Statistical estimate of seismic loading

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Statistical estimate of seismic loading

Statistische Ermittlung seismisch bedingter Belastungen

Previsão estatística de solicitações sísmicas

Prévision statistique des sollicitations séismiques

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1. *Introduction.*

Among the various dynamic loads acting upon structures, those due to seismic action are fundamental in many parts of the world. These loads, in many cases, are important enough to be paramount in the design of structures.

In order to define these loads, it is fundamental to be able to predict the occurrence of an earthquake of a given intensity in a given area and determine the resulting loads occurring in the structures.

The only method which, at present, appears practicable for the prediction of the number of earthquakes of different intensities occurring in a given area is to study the seismic history of the area and to assume its continuity in time. This study should, of course, be accompanied by a knowledge of the geophysics of the area.

Regarding the estimation of the loads produced by an earthquake of a given intensity in a structure, it may be admitted that once its characteristics (its accelerogram) are known, the theory of vibrations allows a sufficiently reliable solution of the problem.

It happens, however, that data allowing the prediction of the form of the accelerogram are not available and, as Housner (1) says, «the chief similarity between different acceleration records lies in the marked irregularity exhibited by all».

(1) HOUSNER G. W. — *Characteristics of Strong-motion Earthquakes*, Bulletin of the Seismological Society of America, Vol. 37, no. 1, Jan. 1947, pag. 19.

Besides this, the actual definition of the intensity of an earthquake constitutes a problem in itself. There are not yet generalised objective methods allowing the intensity of earthquakes to be defined and it is customary to make use of conventional scales, such as the Modified Mercalli Scale, for that purpose.

From the point of view of probabilities, the problem lies in the estimation of the probability of a structure being subjected to loads equal to or greater than a given value during a given period of time. The intensity of the loads may be defined by the maximum displacement undergone by points of the structure. The problem can thus be reduced to the calculation of the probablity of the occurrence of displacements greater than a given value during a given period of time.

This calculation has to be based upon the probabilities of the occurrence of earthquakes of different intensities and upon the probability, of the displacements being equal to or greater than given values, for an earthquake of given intensity.

2. *Statistical behaviour of a structure due to an earthquake*

Once the accelerogram of an earthquake is known, it is easy to demonstrate that the displacement of any of the points of a structure in relation to the ground may be expressed by the integral

$$X = \int_0^t \ddot{x}(\tau)\, \varphi(t - \tau)\, d\tau$$

where $\ddot{x}(\tau)$ is the acceleration of the ground, given by the accelerogram of the earthquake, and $\varphi(t - \tau)$ is a function which depends solely on mechanical characteristics of the structure.

Bearing in mind the irregularity of the accelerograms referred to, Housner (²) and later Goodmann, Rosenblueth and Newmark (³) assumed that, during an earthquake, the variation of the acceleration with time takes place in pulses and in such a way that one of the following characteristics may be considered as random: direction, intensity or spacing in time. The aleatory characteristics of the accelerograms will, of course, be reflected in the behaviour of the structures.

Goodman, Rosenblueth and Newmark determine the expected value and the standard deviation of the displacements of a structure when it is subjected to an earthquake of a given intensity.

In order to define the intensity of an earthquake, in any particular place, they consider a value K, such as

$$K^2 = \frac{2}{\pi} E(n)\, E(\mu^2)$$

(²) See loc. cit.

(³) GOODMAN, L. E. ROSENBLUETH, J. M. and NEWMARK, N. M — *Aseismic Design of Elastic Structures Founded on Firm Ground*, Proceedings American Society of Civil Engineers, Vol. 79, Separate no. 349, November, 1953.

where E (n) is the expected number of pulses and E (μ^2) the expected value of the square of the intensity of each pulse.

Hence, they show that the expected value of the modulus of displacement, |X|, at the end of the time t, is

$$E(|X|) = K\left[\frac{1}{t}\int_0^t \varphi^2(t-\tau)\, d\tau\right]^{1/2}$$

In the same way, the expected value of the maximum of the modulus X which occurs during an earthquake can be written as a product of K and of a function which depends solely on the mechanical characteristics of the structure.

The authors quoted also show that the maximum values of X have standard deviations proportional to their expected values, or in other words, constant coefficients of variation.

The analysis referred to leads, to the assumption that during an earthquake of intensity K, the maximum displacements undergone by points of a structure have expected values E (X) and standard deviations D (X) given by

$$E(X) = K\psi$$
$$D(X) = c\,E(X) = c\,K\,\psi$$

where ψ depends solely on the mechanical characteristics of the structure.

It should be noted that it is further possible to consider the statistical behaviour due to the variation of the mechanical properties of the materials of the structure itself, the principal consequence of which would be to increase the value of c.

3. *Statistical definition of the seismicity of an area*

It is assumed that, for a given area and interval of time, it is possible to define the probablities of seismic occurrences of given intensities and that these probabilities satisfy Poisson's distribution.

As it is known, λ being the expected number of phenomena, the probability of one or more of these taking place is given by

$$P_r\left\{N \geqslant 1\right\} = 1 - e^{-\lambda}$$

For a given area it is of interest to know the number of expected earthquakes of different intensities in a given unit of time.

Defining the intensity of earthquakes by the given value K, the problem lies in determining how the expected number of earthquakes would vary in relation to K.

Letting λ be the expected number of earthquakes of intensity equal to or greater than K, it becomes necessary to define the function λ (K). There are not yet sufficient data to define this function accurately, as, sofar as is known, the values of K corresponding to the accelerograms obtained experimentally have not been calculated.

One of the main undertakings of interest is the relating of Mercalli intensities to the values of K, in order to benefit from the great quantity of information related to the former.

Meanwhile, from the results presented by Gutenberg and Richter (4) it is possible to establish a form for the function λ (K) which will be used later.

Noting that K^2 is a measure of energy per unit volume, and bearing in mind the equations suggested by Gutenberg and Richter relating, on the one hand the expected number of earthquakes with their magnitude, and on the other the magnitude with the energy developed at the epicenter, the form of the function becomes approximately

$$\lambda(K) = \frac{\alpha}{K}$$

The value α measures the seismicity of the area. It should be noted that the form of λ (K) could vary from area to area, but the analysis carried out by Gutenberg and Richter shows that, for distinct areas, the law of variation of the expected number of earthquakes with their intensity appears in approximately the same form, the only variant being a value which measures the seismicity of the area.

To obtain the number of expected earthquakes in an interval of time τ, the expected number of earthquakes, λ, corresponding to unit time, must be multiplied by τ.

It should be noted further that $d\lambda$ measures the expected number of earthquakes of an intensity lying between K and $K + dK$.

4. *Statistical estimate of seismic loading*

To calculate the expected number of times that the displacement X will exceed x, referred to a unit interval of time (5), it will be necessary to calculate the integral

$$I = \int_0^\infty \phi(x, K)\, d_K \lambda(K)$$

where ϕ (x, K) is the probability of the displacements being equal to or greater than x, for an earthquake of intensity K, and λ (K) the expected number of earthquakes whose intensity is equal to or greater than K.

As is seen in 2, ϕ (x, K) should be such that $E(x) = K\psi$ and $D(x) = c\,K\,\psi$.

(4) GUTENBERG, B. and RICHTER, C. F. — *Seismicity of the Earth and Associated Phenomena*, Princeton University Press, Princeton, New Jersey, 1954.

(5) FERRY BORGES, J. — *O Dimensionamento de Estruturas*, Laboratório Nacional de Engenharia Civil, Publicação n.° 54, Lisboa, 1954.

Assuming that the distribution of X is normal, then

$$\phi(\varkappa, K) = \int_{\varkappa}^{\infty} \frac{1}{\sqrt{2\pi}\ c\, K\, \psi} e^{-\frac{(\varkappa - K\psi)^2}{2\, c^2\, K^2\, \psi^2}} d\varkappa$$

and, according to 3, $\lambda(K) = \frac{\alpha}{K}$, $d_K\, \lambda(K) = \frac{-\alpha}{K^2}\, dK$

It is then necessary to calculate the integral

$$I = \int_{o}^{\infty} \int_{\varkappa}^{\infty} \frac{-\alpha}{K^2} \frac{1}{\sqrt{2\pi}\ c\, K\, \psi} e^{-\frac{(\varkappa - K\psi)^2}{2\, c^2\, K^2\, \psi^2}} d\varkappa\, dK,$$

whose value is

$$I = \frac{\alpha\psi}{\varkappa} \left[\frac{c}{2\pi} e^{-\frac{1}{2c^2}} + \phi\left(\frac{1}{c}\right) \right]$$

$$= \frac{\alpha\psi}{\varkappa} \beta$$

where β is only a function of the coefficient of variation c and $\phi\left(\frac{1}{c}\right)$ represents the cumulant of a normal distribution of mean zero and unit standard deviation.

For the values of the coefficient of variation c between 0 and 0.5, β differs from unit by less than 0.5 %.

According to the analysis of Goodmann, Rosenblueth and Newmark, the randomness of seismic loadings leads to values of c of about 10 %. So in practice β can be taken equal to unity without introducing any appreciable error. Hence

$$I = \frac{\alpha\psi}{\varkappa}$$

Thus for a structure whose response is characterised by the value of ψ, the expected number of times that the displacements of the structure will be equal to or greater than $\varkappa$ is directly proportional to the seismicity of the area (defined by α) and inversely proportional to $\varkappa$. This is a simple result.

To calculate the expected number for a time interval τ, I is multiplied by τ, as is well known.

Let us consider another interpretation that can be given to the value of I.

By definition $\lambda = \frac{\alpha}{K}$ and $E(X) = K\psi$, from which

$$\lambda = \frac{\alpha\psi}{E(X)}$$

and when $\varkappa = E(X)$

$$I = \lambda$$

This result can be stated as follows:

For a given structure, the expected number of times the maximum displacements will be equal to or greater than $\varkappa$ is equal to the expected number of earthquakes of intensity equal to or greater than that for which the maximum displacements have a mean value $\varkappa$.

As has been said, knowing the expected number I, it is easy to to calculate the probability of one or more displacements taking place in a given time which correspond to the expected number I:

$$P_r\left\{X \geqslant \varkappa / \tau\right\} = 1 - e^{-I\tau} = 1 - e^{\frac{\psi \alpha \tau}{\varkappa}}$$

It should be noticed that the equality between λ and I, when $\varkappa = E(X)$, would be obvious if $D(X) = 0$; that is, if there were no randomness in the behaviour of the structure subject to an earthquake of a given intensity. As it has been seen, such randomness will only influence the results through the coefficient β, which is practically equal to unity.

5. *Conclusions*

The fundamental problem dealt with in this paper consists in calculating the probability of ruin of a structure when subjected to seismic action.

The following procedure should be followed in practice for the resolution of this problem: (i) to define the maximum value of the displacements of points of a structure which are considered to cause ruin, and, (ii) to calculate the intensity of the earthquake which, on the average, causes this maximum displacement. The probability of ruin of a structure will be the probability of the occurrence of an earthquake of an intensity equal to or greater than the above.

This probability is obviously referred to a given time interval and a given area.

In general it is simpler to substitute this probability by an expected number.

The simplicity of the result obtained derives from the assumption that the expected number λ is inversely proportional to the intensity K, an hypothesis which is considered reasonable but which needs to be confirmed.

It was also assumed that for an earthquake of given intensity the maximum displacements of a structure satisfy Gauss's distribution. From a purely theoretical point of view it would perhaps be more suitable to assume its distribution to be in agreement with a law of extreme values.

It should be noted, however, for the present, that the lack of of precision in observed values does not justify the adoption of more elaborate statistical techniques.

SUMMARY

In order to estimate seismic loadings, the information referring to the seismicity of the area has to be combined with that referring to the behaviour of structures subject to seismic action.

Assuming an earthquake to be a group of random pulses, it is possible, in accordance with the theory of vibrations, to calculate the probability of a given structure undergoing displacements greater than a given value.

On the other hand it is assumed that the expected number of earthquakes, of an intensity equal to or greater than a given value, is inversely proportional to this intensity.

In this paper it is shown that the expected number of times the maximum loading reaches a certain value is equal to an expected number of earthquakes. This expected number is the number of earthquakes whose intensity is greater than that which, on the average, produces the given loading.

ZUSAMMENFASSUNG

Damit die seismisch bedingten Belastungen eines Bauwerks in Rechnung gesetzt werden können, muss man die in der betreffenden Gegend gemachten seismischen Beobachtungen mit dem Verhalten von Bauwerken welche Erdstösse erlitten haben, zusammentragen.

Wenn ein Erdstoss einer Gesamtheit zufälliger Impulse gleichgesetzt wird, lässt sich nach der Schwingunsgstheorie die Wahrscheinlichkeit berechnen, wonach die Verschiebung eines Bauwerkes grösser als ein gegebener Wert ausfallen wird.

Anderseits darf angenommen werden, dass der Erwartungswert für die Anzahl Erdstösse, welche eine bestimmte Stärke überschreiten, umgekehrt proportional zu dieser Stärke ist.

Es wird gezeigt, dass trotz der Zufälligkeiten im Verhalten eines Bauwerks gegenüber Erdstössen der Erwartungswert für die Anzahl der Ueberschreitungen einer bestimmten Stärke gleich dem Erwartungswert der Anzahl der Erdstosswiederholungen wird, deren Stärke zu Beanspruchungen führt, welche mindestens grösser als der in Rechnung gesetzte Wert ausfallen.

RESUMO

Para se poderem prever as solicitações causadas pelos abalos sísmicos, devem combinar-se os dados relativos à sismicidade da região estudada com os relativos ao comportamento das estruturas submetidas a sismos.

Assimilando um sismo a um conjunto aleatório de impulsos, é possível calcular, pela teoria das vibrações, a probabilidade que o deslocamento de uma estrutura tem de exceder determinado valor.

Por outro lado, admite-se que o valor esperado do número de sismos de intensidade superior a determinado valor é inversamente proporcional a esse valor.

Demonstra-se aqui que, apesar da aleatoriedade do comportamento das estruturas sob a acção dos sismos, o valor esperado da repetição de solicitações superiores a um dado valor é igual ao valor esperado da repetição de sismos cuja intensidade produz, em média, solicitações superiores ao valor considerado.

RÉSUMÉ

Afin de prévoir les sollicitations sismiques, il faut combiner les données concernant la sismicité de la région à celles concernant le comportement des ouvrages soumis à l'action des séismes.

En assimilant un séisme à un ensemble aléatoire de pulsations, on peut calculer, par la théorie des vibrations, la probabilité qu'aura le déplacement d'un ouvrage d'être supérieur à une valeur donnée.

Par ailleurs, il y a raison d'admettre que l'espérance mathématique du nombre de séismes dépassant une certaine intensité, est inversément proportionelle à cette intensité.

On démontre ici que, malgré le caractère aléatoire du comportement des charpentes sous l'action des séismes, l'espérance mathématique de la fréquence des sollicitations dépassant une certaine valeur, est égale à l'espérance mathématique du nombre de répétitions des séismes dont l'intensité produit, en moyenne, des sollicitations supérieures à la valeur considérée.