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Aerodynamic stability of suspension bridges under wind action

Aerodynamische Stabilität von Hängebrücken unter der Windeinwirkung

Estabilidade aerodinâmica das pontes suspensas submetidas à acção do vento

La stabilité aérodynamique des ponts suspendus sous l'action du vent

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Tokyo

In 1942, the writer submitted a paper [1] on the stability of the free torsional oscillation of a suspension bridge suggesting that the single-noded torsional oscillation is fatal for the bridge and the frequency variation has a great significance for the destiny of a span. In 1948, Dr. Fr. Bleich directed his attention to the fact that the frequency is not constant.

The present paper deals with the forced oscillation of the suspension bridge due to an alternating aerodynamic force with a frequency ω, and shows the resonance diagram or dynamic magnifier as a function of ω and wind velocity V. The resonance diagram gives the stability range of suspension bridges.

The results obtained from the forced torsional oscillation and forced coupled oscillation of deflectional and torsional motion are summarized as follows.

The torsional frequency of a suspension bridge under wind action is a function of wind velocity, and the stiffening girder becomes unstable at a certain velocity. Especially, the action of drag coefficient which has been omitted by other investigators plays an important rôle in the analysis.

The formula for critical wind velocity is given short and clear. According to the formula, the torsional stiffness and the flexional stiffness have a great influence on the critical wind velocity, and the greater the dead load the greater the effective stiffness ot the suspension bridges.

As to the profile of a cross section of stiffening girder (including a floor part), it is preferable to choose the section which has minimum vortex discharge and positive lift curve for positive angle of attack. Streamlining the cross section of a stifening girder often results in a decrease of critical wind velocity.

Regarding the slope of aerodynamic moment coefficient, the writer would like to choose its value as small as possible and negative for positive angle of attack.

The so called «Section-model experiment» can not play the part of «Full-model experiment» in the higher wind velocity range.

To check up the adequacy of this analysis, the writer has conducted a wind tunnel test in a small scale, in cooperation with prof. M. Yasumi.

In the experiment, slow single noded torsional oscillations were recognized, when the wind velocity approached the predicted critical wind velocity. The existence of slow oscillations with long period seems to back up the writer's analysis.

NOTATIONS

l .... Span length.
f .... Cable sag.
b .... Distance between cables or width of stitffening girder.
φ .... Torsional angle of the girder which corresponds to the «angle of attack».
u .... Deflection of the girder.
$2m$ ...... Dead load per unit length of bridge.
H .... Horizontal component of cable tension due to dead load $H = ml^2/8f$.
Θ/l ...... Polar moment of inertia of stiffening girder per unit length.
EI ...... Flexional rigidity of stiffening girder.
GK ...... Torsional rigidity of stiffening girder.
EJ ...... Reduced flexional rigidity of suspension bridge.
$\underline{GK}$ ...... Reduced torsional rigidity of suspension bridge.
V .... Wind Velocity.
V_k .... Critical wind velocity.
ω_φ .... Natural torsional circular frequency of suspension bridge. (wind off).
N_φ .... Natural torsional frequency of suspension bridge. (wind off).
ω_u ...... Natural deflectional circular frequency of suspension bridge. (wind off).
N_u ...... Natural deflectional frequency of suspension bridge. (wind off). $N_u = \omega_u / 2\pi$
ω .... Circular frequency of an alternating aerodynamic force which acts on stiffening girder.
ρ .... Air density $\rho = 0{,}125 \left(\frac{kg - sec^2}{m^4} \right)$
g .... Acceleration of gravity.
C_l .... Lift coefficient of stiffening girder.
C_d .... Drag coefficient of stiffening girder.
C_m .... Aerodynamic moment coefficient of stiffening girder.
S .... $\left(\frac{d C_l}{d \varphi} \right)_{\varphi = 0}$
S_t .... $\left(\frac{d C_m}{d \varphi} \right)_{\varphi = 0}$
p .... Stagnation pressure; $p = \frac{1}{2} \rho V^2$

μ .... Coefficient concerning lift coeff. and drag coeff. For a singel-noded oscillation, it yelds to

$$\mu^2 = 1 + \frac{\sqrt{128}}{4\pi^2}\frac{S}{C_d}$$

$\mathcal{K}$ .... Dynamic magnifier
j .... Imaginary unit ($= \sqrt{-1}$)

The other letter symbols used in this paper are defined where they first appear.

I. *Fundamental Equations.*

The subsequent analysis is based on the following assumptions.

1) The bridge is a single span and its span length is l.
2) The direction of wind is horizontal and perpendicular to the bridge axis.
3) The effect of towers and side spans are not considered. (Fig. 5).
4) For the sake of simplicity, the distance between cables is assumed to be equal to the width of the stiffening girder.
5) Flexional rigidity (EI) and torsional rigidity (GK) of the stiffening girder are considered constant.
6) The bridge is considered as an elastic structure.

The present paper deals with symetrical single span suspension bridge in which the effect of towers and side spans are not considered. For simplicity's sake, the direction of wind is assumed to be horizontal & perpendicular to the bridge axis. As can be seen from Fig. 1, the bridge section is assumed to be H- shaped, that is, the floor part lies in the middle of the stiffening girder (including the truss type). It seems easys to modify the analysis, so the writer has adopted the above mentioned assumption.

In deriving the fundamental equations of the twisted and deflected stiffening girder, the writer uses the system of fixed coordinate axis x, y, z directed as shown in the figure. He also takes at the centroid of any cross section of the stiffening girder the system of coordinate axis ξ, η, ζ, such that ξ and η are in the direction of the principal axis of the cross section and ζ is in the direction of the tangent to the center line of the stiffening girder after deformation.

The deformation of the girder is defined by the two components u and v of the displacement of the centroid of the cross section in the x- and y- directions and by the angle φ of which the cross section rotates. In the subsequent analysis the horizontal displacement v is neglected.

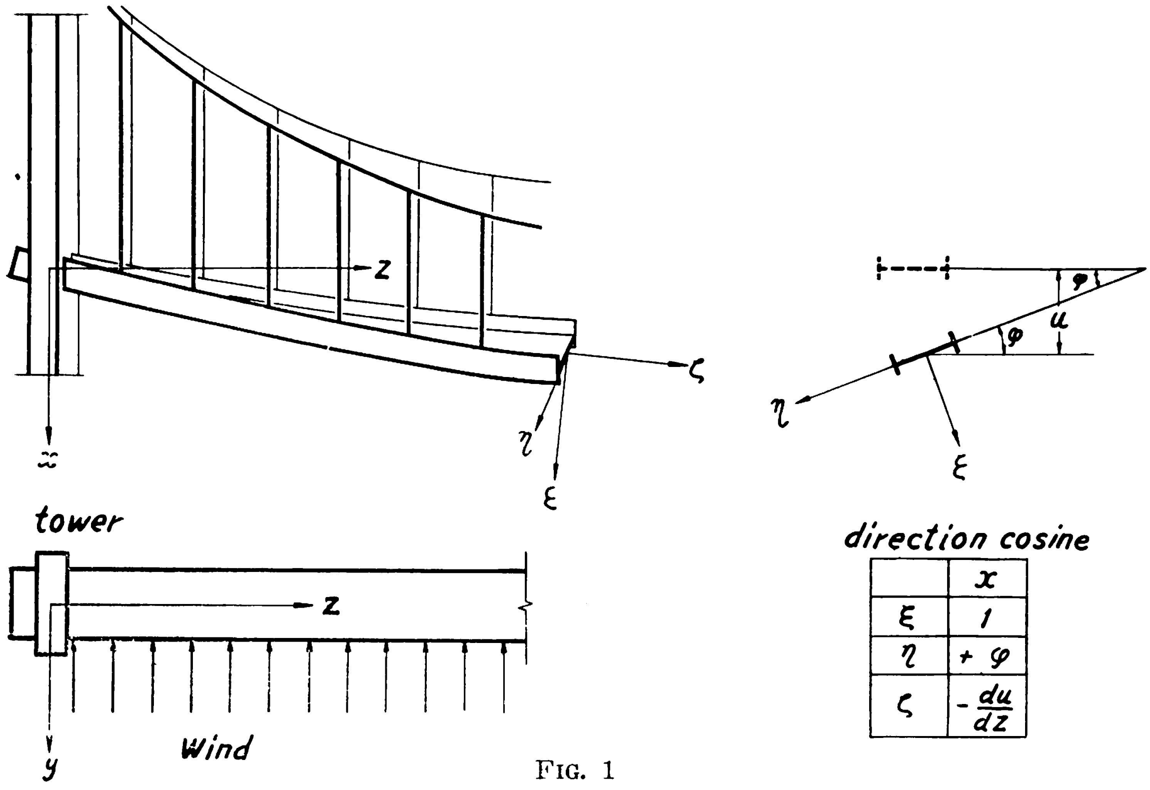
The aerodynamic forces acting on the stiffening girder per unit length, are expressend as follows.

$$\left.\begin{aligned} &\text{Drag,} && W = C_d\, pb \\ &\text{Lift,} && L = C_l\, pb \\ &\text{Torque,} && T = C_m\, pb^2 \end{aligned}\right\} \tag{1}$$

The relations between the aerodynamic coefficients and the angle of incidence are assumed as follows.

$$\left.\begin{array}{ll}\text{For lift coefficient} & C_l = S\,\varphi \\ \text{For aerodynamic torque coef.} & C_m = S_t\,\varphi\end{array}\right\} \qquad (2)$$

Considering now the oscillation of stiffening girder (including a truss type), it is evident that if the girder oscillates, it becomes slightly twisted, and a wind blowing perpendicularily to the span produces aero-



	x
ξ	1
η	+ φ
ζ	$-\frac{du}{dz}$

FIG. 1

dynamic forces. The aerodynamic torque (moment) around the axis ζ per unit length of the span is given by the following expression, considering the relative vertical velocity of the girder,

$$S_t\,pb^2\,(\varphi - \frac{1}{V}\,\frac{du}{dt})$$

Besides this aerodynamic torque, there is another important factor which has a great influence on the stability of a suspension bridge. A girder exposed to wind, is subjected to a lateral wind pressure which corresponds to the drag in an airfoil. The effect of the bending moment due to wind pressure after deformation of the span must be considered, as it then has a component which tends to increase the twist, when the girder is twisted slightly. Calling the bending moment $\mathfrak{M}$, the additional torque about the axis due to the wind pressure W in (1), is represented by $\mathfrak{M}\, du/dz$.

The aerodynamic torque and the above mentioned additional torque are resisted by the torsional stiffness of the span and the tension of the cables. So, considering next the equilibrium condition of the twisted and deflected stiffening girder in Fig. 2. The restoration torque per unit length of stiffening girder due to cable tension is

$$\frac{Hb}{2}\left[\frac{d^2 u_2}{dz^2}-\frac{d^2 u_1}{dz^2}\right]$$

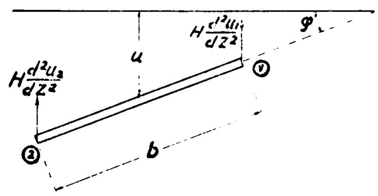


FIG. 2

where, H is the horizontal component of the cable-tension due to dead load m per cable. Substituting the relations;

$$u_1 = u - \frac{b}{2}\varphi \quad \text{and} \quad u_2 = u + \frac{b}{2}\varphi$$

we obtain,

$$\frac{Hb}{2}\left[\frac{d^2 u_2}{dz^2}-\frac{d^2 u_1}{dz^2}\right] = \frac{Hb^2}{2}\frac{d^2\varphi}{dz^2}$$

Furthermore, considering the shear effect and the torsional rigidity of the girder, the restoration torque is given by the equation:

$$\left(GK + \frac{Hb^2}{2}\right)\frac{d^2\varphi}{dz^2} - \frac{EI\,.\,b^2}{4}\frac{d^4\varphi}{dz^4}$$

where, EI is the flexional rigidity and GK is the torsional rigidity of the stiffening girder.

Now, the differential equation for torsional oscillation is expressed by the equation:

$$\frac{\Theta}{1}\frac{\partial^2\varphi}{\partial t^2} = \left(GK + \frac{Hb^2}{2}\right)\frac{\partial^2\varphi}{\partial z^2} - \frac{EI\,.\,b^2}{4}\frac{\partial^4\varphi}{\partial z^4} - \frac{\partial^2 u}{\partial z^2}\mathfrak{M} + S_t\, pb^2\left(\varphi - \frac{1}{v}\frac{\partial u}{\partial t}\right) \tag{3a}$$

in which, Θ is the polar moment of inertia of the cross section of the stiffening girder.

In solving the eq. (3a), it is assumed that

$$\left.\begin{array}{l}\varphi = A\,.\,\sin\lambda z \\ \lambda = \dfrac{n\pi}{l}\;(n=2,3\;\ldots.)\end{array}\right\} \tag{4a}$$

Then eq. (3a) is equivalent to the equation;

$$\frac{\Theta}{1}\frac{\partial^2 \varphi}{\partial t^2} = \underline{GK}\frac{\partial^2 \varphi}{\partial z^2} - \frac{\partial^2 u}{\partial z^2}\mathfrak{M} + S_t\, pb^2\left[\varphi - \frac{1}{v}\frac{\partial u}{\partial t}\right] \qquad (3b)$$

Where,

$$\underline{GK} = GK + \frac{b^2}{4}\lambda^2\left(EI + \frac{2H}{\lambda^2}\right) \qquad (5a)$$

$$\text{for } n = 2,\ \underline{GK} = GK + \frac{\pi^2 b^2}{l^2}\left(EI + \frac{l^2}{2\pi^2}H\right) \qquad (5b)$$

At first, we consider the torsional oscillation, neglecting the term containing du/dt.

There is some disturbance, such as Kármán vortex or fluctuations of wind intensity, which offer a chance to produce an alternating aerodynamic force. It can be assumed that the alternating torque acting on the girder is $T_o \sin \omega t$. T_o may be a function of wind velocity V, but at present very little is known about T_o. However, it is interesting to study rapidly the behavior of the bridge under the action of $T_o \sin \omega t$.

Then, the fundamental equation has the following expression, with the damping coefficient inserted.

$$\frac{\Theta}{1}\frac{\partial^2 \varphi}{\partial t^2} + C\frac{\partial \varphi}{\partial t} - \underline{GK}\frac{\partial^2 \varphi}{\partial z^2} + \frac{\partial^2 u}{\partial z^2}\mathfrak{M} - S_t\, pb^2\varphi = T_o \sin \omega t \qquad (6)$$

As eq. (6) contains the term d^2u/dz^2 we consider the deflection of the span. A deflection of a suspension bridge under a certain uniform load w is given the eq.:

$$EI\frac{d^4u}{dz^4} - 2H\frac{d^2u}{dz^2} = w$$

As long as we assume that

$$u = B \cdot \sin \lambda z$$

$$\lambda = \frac{n\pi}{l}\ (n = 2,3\ \ldots) \qquad (4b)$$

the above equation is equivalent to the following expression.

$$\left(EI + \frac{2H}{\lambda^2}\right)\frac{d^4u}{dz^4} = w$$

Hence the reduced flexional rigidity of a suspension bridge is:

$$EJ = EI + \frac{l^2}{(n\pi)^2}2H \qquad (7a)$$

For n = 2,

$$EJ = EI + \frac{l^2}{2\pi^2} H \tag{7b}$$

Then the reduced torsional rigidity, $\underline{GK}$ is

$$\left.\begin{aligned} \underline{GK} &= GK + \frac{(n\pi)^2 b^2}{4 l^2} EJ \\ n = 2, \quad \underline{GK} &= GK + \frac{\pi^2 b^2}{l^2} EJ \end{aligned}\right\} \tag{5c}$$

The subsequent analysis is limited to the case, n = 2, that is $\lambda = 2\pi/l$, because the one-noded torsional oscillation is fatal to the span.

As the η -component of bending moment $\mathfrak{M}$ due to wind pressure $\mathfrak{M}\,\varphi$, the deflection of the span is defined by the eq.

$$\left(EI + \frac{2H}{\lambda^2}\right)\frac{d^2u}{dz^2} = EJ\frac{d^2u}{dz^2} = -\mathfrak{M}\,\varphi$$

Furthermore, additional deflection due to lift force L can be considered. Considering this effect, the above equation becomes

$$EJ\frac{d^2u}{dz^2} = -\left(1 + \frac{Spb}{\lambda^2\mathfrak{M}}\right)\mathfrak{M}\varphi$$

and introducing the symbol

$$\mu^2 = 1 + \frac{Spb}{\lambda^2\,\mathfrak{M}} \tag{8a}$$

Then,

$$EJ\frac{d^2u}{dz^2} = -\mu^2\mathfrak{M}\varphi \quad \text{or} \quad \frac{d^2u}{dz^2} = -\mu^2\frac{\mathfrak{M}}{EJ}\varphi \tag{9}$$

The bending moment $\mathfrak{M}$ due to wind pressure is given approximately by the equation [1, 2];

$$\mathfrak{M} = \frac{C_d\,pbl^2}{\sqrt{128}} \tag{10}$$

And eq. (8a) yields to;

$$\mu^2 = 1 + \frac{\sqrt{128}}{4\pi^2}\frac{S}{C_d} \tag{8b}$$

Substituting Eq. (9) into eq. (6), and considering eq. (4a),

$$\frac{\Theta}{l}\frac{\partial^2\varphi}{\partial t^2} + C\frac{\partial\varphi}{\partial t} - \underline{GK}\frac{\partial^2\varphi}{\partial z^2} - \mu^2\frac{\mathfrak{M}^2}{EJ}\varphi - S_t\,p\,b^2\varphi = T_o \sin\omega t$$

or

where,

$$\left.\begin{aligned} &\frac{\Theta}{l}\frac{\partial^2\varphi}{\partial t^2}+C\frac{\partial\varphi}{\partial t}+K\varphi=T_o \quad \sin\omega t \\ &K=\left[\underline{GK}\lambda^2-\mu^2\frac{\partial n^2}{EJ}-S_t p b^2\right] \end{aligned}\right\} \tag{11}$$

Its solution for $T_o = O$;

$$\left.\begin{aligned} &\varphi=e^{-\frac{C}{2\Theta/l}t}\cdot\left[A_1 \sin g t+A_2 \cdot \cos g t\right] \\ &g=\sqrt{\frac{K}{\frac{\Theta}{l}}-\frac{C^2}{4\left(\frac{\Theta}{l}\right)^2}} \end{aligned}\right\} \tag{12}$$

Particular Solution of eq. (11) is;

$$\varphi=\frac{T_o \cdot \sin(\omega t-\varphi_o)}{\sqrt{\left(K-\frac{\Theta}{l}\omega^2\right)^2+(C\omega)^2}} \tag{13}$$

where,

$$\tan\varphi_o=\frac{C\omega}{K-\frac{\Theta}{l}\omega^2} \tag{14}$$

The critical damping ν is given by the equation,

$$\nu^2=4\frac{\Theta}{l}K \tag{15}$$

As the natural torsional frequency of the span in still air ($\nu = O$) is,

$$\omega_\varphi^{\,2}=\frac{GK\cdot\lambda^2}{\Theta/l} \tag{16}$$

the critical damping in still air ν_o is exppressed by the equation;

$$\nu_o=2\frac{\Theta}{l}\omega_\varphi \tag{17}$$

With eq. (17), the solution (13) becomes,

$$\varphi = \frac{T_o \sin(\omega t - \varphi_o)}{\frac{\Theta}{l} \cdot \omega_\varphi^2} \cdot \frac{\omega_\varphi^2}{\sqrt{\left\{\omega_\varphi^2 - \omega^2 - \frac{S_t \rho b^2}{\Theta/l} - \frac{\mu^2 \mathcal{H}^2}{EJ \cdot \Theta/l}\right\}^2 + 4\left(\frac{C}{\nu_o}\right)^2 \omega_\varphi^2 \cdot \omega^2}}$$

From this equation, the dynamic magnifier $\mathcal{H}$ is obtained.

$$\mathcal{H}(\omega, V) = \frac{1}{\sqrt{(1 - Y^2 - \bar{A}X - \bar{B}X^2)^2 + 4\left(\frac{C}{\nu_o}\right)^2 Y^2}} \tag{18}$$

where,

$$\left\{\begin{array}{ll} X = \left(\frac{V}{\omega_\varphi . b}\right)^2, & Y = \frac{\omega}{\omega_\varphi} \\ \bar{A} = \frac{S_t \rho b^2}{2\frac{\Theta}{l}} . b^2, & \bar{B} = \frac{(\mu C_d \rho b l^2)^2}{512 EJ . \frac{\Theta}{l}} . \omega_\varphi^2 b^4 \end{array}\right. \tag{19}$$

$\mathcal{H}$ is a function of ω and V. Neglecting the damping term,

$$\mathcal{H}_1 = \frac{1}{1 - Y^2 - \bar{A}X - \bar{B}X^2} \tag{20}$$

When the denominator in eq. (20) diminishes until zero value, $\mathcal{H}_1$ becomes infinite, and the wind velocity for ω is obtained from the equation.

$$1 - Y^2 - \bar{A}X - \bar{B}X^2 = O$$

or

where,

$$\left.\begin{array}{l} BV_\omega^4 + AV_\omega^2 - (\omega_\varphi^2 - \omega^2) = O \\ A = \frac{S_t \rho b^2}{2\frac{\Theta}{l}} \qquad B = \frac{(\mu C_d \rho b l^2)^2}{512\frac{\Theta}{l} EJ} \end{array}\right| \tag{21}$$

V_ω ...... Wind velocity for ω
ω ...... Observed torsional Osc. (rad/sec)
ω_φ ...... Natural torsional Osc. (rad/sec)

In eq. (21) the term involving V^4 is the dominating factor at high wind velocity. Consequently, it is expected that the observed frequency ω becomes small as V increases. At the critical state, ω becomes zero. The critical wind velocity V_k is obtained for $\omega = 0$ in eq. (21), that is

$$BV_k^4 + AV_k^2 - \omega_\varphi^2 = O \tag{22}$$

This critical condition is also obtained from the equation of free oscillation;

$$\frac{\Theta}{l}\frac{d^2\varphi}{dt^2}+\left[\underline{GK}\,\lambda^2-\mu^2\frac{\mathfrak{M}^2}{EJ}-S_t\,pb^2\right]\varphi=O \tag{23a}$$

Its solution is,

$$\varphi=A_1\sin qt+A_2\cos qt$$

$$\text{where,}\qquad q=\sqrt{\frac{\dot{K}}{\Theta/l}} \tag{23b}$$

For q = 0 the same condition is obtained with eq. (22). Neglecting the second term in eq. (22);

(For one-noded tor. osc., $\lambda=\frac{2\pi}{l}$)

$$V_k{}^2=\frac{4\pi\sqrt{128}\,\sqrt{EJ\,.\,\underline{GK}}}{\mu\,Cd\varrho\,bl^3} \tag{24a}$$

This condition corresponds to the stability condition which is obtained from the following equation.

$$\underline{GK}\frac{d^2\varphi}{dz^2}+\mu^2\frac{\mathfrak{M}^2}{EJ}\varphi=O \tag{25}$$

The above equation is equal to a torsional buckling (Kippung) of a suspension bridge.

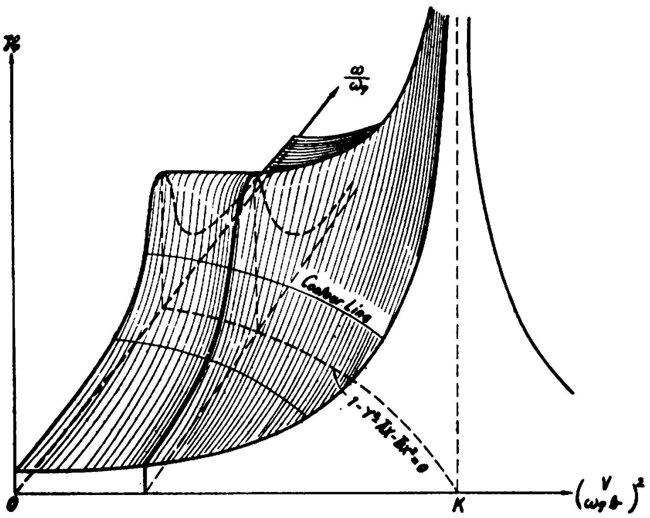


FIG. 3

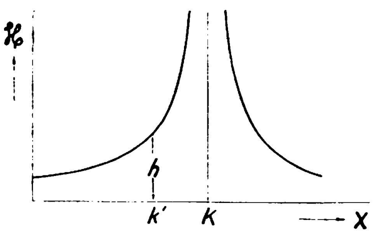


FIG. 4

Again we return to the eq. (18). The function $\mathcal{K}$ varies as shown in Fig. 3. The locus of the crest projected on XY-plane is given by $\partial\mathcal{K}/\partial x=O$. Its equation is;

$$1-Y^2-\bar{A}X-\bar{B}X^2=O \tag{26}$$

which corresponds to the eq. obtained from eq. (23)

$$\frac{\Theta}{l} q^2 = \underline{GK} \lambda^2 - S_t pb^2 - \mu^2 \frac{\mathcal{M}^2}{EJ}$$

Theoretically, the height of the peak at point K in Fig. 3 is large, meaning that a damping term is inactive at this point. The critical velocity V_k given by eq. (22) corresponds to point K. The curve on $\mathcal{H}$-Y plane is a dynamic magnifier or a resonance diagram when wind dies away.

We can sketch a contour line of the $\mathcal{H}$-diagram by cutting it with a horizontal plane of height h. And its equation is

$$(1 - Y^2 - \bar{A}X - \bar{B}X^2)^2 = \frac{1}{h^2} - 4\left(\frac{C}{\nu_0}\right)^2 Y^2 \tag{27}$$

The curve on $\mathcal{H}$-X plane (Fig. 4) refers to the stability of suspension bridge. At point K in fig. 4, the amplitude of torsional oscillation becomes infinitely large, but practically, the amplitude will be sufficiently large at point K' in the figure and the bridge will be destroyed by wind. As we lack information about the disturbing force T_o, it is difficult to determine the point K' theoretically. But it is possible to determine it experimentally, and we can determine the value of h in fig. 4.

If the value of h (the magnitude of $\mathcal{H}$ at wrecking-point K') is known, the corresponding wind velocity is calculated by the equation;

$$X = \frac{-\bar{A} \pm \sqrt{\bar{A}^2 + 4\bar{B}\left(1 - \frac{1}{h}\right)}}{2\bar{B}} \tag{28a}$$

Neglecting the term $\bar{A}$, (This is on the safety side as long as $S_t < O$)

$$X = \frac{1}{\sqrt{\bar{B}}} \cdot \sqrt{1 - \frac{1}{h}} \tag{28b}$$

And the following equation is obtained instead of eq. (24a)

$$V_k{}^2 = \frac{4\pi \sqrt{128} \sqrt{EJ \cdot \underline{GK}}}{\mu\, C_d\, \rho\, bl^3} \cdot \sqrt{1 - \frac{1}{h}} \tag{24b}$$

The above equation is also transformed, assuming that

$$\underline{GK} \doteqdot \pi^2 \frac{b^2}{l^2} EJ \tag{29}$$

$$\left(\frac{V_k}{N_\varphi b}\right)^2 = \left(\frac{2r}{b}\right)^2 \cdot \frac{\frac{2m}{g}}{\rho\, b^2} \cdot \frac{\sqrt{128}}{\mu\, C_d} \sqrt{1-\frac{1}{h}} \tag{30a}$$

where, $r^2 = \frac{\Theta/l}{2\,m/g}$ r; radius of gyration

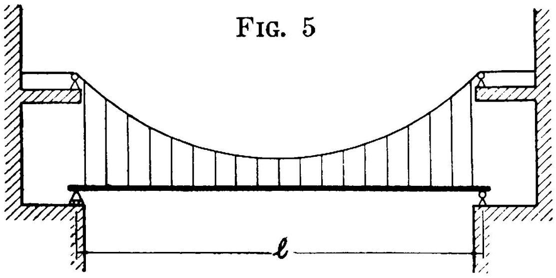
If we represent the wind pressure W by the equation,

$$W = k\,\rho\,F\,V^2 \tag{31}$$

k .... Coefficient.
F .... Area of exposed surface per unit length of the stiffening girder.

then eq. (30a) is expressed as follows.

FIG. 5



$$\left(\frac{V_k}{N_\varphi b}\right)^2 = \left(\frac{2r}{b}\right)^2 \frac{\frac{2m}{g}}{\rho F b} \frac{\sqrt{128}}{2k\mu} \sqrt{1-\frac{1}{h}} \tag{30b}$$

II. *Full Model Test in small scale* (in cooperation with prof. M. Yasumi).

To check up the adequacy of the analysis reported in the preceeding article, wind tunnel tests were carried out in cooperation with prof. M. Yasumi (Univ. of Osaka) since 1949. However, it required several years, the cost of the tests being comparatively high.

The measurements of aerodynamic characteristics of the stiffening girder sections were conducted at the University of Tokyo. The tests on model suspension bridges were carried out in a 3. 5m Wind Tunnel at the University of Osaka. The model suspension bridgs was small scale. (Fig. 5). Its span length was 3m, its width 4cm, and sag/span ratio was 1/10. The girder sections were H-shaped with a depth-width ratio of 0.10, 0.15, and 0.20. The girders were made of brass plate, and thin lead plates were added as weights.

The behavior of suspension bridge under statical lateral loading was also investigated. The studies showed that the natural frequency of the suspension bridge under the action of lateral load decreases with increasing loadings. Fig. 6 shows a test of torsional buckling (Kippung) of a suspension bridge due to lateral load.

One of the results obtained from wind tunnel tests is as follows.

Model - N.° 1; $2m = 5.94$ gr/cm $\quad \frac{\Theta}{l} = 0.00775$ gr-sec

$EJ = 7686 \times 10^3$ gr-cm^2 $\quad GK = 162 \times 10^3$ gr-cm^2

$C_d = 0.243$ $\quad \overline{S} = 5.64$

$S_t = -0.482$ $\quad \mu = 2.77$

$\omega_\varphi = 95.7$ rad/sec (calculated)

$N_\varphi = 15.2$ cyc/sec (»), 15.5 cyc/sec (observed)

The critical velocities obtained from test were 11.9 m/sec and 11.8 m/sec. Observations of torsional frequencies were made by using an electrical-resistance gage. One of its records on an oscillograph is

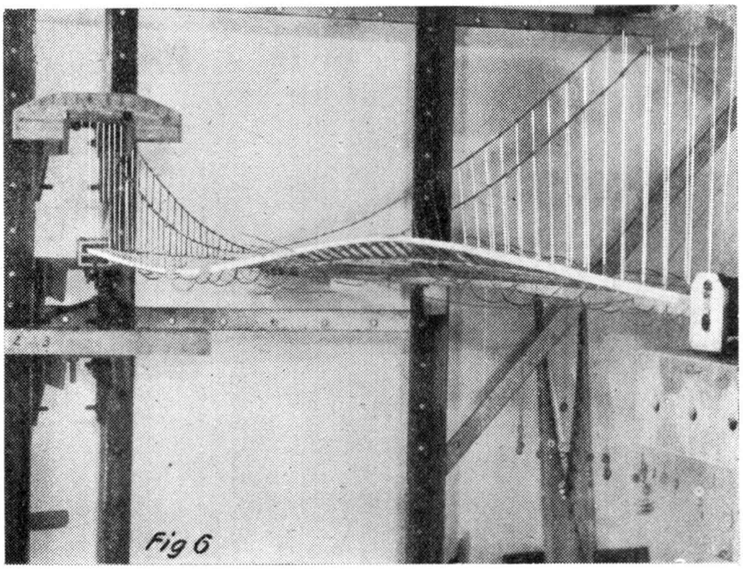


FIG. 6

shown in fig. 7, and it indicates a long period of torsional oscillation.

When the velocity approached the predicted critical wind velocity,

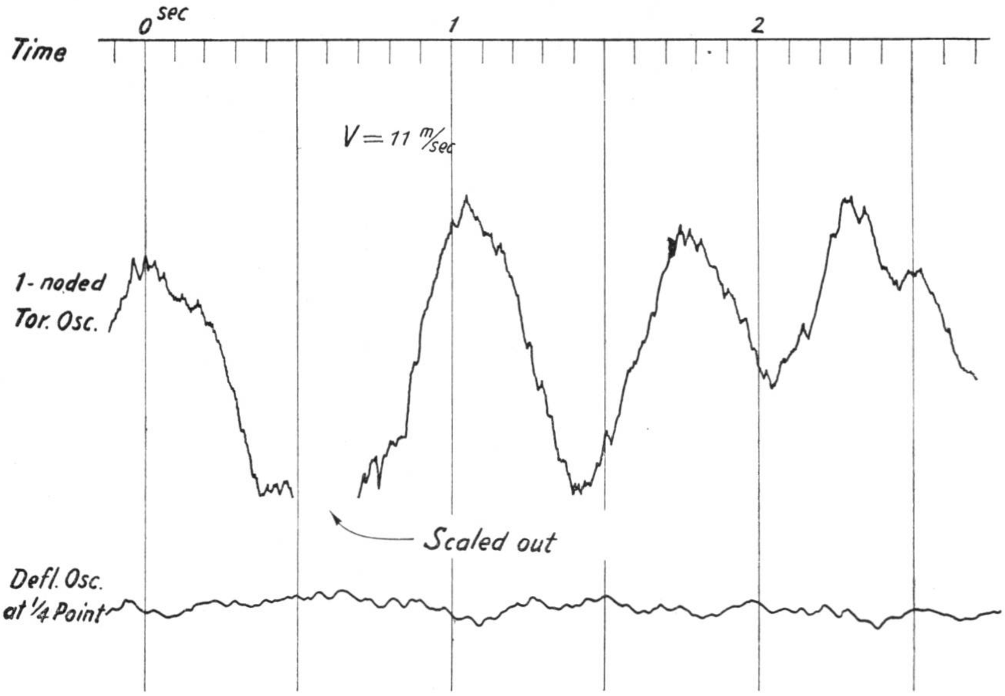


FIG. 7

slow torsional oscillations (One-noded osc.) were observed with naked eyes, which differed from the natural frequency in still air. The photograph (Fig. 8) shows the instant when the span was wrecked by wind.

The last shape of the bridge is similar to the form of the torsional buckling in Fig. 6. Fig. 9 shows the $\mathcal{H}$-diagram of the model suspension bridge calculated from eq. (18). Two contour lines for $\mathcal{H} = 2$ and $\mathcal{H} = 4$ are shown in figure. The critical points obtained from tests are indicated with two flags.

$\mathcal{H}$-X curve for Model N.° 1 is given in Fig. 10, in which the wrecking point (V = 11.9 m/sec) is shown as X = 9.65.

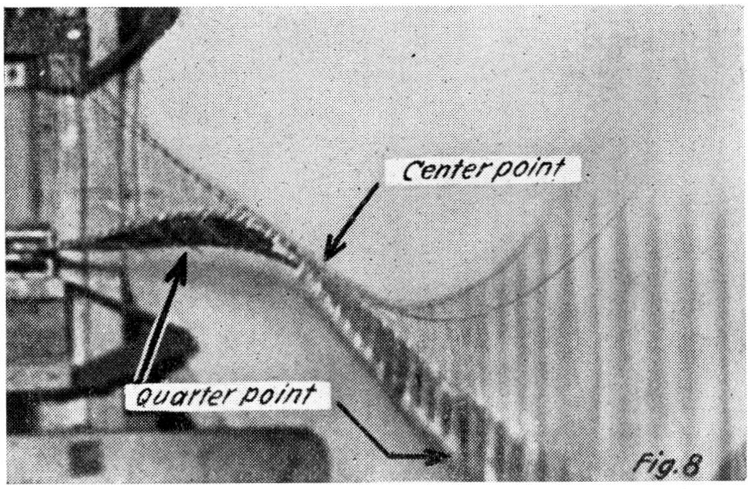


FIG. 8

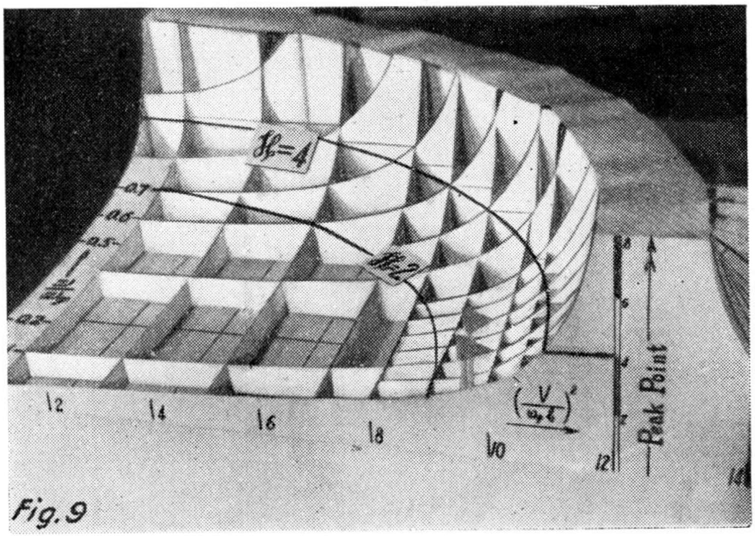


FIG. 9

Neglecting the term $\bar{A}$ and assuming h = 3.5 in eq. (24b) the critical velocity is $V_k = 12.2$ m/sec which compares to the observed value 11.9 m sec.

Experimental results on 4 models are given in Table-1 and Table-2. Fig. 11 shows one of the records of Torsional Oscillation. (Model N.° 3) A mean value of «h» obtained from the narrowly confined studies is 3.48. To decide the correct value of «h» (Dynamic magnifier at wrecking instant) more experimental studies will be required.

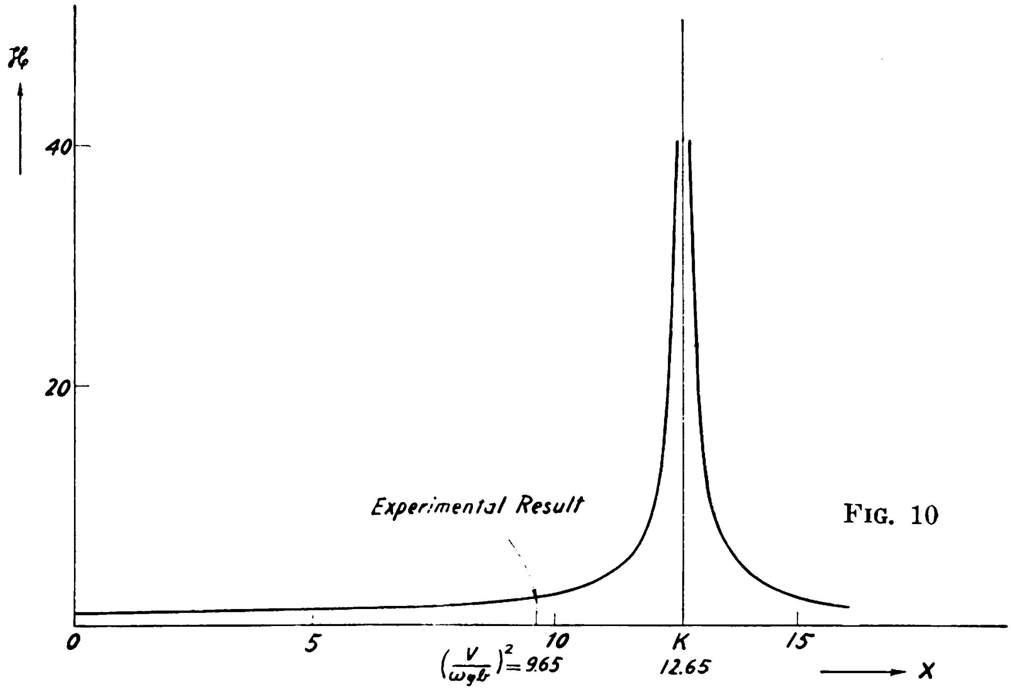


FIG. 10

TABLE 1

Model N.°	N.° 1	N.° 2	N.° 3	N.° 4
d/b	0.20	0.20	0.15	0.10
C_d	0.243	0.243	0.187	0.137
S	5.64	5.64	5.74	8.62
S_t	- 0.482	- 0.482	- 0.504	- 0.515
μ	2.766	2.766	3.130	4.362
m gr/cm	2.971	2.828	2.569	2.848
$(2m/g) \times 10^{-3}$ $\frac{gr - sec^2}{cm^2}$	6 063	5.771	5.243	5.812
$(\Theta/l) \times 10^{-2}$ gr - sec²	0.775	0.807	0.619	0.698
H kg	1.114	1.061	0.963	1.068
EI kg - cm²	2607	3483	927	361
EJ kg - cm²	7686	8321	5318	5231
GK kg - cm²	149	93	75	114
GK kg - cm²	162	108	84	123
b/2r	1.77	1.69	1.84	1.82
N_φ cyc/sec (calculated)	15.2	12.2	12.3	14.0
$\frac{\sqrt{EJ \cdot GK}}{\mu C_d}$ kg - cm²	1660	1410	1142	1342
$\frac{4\pi \sqrt{128} \sqrt{EJ \cdot GK}}{\mu C_d \rho b l^3}$ m²/sec²	174.84	148.53	120.26	141.36

Note: d... depth of stiffening girder.

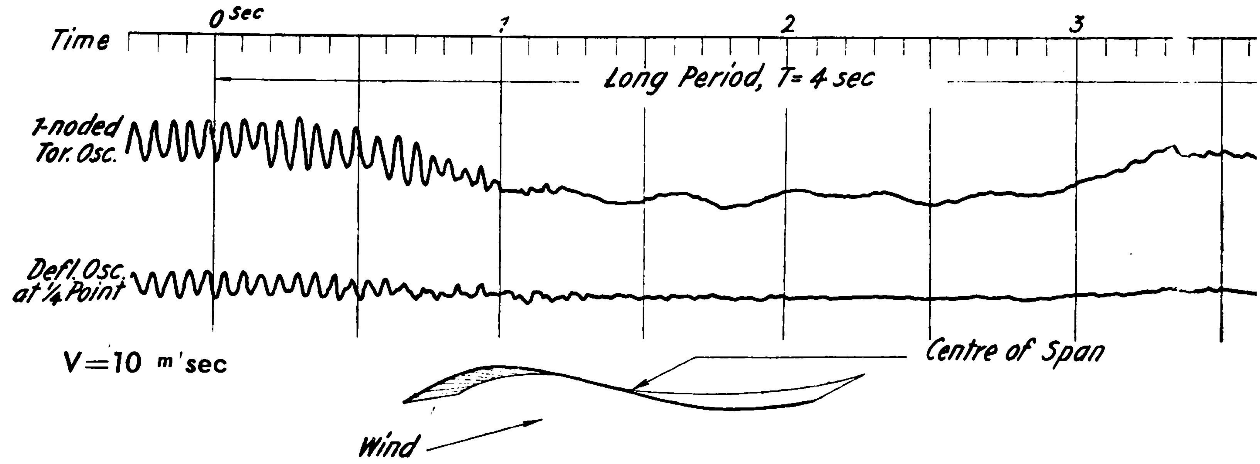


TABLE 2

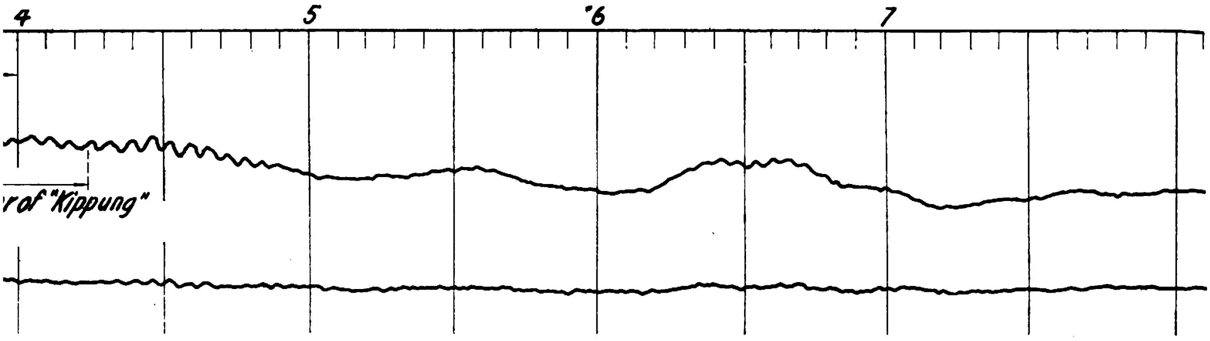
Model N.°	N.° 1	N.° 2	N.° 3	N.° 4
Observed critical velocity in m/sec	11.9	about 10.9	10.3	11.0
$\sqrt{\frac{4\pi \sqrt{128} \sqrt{EJ \cdot GK}}{\mu \, C_d \, \rho \, b \, l^3}}$ (m/sec)	13.2	12.2	11.0	11.9
Value of «h» obtained from eq. (24b)	2.9	2.8	4.5	3.7
$\sqrt[4]{1 - \frac{1}{h}}$	0.90	0.89	0.94	0.93

Mean value of «h» obtained from experiments is 3.48.

TABLE 3

Comparison of predicted and observed values

Model N.°	N.° 1	N.° 2	N.° 3	N.° 4
$V_k = \sqrt{\frac{120 \sqrt{EJ \cdot GK}}{\mu \, C_d \, \rho \, b \, l^3}}$ (m/sec)	12.2	11.2	10.1	10.9
Observed critical velocity (m/sec)	11.9	10.9	10.3	11.0



Substituting h=3.48 in eq. (24b) the following expression is obtained.

$$V_k^2 = \frac{120\sqrt{EJ \cdot GK}}{\mu C_\alpha \rho b l^3} \tag{32}$$

We can estimate the necessary stiffness of a stiffening girder, by introducing a proper factor of safety. Flexional stiffness of a suspension bridge is approximately given by the equation, [substituting V = 60 m/sec and considering eq. (29)].

$$EJ = 1.8 \ (\mu C_d l^4) \tag{33}$$

which corresponds to the equation;

$$V_k^2 = \frac{80\sqrt{EJ \cdot GK}}{\mu C_\alpha \rho b l^3}$$

III. *Coupled Oscillation.*

We consider next the coupled oscillation of torsional and deflectional vibration in suspension bridges. The critical condition is also obtained from the equation of free vibration.

In 1947 the writer developed [3] the coupled oscillation according to Mr. Theodorsen, but in the present article he considers the aerodynamic forces as in the preceeding articles in order to obtain a general outlook.

The equation of flexional oscillation of the span is introduced by remarking that the η -component of the bending moment due to wind pressure is $\varphi \mathfrak{M}$. [3, 4] And considering the relation

$$\frac{d^2\varphi}{dz^2} \doteq \frac{\mathfrak{M}}{GK}\frac{d^2u}{dz^2}$$

the fundamental differential equation of suspension bridge is represented approximately as follows:

$$\frac{2m}{g}\frac{d^2u}{dt^2}+\underset{\cdot}{E}u-S'pb\varphi+D\frac{du}{dt}=O$$

$$\frac{\Theta}{1}\frac{d^2\varphi}{dt^2}+\underset{\cdot}{K}\varphi+S_t\,p\frac{b^2}{V}\frac{du}{dt}+C\frac{d\varphi}{dt}=O \qquad (34)$$

where,

$$\underset{\cdot}{E}=EJ.\lambda^4-\frac{\mathfrak{M}^2}{\underline{GK}}.\lambda^2$$

$$\underset{\cdot}{K}=\underline{GK}.\lambda^2-\frac{\mathfrak{M}^2}{EJ}-S_t\,pb^2$$

$$D=\frac{S'pb}{V}+\Delta \quad (\Delta;\ \text{damping coeff.})$$

$$S'=S+C_d$$

The solution is assumed to be of a sinusoidal form, then the characteristic equation has the quartic form, neglecting the damping coefficient.

$$x^4+ax^3+bx^2+cx+d=O$$

$$a=\frac{s'pb}{\frac{2m}{g}V} \qquad b=\frac{\underset{\cdot}{E}}{\frac{2m}{g}}+\frac{\underset{\cdot}{K}}{\frac{\Theta}{1}} \qquad (35)$$

$$c=\left[\frac{s'pb}{V}\underset{\cdot}{K}+s's_t\frac{p^2b^3}{V}\right]\frac{1}{\frac{2m}{g}\frac{\Theta}{1}}$$

$$d=\frac{\underset{\cdot}{K}\underset{\cdot}{E}}{\frac{2m}{g}\frac{\Theta}{1}}$$

The principal stability conditions derived from eq. (35) may be summarized as follows.

$$\left.\begin{array}{ll}\text{i)} & S'=S+C_d>0\\ \text{ii)} & S_t<0\\ \text{iii)} & \text{eq. (24a)}\end{array}\right\} \qquad (36)$$

In analysing the forced oscillation of the span, the writer has assumed the disturbing force shown in the following equations, according to the eq. (6).

$$\left.\begin{aligned}
&\Phi \ddot{\varphi} + K \varphi + C \dot{\varphi} + F_t \dot{U} = T_o \ \sin \omega t \\
&M \ddot{U} + EU - F \varphi + D\dot{U} = \frac{e}{b} T_o \ \sin \omega t
\end{aligned}\right.$$

where,

$$\left.\begin{aligned}
&\Phi = \frac{\Theta}{l} \qquad M = \frac{2\,m}{g} \\
&K = \underline{GK} \,.\, \lambda^2 - \frac{\mathfrak{M}^2}{EJ^{\wedge}} - S_t\ pb^2 \\
&E = EJ \,.\, \lambda^4 - \frac{\mathfrak{M}^2}{\underline{GK}} \,.\, \lambda^2 \\
&F_t = \frac{s_t\, p\, b^2}{V} \qquad F = s'pb \\
&D = \frac{s'pb}{V} + \Delta
\end{aligned}\right\} \quad (37)$$

The particular solution is, with the notation $j = \sqrt{-1}$,

$$\varphi = \frac{[(E - M\omega^2) + j\omega\varepsilon]\, T_o}{(K - \Phi\omega^2)(E - M\omega^2) - \omega^2 CD + j\omega\,[D(K - \Phi\omega^2) + C(E - M\omega^2) + FF_t]} \tag{38}$$

where, $\varepsilon = D - F_t\ e/b$

The magnitude of φ is defined by the equation,

$$\left(\frac{\varphi}{T_o}\right)^2 = \frac{(E - M\,\omega^2)^2 + \omega^2\,\varepsilon^2}{[(K - \Phi\omega^2)(E - M\omega^2) - \omega^2 CD]^2 + \omega^2[D(K - \Phi\omega^2) + C(E - M\omega^2) + FF_t]^2} \tag{38a}$$

Neglecting the damping factor Δ and C;

$$\left(\frac{\varphi}{T_o}\right)^2 = \frac{1}{\left(\frac{\Theta}{l}\right)^2 \omega_\varphi^4} \cdot \frac{(Z^2 - Y^2 - B'X^2Z^2)^2 + \frac{Y^2}{(2m/g)^2}\frac{\varepsilon^2}{\omega_\varphi^2}}{(1 - Y^2 - \bar{A}X - B'X^2)^2(Z^2 - Y^2 - B'X^2Z^2)^2 + \frac{Y}{(2m/g)^2}\frac{1}{\omega_\varphi^2}\left(\frac{S'pb}{V}\right)^2(1 - Y^2 - B'X^2)^2} \tag{38b}$$

where,

$$X = \left(\frac{V}{\omega_\varphi\, b}\right)^2 \qquad Y = \frac{\omega}{\omega_\varphi}$$

$$\varepsilon = D - F_t\ \frac{e}{b}$$

$$\bar{A} = \frac{S_t\ \rho\ b^2}{2\ \Theta/l} \,.\, b^2 \qquad B' = \frac{(Cd\ \rho\ bl^2)^2}{512\,EJ \,.\, \Theta/l} \,.\, \omega_\varphi^2\ b^4$$

The managing terms are the last term in the denominator and in the numerator. Hence the dynamic magnifier is represented by,

$$\mathcal{K}^2 = \frac{(S' - e S_t)^2}{(S')^2} \; \frac{1}{[1 - Y^2 - B'X^2]^2} \tag{39}$$

Comparing with eq. (20), eq. (39) is multiplied by a coefficient which involves an aerodynamic coefficient and does not include an $\overline{A}$-term. Furthermore, eq. (39) shows again that the «Kipperscheinung» due to wind pressure plays an important part in stability problems of suspension bridges.

If eq. (39) is correct in spite of involving some assumptions and a certain approximation, it seems to be preferable to choose a section which has the characteristic $S_t > 0$, but this condition is against eq. (36). As our experimenal studies have been restricted, it is difficult, presently to clear this point definitely. The writer never theless recomends, at present, to choose the value S_t as $S_t \doteqdot 0$, and when this is impossible, $S_t < 0$.

IV. *Section-Model Experiment.*

As the full size model test is expensive, a «section model test» is often carried out. Let us consider the section model with a fixed center of rotation as shown in Fig. 12. Spring constant per unit length per one side, is noted K. The fundamental equation is:

$$\frac{\Theta}{l}\frac{d^2\varphi}{dt^2} + \left[\frac{Kb^2}{2} - S_t p b^2\right]\varphi + C\frac{d\varphi}{dt} = T_0 \sin \omega t \tag{40}$$

While, the equation of torsional oscillation in suspension bridge is represented by eq. (11)

$$\frac{\Theta}{l}\frac{d^2\varphi}{dt^2} + \left[\underline{GK} \cdot \lambda^2 - \mu^2 \frac{\mathfrak{M}^2}{EJ} - S_t p b^2\right]\varphi + C\frac{d\varphi}{dt} = T_0 \sin \omega t \tag{11}$$

Comparing eq. (40) and (11), it appears that a section model may be made to correspond to a full size model, as long as the term $\mathfrak{M}$ is inactive. Hence, in the comparatively low velocity range, a section model can play the part of a full size model. But in high wind velocity range, the wind pressure cannot be neglected. As the eq. (11) shows, the reduced spring constant of this system is not constant but decreases with increasing wind velocity, due to $\mathfrak{M}$ including the velocity V.

Dynamic magnifier of eq. (40) is,

$$\mathcal{K}_s = \frac{\omega_\varphi^2}{\sqrt{\left[\omega_\varphi^2 - \omega^2 - \frac{S_t p b^2}{\Theta/l}\right]^2 + 4\left(\frac{C}{\nu_0}\right)^2 \omega_\varphi^2 \, \omega^2}} \tag{41a}$$

Where, ν_o is the critical damping in still air. Neglecting the damping term, we obtain:

$$\mathcal{K}_s = \frac{\omega_\varphi^2}{\omega_\varphi^2 - \omega^2 - \dfrac{S_t \rho b^2}{2\,\Theta/l} V^2} \qquad (41b)$$

Making the denominator of eq. (41b) equal to zero, the critical condition of the system is introduced;

$$\omega_\varphi^2 - \omega^2 - \frac{S_t \rho b^2}{2\,\Theta/l} V^2 = O \qquad (42)$$

To check the adequacy of the above analysis, data introduced by Dr. Fr. Bleich [5] will be mentionned here.

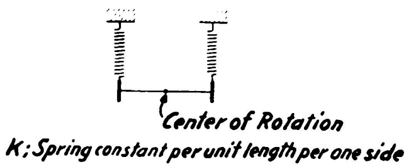


FIG. 12

$b = 90$ ft. $\qquad r = 27.8$ ft.

$$\frac{2m}{g} = 555 \, \frac{\text{lb.sec}^2}{\text{ft}^2}$$

Configuration I	Natural torsional frequency	ω_φ	$= 0.929$ rad/sec
	Observed torsional frequency for $V\omega_1$	ω_1	$= 0.856$ rad/sec
	Observed wind velocity	V_{ω_1}	$= 94$ ft/sec
Configuration II	Natural torsional frequency	ω_φ	$= 1.74$ rad/sec
	Observed torsional frequency for $V\omega_2$	ω_2	$= 1.67$ rad/sec
	Observed wind velocity	V_{ω_2}	$= 127$ ft/sec

Substituting the numerical values of configuration I in the eq. (42):

$$\frac{S_t}{\Theta/l} = \frac{2\,(\omega_\varphi^2 - \omega_1^2)}{\rho\, b^2 V^2} = \frac{2\,[(0.929)^2 - (0.856)^2]}{2.378 \times 10^{-3} . (90)^2 . (94)^2}$$

$$= \frac{1.531}{10^6} \left(\frac{1}{\text{lb} - \text{sec}^2} \right)$$

Substituting the above value and the values of configuration II in eq. (42) again, the predicted critical wind velocity for configuration II will be $V = 127.3$ ft/sec.

The observed figure is 127 ft/sec, which corresponds to the calculated value.

V. ***General Considerations.***

The present analysis is based on the assumption that the aerodynamic alternating torque is represented by $T_0 \sin \omega t$. There seems to exist many opportunities of resonance between the frequency ω — it does not always follow that the aerodynamic force is a source of disturbance with a frequency ω — and the frequency of the suspension bridge, as the frequency of the suspension bridge, under wind action is not constant, as shown in Fig. 3.

Kármán vortex or fluctuation of wind intensity is considered to be the origin of the alternating aerodynamic torque. In any case, minimizing a vortex discharge is an efficient step to do away with a certain amount of oscillation. The term T_0 remains untouched, as the writer knowlege of aerodynamics is restrained.

It is interesting to note that the frequency of the alternating torque which determines the fate of a suspension bridge is a slow frequency, as the dynamic magnifier has an infinite height at point K in Fig. 3. Due to this, the records of torsional oscillations obtained from the experiment at the collapse of the bridge, as can be seen in Fig. 7 or 11, are the interesting figures.

Supposing that there exists no aerodynamic alternating force, — when the wind velocity approaches the critical velocity V_k, the suspension bridge will be wrecked by wind pressure. This phenomenon is known as the «Kipperscheinung», as shown in eq. (25).

Considering this, the aerodynamic alternating torque $T_0 . \sin \omega t$ is introduced as a matter of convenience to obtain the critical velocity. The suspension bridge apparently produces a play in costume of «vibration» before the limelight of «aerodynamic disturbance», but the producer behind the stage seems to be the «Kipperscheinung» or torsional buckling of a suspension bridge.

The critical wind velocity for the fatal singlenoded torsional oscillation is given by the eq. (24b).

$$V_k^2 = \frac{4\pi \sqrt{128} \sqrt{EJ . \underline{\underline{GK}}}}{\mu C_d \rho b l^3} \sqrt{1 - \frac{1}{h}} \tag{24b}$$

From this equation the necessary stiffness of a stiffening girder may also be calculated. Increasing dead load results in an increase of the effective stiffness of a suspension bridge, thus the dead load plays the part of a flexional stiffness EI of the stiffening girder as slown in eq. (7b) and (5c).

$$EJ = EI + \frac{l^2}{2\pi^2} H \tag{7b}$$

$$\underline{\underline{GK}} = GK + \frac{\pi^2 b^2}{l^2} EJ \tag{5c}$$

The use of both top and bottom lateral systems in the stiffening girder (truss type) increases the GK in eq. (5c).

Installing of center diagonal ties is an effective means of preventing single-noded torsional motion, and in this case the stability conditions for $\lambda = 3\pi/l$ and etc. make their appearance.

As can be seen from eq. (24b), a small value of μ raises the critical wind velocity. Streamlining the shape of the cross section of a stiffening girder seems to increase the value of coefficient μ which contains coefficient S.

To prevent the dangerous self-excited vibration of the span, requires $S' = S + C_d > 0$

Regarding the coefficient S_t, the writer recommends to choose a value $S_t = 0$, if possible.

Eq. (24b) is transformed into the following expression.

$$\left(\frac{V_k}{N_\varphi b}\right)^2 = \left(\frac{2r}{b}\right)^2 \times \frac{\frac{2m}{g}}{\rho b^2} \times \frac{\sqrt{128}}{\mu C_d} \sqrt{1 - \frac{1}{h}} \tag{30a}$$

This expression offers some resemblance to the experimental results obtained by prof. Farquharson [6], but the critical wind velocity defined by prof. Farquharson corresponds to the starting point of the catastrophic torsional oscillation, whereas the critical velocity V_k in the present paper corresponds to the last instant of the bridge oscillation.

Substituting the numerical value for the New Tacoma bridge in eq. (32), assuming $h = 3.48$;

$l = 2800$ ft. $\quad b = 60$ ft.

$S = 1.54 \quad C_d = 0.25$ (7)

$m = 4339$ lb/ft/cable $\quad H = 15.187 \times 10^6$ lb

$EI = 277.53 \times 10^{10}$ lb — ft² (8)

$EJ = 8.807 \times 10^{12}$ lb — ft²

$$GK \doteq \frac{\pi^2 b^2}{l^2} EJ = 39.91 \times 10^9 \quad \text{lb — ft}^2$$

$\mu = 1.66$

From eq. (32) the critical wind velocity is:

$$V_k = 234 \text{ ft/sec} = 160 \text{ mile/hour} = 71 \text{ m/sec}$$

Finally, the writer has much pleasure in expressing his grateful thanks to Emeritus prof. Y. Tanaka who has kindly afforded great facilities for investigation of the present problems. (22, April, 1955)

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SUMMARY

The frequency of the fatal single noded torsional oscillation of a suspension bridge, under forced vibration due to aerodynamic alternating force, is different from the natural frequency (wind off), as the drag coefficient of the stiffening girder plays an important part in the higher wind velocity range.

The critical wind velocity V_k (m/sec) is expressed approximately by the equation:

$$V_k^2 = \frac{120\sqrt{EJ \cdot GK}}{\mu\, C_d\, \rho\, b\, l^3} \tag{1}$$

In practice, it will be reasonable to take a correct safety factor. Then the necessary stiffness of a stiffening girder (including truss type) can be estimated.

The flexional stiffness of suspension bridge is approximately given by the equation obtained by introducing V = 60 m/sec into eq. (1).

$$EJ = 1.8\ (\mu\, C_d\, l^4) \quad \text{in Kgm}^2 \tag{2}$$

which corresponds to the equation,

$$V_k^2 = \frac{80\sqrt{EJ \cdot GK}}{\mu\, C_d\, \rho\, b\, l^3} \tag{3}$$

The greater the dead load, the greater the effective stiffness of a suspension bridge.

As to the shape of cross-section of the stiffening girder, it is preferable to choose the section which has minimum vortex discharge and $S' = S + C_d > 0$. Streamlining the cross section of a stiffening girder seems to bring an increase of the coefficient μ which involves a lift coefficient and a drag coefficient.

Regarding the coefficient S_t, the writer recommends at present to choose its value $S_t \doteqdot 0$, and when this is impossible, $S_t < 0$.

As to the adequacy of the numerical factor in eq. (1) or (3) further experimental research is expected.

ZUSAMMENFASSUNG

Die unter der Einwirkung einer wechselnden Windkraft auftretende Frequenz der gefährlichen, einknotigen Torsionsschwingung einer Hängebrücke ist verschieden von der Eigenfrequenz, da der Widerstandskoeffizient des Versteifungsträgers bei hohen Windgeschwindigkeiten eine wichtige Rolle spielt.

Die kritische Windgeschwindigkeit V_k (m/sek) wird durch folgende Näherungsgleichung erfasst:

$$V_k^2 = \frac{120 \sqrt{EJ \cdot GK}}{\mu\, C_d\, \rho\, b\, l^3} \qquad (1)$$

In der Praxis ist ein entsprechender Sicherheitsfaktor bei der Auswertung der obigen Gleichung zu wählen und damit ist auch die erforderliche Steifigkeit des Versteifungsträgers bestimmt (auch für Fachwerkträger).

Die wirksame Biegungssteifigkeit einer Hängebrücke wird annähernd durch folgende Gleichung gegeben, die durch Einsetzen von V = 60 m/sek in Gl. (1) folgt:

$$EJ = 1.8\ (\mu\ C_d\ l^4) \text{ in kgm}^2 \qquad (2)$$

Diese entspricht der folgenden Gleichung:

$$V_k^2 = \frac{80 \sqrt{EJ \cdot GK}}{\mu\, C_d\, \rho\, b\, l^3} \qquad (3)$$

Eine grössere ständige Last ergibt eine grössere wirksame Steifigkeit der Hängebrücke.

Es ist noch wünschenswert, am Versteifungsträger einen Querschnitt zu wählen mit minimaler Wirbel-Ablösung und mit $S' = S + C_d > 0$.

Wird der Querschnitt des Trägers stromlinienförmig gestaltet, dann scheint es, als ob der Beiwert μ sich vergrössere, der einen Widerstands- und einen Auftriebs-Beiwert enthält.

Hinsichtlich des Koeffizienten S_t möchte der Verfasser dessen Wert mit $S_t \doteqdot 0$ wählen, und wenn dies unmöglich wäre unter den gegenwärtigen Umständen, mit $S_t < 0$.

Ueber die Zweckmässigkeit numerischer Faktoren in Gl. (1) oder (3) sind weitere experimentelle Untersuchungen abzuwarten.

RESUMO

A frequência da oscilação de torsão de nó único que provoca a rotura de uma ponte suspensa sob o efeito de vibrações forçadas produzidas por esforços aerodinâmicos alternados difere da frequência própria (vento nulo), pelo coeficiente de resistência da viga de contraventamento ter um papel importante para valores elevados da velocidade do vento.

O valor aproximado da velocidade crítica do vento V_k (m/seg) é dado pela equação

$$V_k^2 = \frac{120 \sqrt{EJ \cdot GK}}{\mu C_d \rho b l^3} \quad (1)$$

Na prática é conveniente tomar-se um coeficente de segurança adequado. Pode-se então avaliar a rigidez necessária da viga de contraventamento (incluindo as vigas trianguladas).

A rigidez à flexão aproximada de uma ponte suspensa obtém-se partindo da equação (1) na qual V = 60 m/seg

$$EJ = 1{,}8 \, (\mu C_d l^4) \text{ em Kg/m}^2 \quad (2)$$

que corresponde à equação

$$V_k^2 = \frac{80 \sqrt{EJ \cdot GK}}{\mu C_d \rho b l^3} \quad (3)$$

Quanto maior é o peso próprio, maior é a rigidez efectiva de uma ponte suspensa.

No que se refere à forma da secção transversal da viga de contraventamento, convém escolher a que dá o vórtice mínimo e para a qual $S' = S + C_d > 0$. A adopção de formas aerodinâmicas para essa secção transversal parece causar um acréscimo do coeficiente μ que compreende um coeficiente de sustentação e um coeficiente de resistência.

No que diz respeito ao coeficiente S_t, o autor recomenda, que, por enquanto, se escolha $S_t = 0$ ou, caso não seja possível, $S_t < 0$.

Quanto à exactidão do valor do coeficiente numérico das equações (1) ou (3), convém esperar os resultados de novos estudos experimentais.

RÉSUMÉ

La fréquence de l'oscillation torsionnelle à noeud unique qui cause la rupture d'un pont suspendu sous l'effet des vibrations forcées produites par des efforts aérodynamiques alternés est différente de sa fréquence propre (vent nul), le coefficient de traînée de la poutre de raidissement jouant un rôle important pour des valeurs élevées de la vitesse du vent.

La valeur critique approximative de la vitesse du vent V (m/sec) est donnée par l'équation:

$$V_k^2 = \frac{120 \sqrt{\underline{\underline{EJ}} . \underline{\underline{GK}}}}{\mu \, C_d \, \rho \, b \, l^3} \qquad (1)$$

Dans la pratique il conviendra de prendre un coefficent de sécurité raisonnable. On pourra alors estimer la rigidité nécessaire de la poutre de raidissement (y compris les poutres triangulées).

La rigidité à la flexion approximative d'un pont suspendu est donnée par l'equation (1) dans laquelle V = 60 m/sec

$$EJ = 1{,}8 \; (\mu \; C_d \; l^4) \text{ en kgm}^2 \qquad (2)$$

qui correspond à l'équation

$$V_k^2 = \frac{80 \sqrt{\underline{\underline{EJ}} . \underline{\underline{GK}}}}{\mu \, C_d \, \rho \, b \, l^3} \qquad (3)$$

Plus le poids propre est important plus la rigidité effective d'un pont suspendu est grande.

Quant à la forme de la section transversale de la poutre de raidissement, il convient de choisir celle donnant le minimum de tourbillon et pour laquelle $S' = S + C_d > 0$. L'adoption de formes aérodynamiques pour cette section transversale semble entraîner une augmentation du coefficient μ qui comprend un coefficient de portance et un coefficient de traînée.

En ce qui concerne le coefficient S_t, l'auteur recommande, dans l'état actuel des choses, de prendre $S_t = 0$ ou, si ceci n'est pas possible $S_t < 0$.

Quant à l'exactitude de la valeur du coefficient numérique des équations (1) ou (3), il convient d'attendre les résultats de nouvelles études expérimentales.

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