fig. 7(*c*). Average shrinkage and difference in shrinkage $\Delta\epsilon$ between the top and bottom surfaces observed in the shrinkage tests on beams made in accordance with Table III. Consistency of concrete—slump test: 9·0 cm.

uniform shrinkage can occur in structural members made of ordinary concrete mixes, primarily when they are over-vibrated to a certain extent.

Even if non-uniform shrinkage can be obviated or reduced by using appropriate methods of concrete manufacture, it should be taken into account in the design of concrete structures.

Order of magnitude of moments and deformations due to non-uniform shrinkage of slabs supported on four sides

The effect of non-uniformly distributed shrinkage is in principle the same as the effect of non-uniform temperature distribution over the thickness of the slab. Consider the three cases in fig. 8 representing different conditions at the supports. In the case No. 1, where the slab is clamped along all four edges, the additional moments are obtained directly. If the slab is simply supported along all four edges (case No. 2)

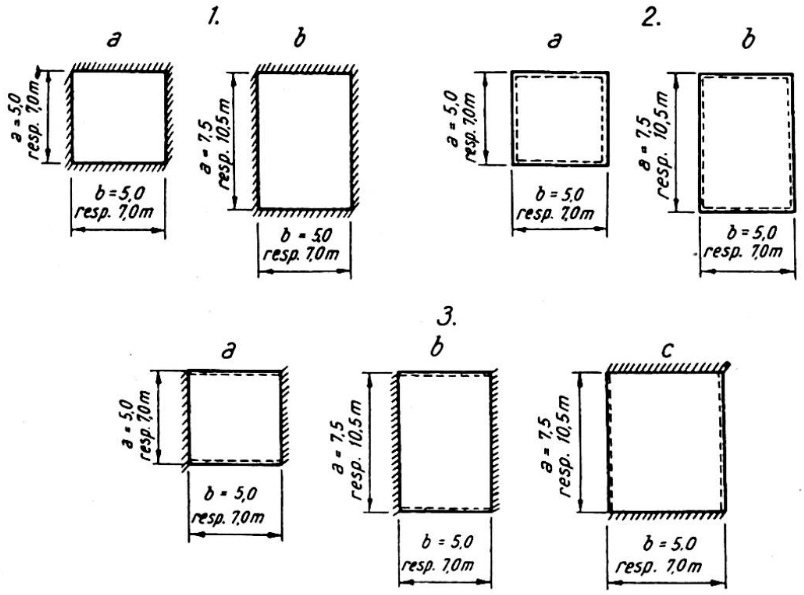

Fig. 8. Conditions at the supports and surface dimensions of concrete slabs subjected to non-uniform shrinkage and dealt with in the examples. See the results given in Tables V to VII

or simply supported along two opposite edges and clamped along the other two edges (case No. 3), then the moments and the deformations can be calculated by means of the elastic theory of plates. This calculation has been carried out by the author.* If the results of this calculation are applied to the slabs shown in fig. 8 (with $\Delta\epsilon=0{\cdot}3$ per thousand and $E_c=10^5$ kg./cm.^{2}), the moments m_0 and m_{max}, the deflections w_{max}, and the calculated stresses $K_{bH}=\frac{m}{H^2}\cdot 6$ given in Tables V to VII are obtained. Alternative calculations have been made for two values of the thickness of slab, viz. $H=15$ cm. and $H=20$ cm.

For comparison, Tables V to VII show the moments, deflections, and stresses due to the total load $q=0{\cdot}8$ t./m.^{2}

TABLE V

Moments and stresses due to non-uniform shrinkage, method of support No. 1. $\Delta\epsilon=0{\cdot}3$ per thousand. $E_c=100{,}000$ kg./cm.^{2} At other values of $\Delta\epsilon$ and E_c, the figures given in columns 3, 5, 6, and 8 should be multiplied by $\frac{\Delta\epsilon\,.\,E_c}{0{\cdot}3\,.\,10^{-3}\,.\,10^5}$

1	2	3	4	5	6	7	8	9	
Method of support	b m.	m_0 kg. $H=15$	w_{max} $H=15$	K_{bH} kg./cm.^{2} $H=15$	m_0 kg. $H=20$	w_{max} $H=20$	K_{bH} kg./cm.^{2} $H=20$	$K_{bH}(q=0{\cdot}8$; t./m.^{2} $H=15$	$H=20$
1a	5·0	−561	0	15	−995	0	15	28	16
	7·0	−561	0	15	−995	0	15	55	31
1b	5·0	−561	0	15	−995	0	15	41	23
	7·0	−561	0	15	−995	0	15	80	45

TABLE VI

Moments, stresses, and deflections due to non-uniform shrinkage, method of support No. 3. $\Delta\epsilon=0{\cdot}3$ per thousand. $E_c=100{,}000$ kg./cm.^{2} At other values of $\Delta\epsilon$ and E_c, the figures given in columns 3, 5, 6, 7 and 8 should be multiplied by $\frac{\Delta\epsilon\,.\,E_c}{0{\cdot}3\,.\,10^{-3}\,.\,10^5}$. The figures given in columns 5 and 9 are independent of E_c, and should be multiplied by $\frac{\Delta\epsilon}{0{\cdot}3\,‰}$ at other values of $\Delta\epsilon$.

1	2	3	4	5	6	7	8	9		10	
Method of support	b m.	m_{max} kg. $H=15$	w_{max} cm. $H=15$	K_{bH} g./cm.^{2} $H=15$	m_{max} kg. $H=20$	w_{max} cm. $H=20$	K_{bH} kg./cm.^{2} $H=20$	w_{max} cm. $q=0{\cdot}8$ t./m^2 $H=15$	$H=20$	K_{bH} kg./cm.^{2} $q=0{\cdot}8$ t./m.^{2} $H=15$	$H=20$
2a	5·0	−280	0·37	−7·5	−500	0·28	−7·5	0·72	0·30	20	11
	7·0	−280	0·72	−7·5	−500	0·54	−7·5	2·76	1·15	39	22
2b	5·0	−426	0·50	−11·4	−762	0·37	−11·4	1·37	0·58	39	22
	7·0	−426	0·97	−11·4	−762	0·73	−11·4	5·26	2·23	77	43

* Nylander, H., "Korsarmerade betongplattor," *Betong*, No. 1, p 3, 1950.

TABLE VII

Moments, stresses, and deflections due to non-uniform shrinkage, method of support No. 3. $\Delta\epsilon=0{\cdot}3$ per thousand. $E_c=100{,}000$ kg. cm.[2] At other values of $\Delta\epsilon$ and E_c, the figures given in columns 3, 4, 6, 7 and 8 should be multiplied by $\frac{\Delta\epsilon \, . \, E_c}{0{\cdot}3 \, . \, 10^{-3} \, . \, 10^5}$. The figures given in columns 5 and 9 are independent of E_c, and should be multiplied by $\frac{\Delta\epsilon}{0{\cdot}3\,‰}$ at other values of $\Delta\epsilon$.

1	2	3	4	5	6	7	8	9	10	11		12	
Method of support	b m.	m_i $H=15$	$m_{i_{mean}}$ $H=15$	w_{max} cm. $H=15$	$K_{bi\,mean}$ $H=15$	m_i $H=20$	$m_{i_{mean}}$ $H=20$	w_{max} $H=20$	$K_{bi\,mean}$ $H=20$	w_{max} cm. $q=0{\cdot}8$; t./m.: $H=15$	$H=20$	$K_{bi\,max}$ $q=0{\cdot}8$; t./m.: $H=15$	$H=20$
3a	5·0	−815	−930	0·079	−25	−1,450	−1,450	0·059	−25	0·36	0·143	−37	−21
	7·0	−815	−930	0·155	−25	−1,450	−1,450	0·116	−25	1·31	0·55	−72	−41
3b	5·0	−640	−760	0,093	−20	−1,130	−1,340	0·070	−20	0·44	0·183	−44	−25
	7·0	−640	−760	0·182	−20	−1,130	−1,340	0·137	−20	1·71	0·70	−86	−49
3c	5·0	−1,010	−1,055	0·104	−31	−1,800	−31	0·078	−31	0·96	0·40	−55	−31
	7·0	−1,010	−1,055	0·204	−31	−1,800	−31	0·153	−31	1·88	0·78	−107	−61

It will be seen from Tables V to VII that a conceivable difference in shrinkage $\Delta\epsilon=0{\cdot}3$ per thousand produces deflections which cannot be disregarded in comparison with the deflections due to the load, primarily in the case of the simply supported slabs, but also in the case of the slabs clamped along two opposite edges.

Furthermore, the tables show that the moments and the stresses at the supports due to non-uniform shrinkage can be of the same order of magnitude as the moments and the stresses caused by the load.

In consideration of these results, the effect of non-uniform shrinkage should not be disregarded when concrete slabs reinforced in two directions are designed in accordance with the theory of elasticity. Since non-uniform shrinkage is no longer of any importance in the stage of failure, as the structure is not affected by shrinkage on account of its low rigidity, shrinkage need not be taken into account in the method of limit design.

In estimating the rigidity of structures, non-uniform shrinkage must be taken into consideration for two reasons, viz. first, because it produces the additional deflection, and second, because it causes an increase in the moments at the supports, and hence facilitates the development of cracks at the supports.

REDUCTION OF NON-UNIFORM SHRINKAGE BY ADDITION OF AN EXTRA LAYER OF COARSE AGGREGATE

To prepare concrete mixes in which the coarse aggregate does not segregate during vibration is probably difficult on the site.

Another method of reducing non-uniform shrinkage, which appears to be relatively practicable, is to spread an additional layer of coarse aggregate on the surface of the concrete after placement, and to subject this layer to vibration so as to mix the coarse aggregate with the freshly poured concrete. This possibility was studied in tests on ten beams differing in thickness and in the grading of the additional layer of coarse aggregate (see Table VIII).

The workability determined by means of consistency tests was 4·6° VB on the average.

TABLE VIII

Data on additional layers of coarse aggregate mixed by vibration with the concrete of beams after placement

Beam No.	Thickness of layer of coarse aggregate, cm.	Weight of layer of coarse aggregate, kg.	Period of vibration sec.	Grading of layer of coarse aggregate, mm.
A_1, A_2	1	2·3	20	4–32
B_1, B_2	3	6·9	400	4–32
C_1, C_2	1	2·3	15	16–32
D_1, D_2	3	6·9	250	16–32
E_1, E_2	0	—	—	—

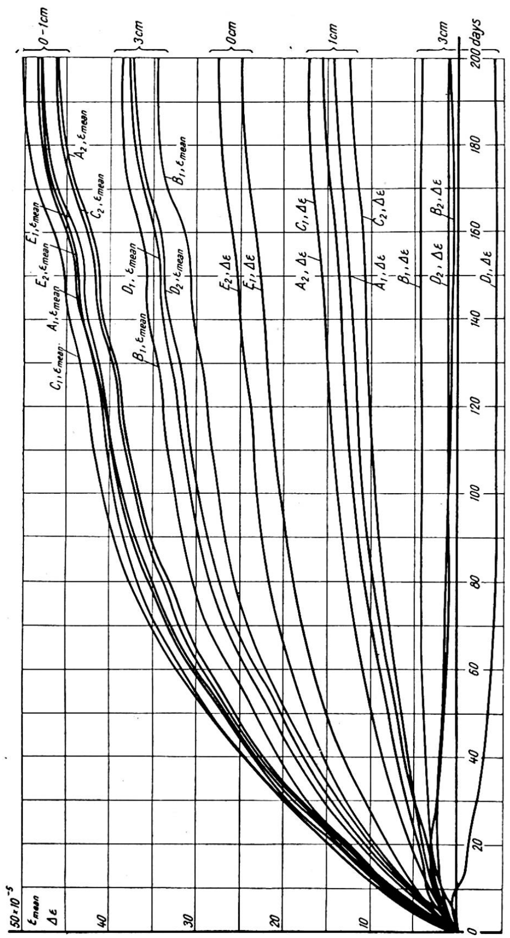

Fig. 9. Average shrinkage and difference in shrinkage between the top and bottom surfaces observed in the shrinkage tests on beams made in accordance with Table VIII

Each beam was 15 cm. in depth, 15 cm. in width, and 90 cm. in length.

The concrete mix used for the beams had the proportions cement: fine aggregate: coarse aggregate=1:4·4:4·4 by weight. The water–cement ratio was 0·75. The gradings of fine and coarse aggregate were close to the ideal curves.

To begin with, the beams were vibrated for two minutes per insertion of internal vibrator in accordance with the same system as in the tests described earlier (fig. 5). After that, the layer of coarse aggregate was gradually pushed down into the concrete by applying the point of the vibrator to a metal sheet spread over the layer. The layer of coarse aggregate was vibrated until cement paste rose to the surface through the layer.

The results of the shrinkage measurements are shown in fig. 9. It will be seen from fig. 9 that the difference in shrinkage between the top and bottom surfaces of the beams was practically zero when the additional layer of coarse aggregate was 3 cm. in thickness. Furthermore, it follows from fig. 9 that $\Delta\epsilon$ obtained in the tests using a layer of coarse aggregate 1 cm. in thickness was about 60% of $\Delta\epsilon$ for beams without any additional layer of coarse aggregate.

The curves representing ϵ_{mean} in fig. 9 indicate that the addition of an extra layer of coarse aggregate has reduced ϵ_{mean} in a proportion which is in agreement with the reduction in $\Delta\epsilon$.

These tests have demonstrated that the addition of an extra layer of coarse aggregate can reduce or eliminate non-uniform shrinkage.

Moreover, the possibilities of using this method on the site have been tested by the author, together with Mr. S. E. Bjerking, in the construction of floor slabs for dwelling-houses in Uppsala. The method was found to be applicable under practical conditions.

Summary

In some tests on vibrated reinforced concrete beams it was shown that the counteractive effect of the reinforcement on shrinkage was of minor importance in comparison with the effect produced by non-uniformly distributed shrinkage owing to the segration of the coarse aggregate. Type tests with non-reinforced beams showed the influence of varying consistency and time of vibration on the non-uniform shrinkage.

The magnitude of moments and deformations due to non-uniform shrinkage in slabs supported on four sides was calculated. It was shown that these moments and deformations at an allowable load can be of the same order of magnitude as the moments and stresses caused by the load, primarily in the case of the simple supported slabs, but also in the case of slabs clamped along two opposite edges.

A method of reducing non-uniform shrinkage was examined. An additional layer of coarse aggregate was spread on the surface of the concrete after placing and this layer was subjected to vibration so that the coarse aggregate was mixed with the freshly poured concrete. It was shown that there are possibilities of reducing or eliminating the non-uniform shrinkage by this method.

Résumé

Certains essais effectués sur des poutres en béton armé vibré ont montré que l'influence exercée par l'armature dans le sens d'une réduction du retrait est plus faible que l'influence qu'exerce la ségrégation des gros grains d'agrégat dans le sens de l'hétérogénéité de ce retrait. Des essais-types sur poutres non armées ont mis en évidence l'influence des variations de la consistance du mélange et de la durée de la vibration sur cette hétérogénéité.

L'auteur a calculé les moments et les déformations qui résultent de l'hétérogénéité du retrait, sur des dalles appuyées de tous côtés. Il a constaté que ces moments et déformations peuvent être du même ordre de grandeur que ceux qui résultent des charges admissibles, tout particulièrement lorsqu'il s'agit de dalles reposant librement sur leurs appuis, mais aussi dans le cas des dalles encastrées sur deux bords opposés.

L'auteur a étudié une méthode qui doit permettre une réduction de l'hétérogénéité du retrait. Elle consiste à répandre une couche additionnelle de gravier sur la surface du béton après mise en œuvre, puis à procéder à un traitement de vibration pour assurer l'incorporation du gravier au béton frais. On a pu constater que cette méthode permet de réduire, voire même de supprimer complètement l'hétérogénéité du retrait.

Zusammenfassung

In einigen Versuchen an vibrierten Eisenbetonbalken zeigte es sich, dass der vermindernde Einfluss der Armierung auf das Schwinden geringer war als der Einflus der Ungleichmässigkeit des Schwindens, herrührend von der Entmischung grobkörniger Zuschlagstoffe. Allgemeine Versuche mit unarmierten Balken zeigten den Einflus veränderlicher Konsistenz und Vibrationsdauer auf das ungleichförmige Schwinden.

Die Momente und Verformungen infolge des ungleichförmigen Schwindens in allseitig aufliegenden Platten wurden berechnet. Es zeigte sich, dass diese Momente und Verformungen von gleicher Grössenordnung sein können wie die Momente und Beanspruchungen aus der zulässigen Belastung, besonders im Falle der frei aufliegenden Platte, aber auch bei der an zwei gegenüberliegenden Rändern eingespannten Platte.

Es wurde eine Methode zur Verminderung des ungleichförmigen Schwindens untersucht. Eine zusätzliche Lage von Kieszusatz wurde über die Oberfläche des eingebrachten Betons gestreut und dann vibriert, so dass sich der Kies mit dem frischen Beton vermischte. Es konnte gezeigt werden, dass durch diese Massnahme das ungleichförmige Schwinden vermindert oder ausgeschaltet werden kann.