Application of the semi-probabilistic method to reinforced concrete structures

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Application of the Semi-probabilistic Method to Reinforced Concrete Structures

Application de la méthode semi-probabiliste aux constructions en béton armé

Bemerkung zur Anwendung einer teilweise auf Wahrscheinlichkeitsrechnung beruhenden Methode auf die Berechnung von Eisenbetonkonstruktionen

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The starting point for our discussion of the safety of reinforced concrete structures will be the statement that by the term "collapse" of such a structure (a column or a bar, in particular), we understand the phenomenon of the crushing of the concrete portion and the attainment of the yield point in the reinforcement bars. According to numerous observations of the collapse of reinforced concrete structures these two phenomena may be considered to appear simultaneously.

Under these conditions the load carrying capacity of a reinforced concrete structure is governed by the value of the ultimate compressive stress of the concrete and the yield point of the reinforcement steel which form the basis for its determination. Both quantities should be regarded as random quantities.

Fig. 1 represents a scheme of the probability curve for the compressive

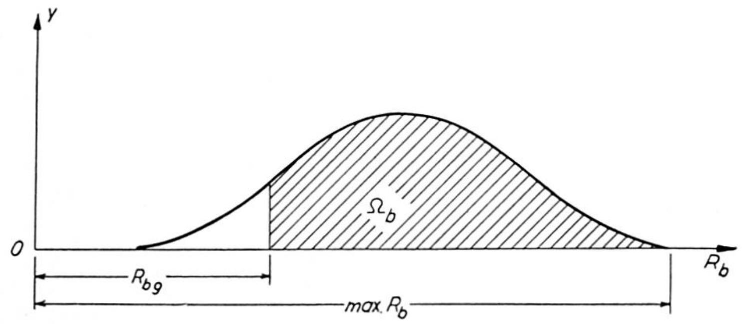

Fig. 1.

strength of the concrete R_b. The shaded area expresses the probability Ω_b that the values of R_b are contained between the limit value R_{bg} and the maximum value max R_b. In other words R_{bg} is the value of the strength of the concrete, below which the strength of concrete of a given quality will not fall.

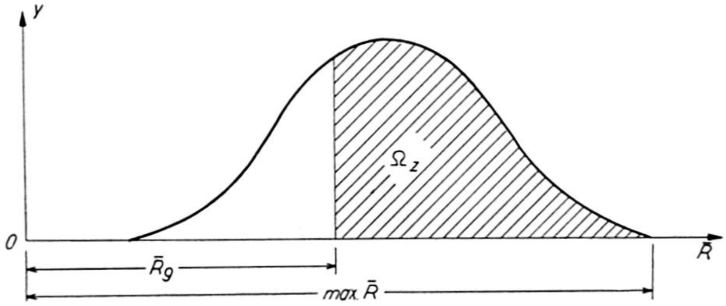

Fig. 2.

Fig. 2 represents the scheme of a probability curve for the yield point of the reinforcement steel. It is assumed that the same curve characterises the yield point of the steel in tension and in compression. The shaded area expresses, in this case, the probability Ω_z that the values $\bar{R}$ are contained between the limit value $\bar{R}_g$ and the maximum value max $\bar{R}$ or, in other words, that it can be stated with the probability Ω_z that the yield point of the reinforcement steel will not fall below $\bar{R}_g$.

In order that the collapse of a reinforced concrete column or beam should not take place it is necessary that the following two independent circumstances should coincide:

A. The ultimate strength of the concrete must be greater than the limit value R_{bg}, Ω_b denoting the probability of this fact.

B. The yield point of the reinforcement steel must be greater than the limit value $\bar{R}_g$, Ω_z denoting the probability of this fact.

In this connection, by virtue of the rule of multiplication of probabilities, we can state, with the probability $\Omega_b \Omega_z$, that if the compressive strength of the concrete is not less than R_{bg} and the yield point of the steel is not less than $\bar{R}_g$, collapse of the structure will not take place.

If, therefore, the safety index or, in other words, the probability that the collapse of a reinforced concrete structure will not take place, is denoted by p, we shall obtain the equation

$$\Omega_b \Omega_z = p. \tag{1}$$

Since the collapse of a reinforced concrete structure takes place at the moment when the ultimate compressive stress in the concrete and the yield point in the steel are simultaneously exceeded there is no reason for assuming different values for the probabilities Ω_b and Ω_z, all the more since their equality ensures the greatest accuracy in the readings of the probability curves.

Therefore, we find that

$$\Omega_b = \Omega_z = \sqrt{p}. \tag{2}$$

In the case of the axial compression of a column the safety index p that is assumed corresponds to the value R_{bg} of the ultimate stress of the concrete and the value $\bar{R}_g$ of the yield point of the steel and the axial force causing the collapse of the column is expressed, in this connection, by the equation

$$N_n = \kappa (A_b R_{bg} + A_z \bar{R}_g), \tag{3}$$

where A_b is the area of the concrete portion of the cross-section of the column A_z — the area of the cross-section of the reinforcement and κ — the reduction coefficient for buckling.

The compressive force acting on the column N_d, the action of which on the structure is admissible, is not equal to N_n, because there are certain circumstances reducing the force N_n. Thus, the force N_d should be compared with the force N_n reduced in an appropriate manner, that is to say it should be assumed that

$$N_d = N_n (1 - \sum \alpha_i'), \tag{4}$$

where the coefficients α_i' denote the limiting relative reductions of the force N_n causing collapse, due to an incomplete satisfaction of each particular condition, for which Eq. (3) was derived and $\sum \alpha_i'$ denotes the relative reduction of N_n due to incomplete satisfaction of all these conditions. The coefficients α_i' are not, in general, of a random character.

A reasoning analogous to that given in Ref. [1] and [2] for the quantities p and α will now be followed for the safety index p and the coefficients α', the following values for α_i' being used:

$\alpha_1' = 0{,}04$ the coefficient of reduction of the force causing collapse, due to the errors in the dimensions of the concrete portion of the cross-section of the column;

$\alpha_2' = 0{,}03$ the coefficient of reduction of the force causing collapse, due to the errors in the transverse dimensions of the reinforcement bars;

$\alpha_3' = 0{,}10$ the coefficient of reduction of the force causing collapse, due to the eccentricity of the compressive force caused by the errors in the dimensions of the cross-section of the column;

$\alpha_4' = 0{,}20$ the coefficient of reduction of the force causing collapse, due to the eccentricity caused by the difference in the temperatures at different points on the surface of the column;

$\alpha_5' = 0{,}10$ the coefficient of reduction of the force causing collapse, due to the deviation of the column from a straight line.

We have $\sum \alpha_i' = 0{,}47$.

Thus, for example, in order to determine the coefficient α_i' two columns are considered: a column 30×30 cm, reinforced with 4 bars of 12,57 cm^2 total cross-sectional area, and a column 45×45 cm, reinforced with 8 bars of 32 cm^2 total cross-section. For the first column we assume that $R_{bg} = 140$ kg/cm^2 and $\overline{R} = 2500$ kg/cm^2; for the second column that $R_{bg} = 170$ kg/cm^2 and $\overline{R} = 3600$ kg/cm^2. Then, from Eq. (3) we obtain $N_n = 127$ kg and $N_n = 360$ kg, respectively.

Assuming, according to the usual standards, that the admissible deviations are ± 10 mm we again compute N_n for a column 29×29 cm and a column 44×44 cm, and we obtain $N_n = 122$ kg and $N_n = 357$ kg, values which give $\alpha_1' = 0{,}05$ and $\alpha_1' = 0{,}03$, respectively, or an average value of $\alpha_1' = 0{,}04$.

The same reasoning is used for the determination of the other coefficients α' for the compression and for the bending of reinforced concrete beams.

On the basis of the method of plastic deformations, it is justifiable to consider the collapse of a reinforced concrete column (which consists in simultaneous crushing of the concrete and attainment of the yield point in the reinforcement) to be identical, as far as the practical effect is concerned, with the phenomenon occuring in a compressed steel bar when the yield point is exceeded over the entire cross-section. Since in the latter case the safety factor was assumed to be $p = 0{,}8$, this value may also be considered to be justified in the case of a reinforced concrete column.

In the case of bending, the fact that the beam is broken when the ultimate compressive stress in the concrete and the yield point in the steel are exceeded simultaneously, leads to the following formula for the collapse moment of the beam:

$$M_n = 0{,}375\, b\, h_1^2\, R_{bg}. \tag{5}$$

This corresponds to the stress diagram in Fig. 3.

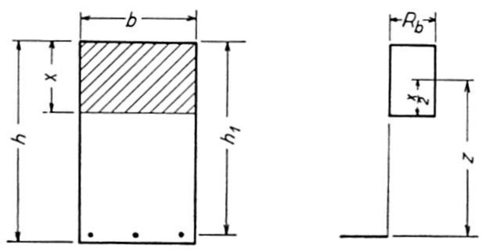

Fig. 3.

The admissible moment in a reinforced concrete beam cannot be considered to be equal to the collapse moment and is expressed by the equation

$$M_d = M_n (1 - \sum \alpha_i') \tag{6}$$

analogous to Eq. (4) for the admissible force in a compressed column.

Bearing in mind the circumstances similar to those enumerated above for the problem of compressed columns, we determine the following coefficients α_i':

$\alpha_1' = 0{,}06$ the coefficient of reduction of the collapse moment due to the errors in the dimensions of the concrete portion of the cross-section of the beam;

$\alpha_2' = 0{,}04$ the coefficient of reduction of the collapse moment due to the errors in the transverse dimensions of the reinforcement bars;

$\alpha_3' = 0{,}06$ the coefficient of reduction of the collapse moment due to the eccentricity and obliqueness of the load;

$\alpha_4' = 0{,}12$ the coefficient of reduction of the collapse moment due to the difference between the temperatures on the upper and the lower surface of the beam and the shrinkage of the concrete.

We have $\sum \alpha_i' = 0{,}28$.

If a reinforced concrete column or beam is subject to better conditions than those used for the determination of the coefficients α_i' some of them may be assumed to be zero, which will increase N_d or M_d. Thus we proceed in a similar manner to that described for steel structures in Ref. [1], where we were concerned with an increase in the admissible stress under favourable conditions. This is discussed in greater detail in Ref. [3].

References

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Summary

The load carrying capacity of a reinforced concrete structure is determined by the ultimate compressive stress of the concrete and the yield point of the reinforcement steel. Both quantities should be regarded as random quantities.

If the compressive strength of the concrete is not less than R_{bg} (the probability Ω_b) and the yield point of the reinforcement steel is not less than $\bar{R}_g$ (the probability Ω_z) then according to the rule of multiplication of probabilities it can be stated that collapse will not take place, with the probability $\Omega_b \Omega_z$.

If p is the safety factor (that is the probability that collapse of the structure will not occur) then, from the equation $\Omega_b \Omega_z = p$, we can determine the admissible force in the column or the admissible bending moment in the beam.

Résumé

La contrainte du béton à la limite de résistance à la compression et la contrainte à la limite d'écoulement de l'acier d'armature ont une importance décisive pour la capacité portante d'une construction en béton armé. Ces deux grandeurs doivent être considérées comme des grandeurs aléatoires.

Si la résistance du béton à la compression n'est pas inférieure à R_{bg} (probabilité Ω_b) et la contrainte à la limite d'écoulement de l'acier d'armature n'est pas inférieure à $\bar{R}_g$ (probabilité Ω_z), alors conformément au théorème sur la multiplication de probabilités, on peut soutenir avec la probabilité $\Omega_b \Omega_z$ que la construction ne s'effondrera pas. Si p est l'indice de sécurité (c. à. d. la probabilité que la construction ne s'effondre pas), on aura l'équation

$$\Omega_b \Omega_z = p$$

qui permet de calculer la force admissible dans le pilier ou le moment admissible pour la poutre.

Zusammenfassung

Für die Tragfähigkeit einer Eisenbetonkonstruktion sind die Betonspannung beim Erreichen der Bruchgrenze und die Eisenspannung beim Erreichen der Fließgrenze von maßgebender Bedeutung. Diese Werte müssen als zufällige Größen betrachtet werden.

Falls die Bruchfestigkeit des Betons kleiner als der Wert R_{bg} (Wahrscheinlichkeit Ω_b) und die Fließgrenze der Bewehrungseisen kleiner als der Wert $\bar{R}_g$ ist (Wahrscheinlichkeit Ω_z), dann wird, nach dem Satz der Multiplikation von Wahrscheinlichkeiten, die Wahrscheinlichkeit, daß keine Katastrophe eintritt, $\Omega_b \Omega_z$ sein. Bezeichnen wir den Sicherheitsindex mit p (d. h. die Wahrscheinlichkeit, daß die Konstruktion hält), so gilt

$$p = \Omega_b \Omega_z,$$

was uns erlaubt, die zulässige Kraft in einer Stütze oder das zulässige Moment in einem Balken zu bestimmen.