Redistribution of stresses in a continuously supported beam, due to creep

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Redistribution of Stresses in a Continuously Supported Beam, due to Creep

Redistribution due au fluage des contraintes dans une poutre sur appui continu

Kräfteumlagerungen in einem Balken auf durchgehender Bettung infolge Kriechen

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I. Introduction

In a so-called "foundation beam", the deformations due to creep will produce a variation of the reaction of the support, and consequently a redistribution of the stresses in the beam will take place.

In the present paper, this redistribution has been studied, taking into account not only the creep of the beam, but also a possible creep of the support. We shall assume linear behaviour of creep, i. e., the case in which creep strains are proportional at any instant to the applied stress.

Generally, linear or non-linear creep behaviour are studied by means of standard creep tests at constant stress; but it is important to note that such tests are not sufficient to assure linear behaviour at different rates of stress.

To admit linear behaviour is the same as to admit the generalized principle of superposition in BOLTZMANN's sense. So, it will be possible to investigate linear behaviour by means of superposition.

For materials like synthetic plastics, many papers, and our own experiences [1] show that the creep of these materials follows very closely the principle of superposition.

For concrete, McHENRY[1]), ROSS, BLACKSTON etc., [2] [3] [4] have shown

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[1]) McHENRY was the first — according to our opinion — to realize that the creep recovery curves could be interpreted like the superposition of the separated effects of

that the superposition method gives excellent results. In Russia, GWOSDEW[2]) and others [5] used this method of work successfully.

In this order of ideas, the theory of creep effects can be deduced from VOLTERRA's theory of hereditary phenomena [6] which naturally implies the BOLTZMANN principle of superposition.

II. General Formulation

According to the previous considerations, it is easy to show that in those kinds of materials the effect of a certain solicitation can be expressed by means of VOLTERRA's mentioned theory. In our particular case it is easy to show [7] that the curvature of a bent beam at the instant t, subject to a variable bending moment $M(t)$ applied at the instant τ_0, can be expressed by

$$\mu = -\frac{\partial^2 y}{\partial x^2} = \frac{M(t)}{EI} + \int_{\tau_0}^{t} M(\tau) f_1(t,\tau)\, d\tau, \tag{1}$$

where E is the elastic modulus of the material of the beam, and I the inertia moment of the cross section.

The function $f_1(t,\tau)$ is called the creep function. It can be easily connected with the curves that we obtain experimentally performing creep tests at different ages of the material, by means of

$$f_1(t,\tau) = -\frac{\partial}{\partial \tau} \bar{\epsilon}_0(t,\tau), \tag{2}$$

where $\bar{\epsilon}_0(t,\tau)$ is the specific creep strain at the time t, obtained when a unit stress was applied at the age τ.

We shall consider also, a linear viscoelastic behaviour of the support. The deflections which will take place when a pressure $q(x,t)$ is applied at the instant τ_0 will be expressed by means of

$$y(x,t) = \frac{q(x,t)}{k} + \int_{\tau_0}^{t} q(x,\tau) f_2(t,\tau)\, d\tau, \tag{3}$$

where $f_2(t,\tau)$ is the creep coefficient of the support, and k the classical coefficient of elastic reaction.

If we call $p(x,t)$ the external load acting on the beam, we can write the following fundamental relation between bending, external loads, and reaction of the support

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an initial load and a negative one of the same intensity, applied at the instant of unload. In such a way — and many experiments confirm the criterion — it is proved that creep in concrete obeys the same law, for increasing and decreasing stresses.

[2]) The very importance of the paper of this author in 1943 is to show that FREYSSINET's ideas about deformation of concrete are congruent with the linear-integral formulation of VOLTERRA's theory of hereditary phenomena.

$$\frac{\partial^2 M}{\partial x^2} = -p(x,t) - q(x,t). \tag{4}$$

Eliminating $y(x,t)$ and $M(x,t)$ between eqs. (1), (3), and (4), we obtain the following integral equation

$$\begin{aligned} E I \int_{\tau_0}^{t} \frac{\partial^4 q}{\partial x^4} f_2(t,\tau)\, d\tau + \frac{E I}{k} \frac{\partial^4 q}{\partial x^4} + E \int_{\tau_0}^{t} q(x,\tau) f_1(t,\tau)\, d\tau + q(x,t) = \\ -p(x,t) - E \int_{\tau_0}^{t} p(x,\tau) f_1(t,\tau)\, d\tau. \end{aligned} \tag{5}$$

To solve this equation we can imagine that functions $p(x,t)$ and $q(x,t)$ are expressed by the following expansions of orthogonal functions

$$\begin{aligned} p(x,t) &= \sum_1^\infty a_i(t)\, \varphi_i(x), \\ q(x,t) &= \sum_1^\infty b_i(t)\, \varphi_i(x), \end{aligned} \tag{6}$$

where $\varphi_i(x)$ are the Eigen-functions of the following differential equation

$$\frac{d^4 \varphi_i}{d x^4} - k_i \varphi_i = 0. \tag{7}$$

It is noted that the φ_i-functions are orthogonal functions, when conditions of free, or hinged or built-in ends are satisfied. Then, the coefficients $a_i(t)$ can be immediately calculated as follows

$$a_i(t) = \frac{1}{D} \int_0^l p(x,t)\, \varphi_i(x)\, dx; \qquad D = \int_0^l \varphi_i^2(x)\, dx. \tag{8}$$

Concerning the $b_i(t)$-coefficients, substituting eqs. (6) into (5) and taking into account (7), the following integral equation for the $b_i(t)$-coefficients can be written

$$b_i(t) + \lambda_i \int_{\tau_0}^{t} b_i(\tau)\, [f_1(t,\tau) + I k_i f_2(t,\tau)]\, d\tau = g_i(t), \tag{9}$$

where $\lambda_i = \dfrac{E}{1 + \dfrac{EI}{k} k_i}$ and $g_i(t) = -\dfrac{1}{1 + \dfrac{EI}{k} k_i} \left[a_i(t) + E \int_{\tau_0}^{t} a_i(\tau) f_1(t,\tau)\, d\tau\right]$.

To solve the preceeding integral equation we shall apply some restrictions to the functions $f_1(t,\tau)$ and $f_2(t,\tau)$ which correspond to two important cases of practical applications.

III. Case of Invariable Creep

We consider the case in which the creep of the materials is independent of the age of loading, that is to say that the creep functions $f_1(t,\tau)$ and $f_2(t,\tau)$ will depend only on the difference of the parameters $(t-\tau)$. In this way the integral eq. (9) has an elegant solution [1] by means of the Laplace transformation, as follows

$$b_i(t) = -\frac{1}{1+\frac{EI}{k}k_i} L^{-1} \frac{L a_i(t)[1+E L f_1(t)]}{1+\lambda_i L[f_1(t)+I k_i f_2(t)]}, \tag{10}$$

where L represents the Laplace transformations defined by

$$L f(t) = \int_0^\infty e^{-st} f(t)\, d t$$

and L^{-1} represents the inverse transformation.

The problem is formally solved in a very general way. If the law of variation of external loads, and the analytical expressions of the creep coefficients are given, we are able to calculate the $b_i(t)$ coefficients, using tables or direct inverting methods, by means of expression (10).

However, the most practical interest is to know the convergence of deflections, contact-pressures, etc., when $t \to \infty$ that is to say the asymptotic behaviour. In order to do this we have to investigate the asymptotic behaviour of the functions $b_i(t)$. A theorem of Paley and Wiener on the Volterra integral equation [8] affirms that the $b_i(t)$ coefficients given by eq. (9) will converge to the limit

$$b_i(\infty) = \frac{\lim\limits_{t\to\infty} g_i(t)}{1+\lambda_i \int_0^\infty [f_1(t)+I k_i f_2(t)]\, d t}, \tag{11}$$

if and only if

$$\lambda_i \int_0^\infty [f_1(t)+I k_i f_2(t)]\, e^{-st}\, d t \neq -1; \quad s \geqq 0.$$

This condition is always fulfilled because λ_i and the integrand are positive. Morover $b_i(\infty)$ will tend to a finite limit, only if $g_i(t)$ tends to a finite one. This last condition is naturally dependant on the convergence of the external loads $p(x,t)$. If we suppose that the external forces tend to the finite limit

$$\lim_{t\to\infty} p(x,t) = p_\infty(x) \tag{12}$$

the $a_i(t)$ coefficients given by eq. (8) will also tend to a finite limit $a_i(\infty)$. Consequently the limit

$$\lim_{t\to\infty} g_i(t)$$

may be obtained by means of the mentioned theorem of Paley and Wiener, using the expressions of $g_i(t)$ given in (9),

$$\lim_{t\to\infty} g_i(t) = -\frac{a_i(\infty)}{1+\frac{EI}{k}k_i}\left[1+E\int_0^\infty f_1(t)\, d t\right]. \tag{13}$$

Substituting (13) into (11) we obtain the following expression for the $b_i(\infty)$ coefficients

$$b_i(\infty) = -\frac{a_i(\infty)}{1+\frac{EI}{k}k_i\frac{1+k\gamma_2}{1+E\gamma_1}}, \tag{14}$$

where γ_1 and γ_2 are

$$\gamma_1 = \int_0^\infty f_1(t)\,dt; \qquad \gamma_2 = \int_0^\infty f_2(t)\,d\tau.$$

If we substitute $b_i(\infty)$ given by (14) in the expression (6) we obtain

$$q(x,\infty) = -\sum_1^\infty \frac{a_i(\infty)}{1+\frac{EI}{k}k_i\frac{1+k\gamma_2}{1+E\gamma_1}}\varphi_i(x). \tag{15}$$

If we solve the purely elastic (classical) problem of the foundation by means of the orthogonal functions (6) we obtain the following expansion

$$q(x,0) = -\sum_1^\infty \frac{a_i(0)}{1+\frac{EI}{k}k_i}\varphi_i(x). \tag{16}$$

Comparing (15) and (16) we find that the asymptotic solution of the visco-elastic problem is coincident with the solution of the purely elastic problem, in which the elastic modulus E and the coefficient of reaction of the support are substituted by the following effective values:

$$E^* = \frac{E}{1+E\gamma_1}; \qquad k^* = \frac{k}{1+k\gamma_2} \tag{17}$$

and the asymptotic values of the external loads, instead of its initial values is used.

Hence, to investigate asymptotic behaviour, when the creep of a material is independent of the age (invariable creep) the so-called method of "effective modulus" is perfectly correct.

IV. Concrete Beam on an Elastic Support

We shall study only the case in which the support is an elastic one, and the loads remain constant after application. The integral eq. (9) will be reduced to

$$b_i(t) + \lambda_i \int_{\tau_0}^t b_i(\tau) f_1(t,\tau)\,d\tau = -\frac{a_i}{1+\frac{EI}{k}k_i}[1+E\bar{\epsilon}_0(t,\tau_0)]. \tag{18}$$

It is known that creep in concrete depends on the age; generally this influence of age is studied by means of standard creep tests at constant stress and different ages of the concrete. We shall have a certain number of tests that will give us the specific creep curves $\bar{\epsilon}_0(t, \tau)$ obtained at different ages τ. Experience shows that diminution of creep with the age is an asymptotic phenomenon.

Many authors have proposed formulas in order to represent analytically the specific creep $\bar{\epsilon}_0(t, \tau)$. Among the most important we shall mention the Dischinger formula [9] which considers the age dependence in the following form. The specific creep of the concrete at the age of loading is represented by means of the following function

$$\theta(t) = \gamma_0 (1 - e^{-\delta t}). \tag{19}$$

Then, it is assumed that creep of concrete, when the load is introduced at a certain time τ, can be represented by means of

$$\bar{\epsilon}_0(t, \tau) = \theta(t) - \theta(\tau). \tag{20}$$

That is to say, as we can see in Fig. 1, the creep produced by a load introduced at the instant τ can be obtained by translating vertically the creep curve given for the reference age $\tau = \tau_0$.

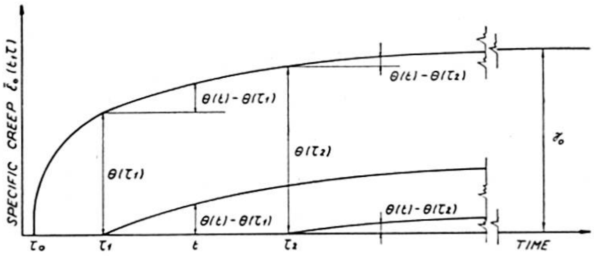

Fig. 1.

This assumption has one inconvenience, namely, the intensity of creep tends to zero when the age of concrete τ tends to infinite. Generally concrete always presents creep, and the experiments show that the intensity of creep is lower when age increases, but tending asymptotically to a limit when $\tau \to \infty$. In other words, function $\bar{\epsilon}_0(t, \tau)$ will tend asymptotically towards a not null function of the type $F(t - \tau)$ when $\tau \to \infty$.

For this reason the Dischinger formula gives always a pessimistic evaluation of the creep effects, when new loads are introduced in the structures at different ages [10].

In order to represent more closely the dependance of creep on age, the following function was proposed [11]

$$\bar{\epsilon}_0(t, \tau) = \psi(\tau) F(t - \tau), \tag{21}$$

where ψ is an always positive function that decreases monotonously towards the finite limit $\psi(\infty) = \gamma_0$ and represents the gradual diminution of creep due to age. Function $F(t - \tau)$ is also positive and monotonously growing towards an upper limit equal to unity, for great values of the parameter $(t - \tau)$. Taking

$$\psi(\tau) = \gamma_0 + \frac{C}{\tau}, \qquad F(t-\tau) = 1 - e^{-\delta(t-\tau)} \tag{22}$$

and choosing convenient constants γ_0, c, δ it is possible to follow as closely as desired the creep of concrete and its dependence on the age. In Fig. 2 this situation is shown. It is noted that γ_0 represents the asymptotic value of the specific creep of an aged concrete, i. e., of a concrete loaded when $\tau \to \infty$.

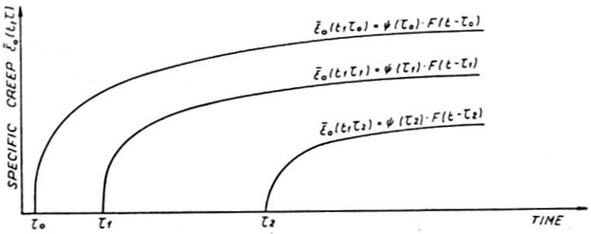

Fig. 2.

The creep function $f_1(t,\tau)$ can be calculated by means of eq. (2) and its values is

$$f_1(t,\tau) = -\frac{\partial}{\partial \tau}[\psi(\tau)(1 - e^{-\delta(t-\tau)})]. \tag{23}$$

Substituting (23) in (18) eventually leads to

$$b_i(t) - \lambda_i \int_{\tau_0}^{t} b_i(\tau)\psi'(\tau)\,d\tau + \lambda_i \int_{\tau_0}^{t} b_i(\tau)(\psi' + \delta\psi)e^{-\delta(t-\tau)}\,d\tau = h_i(t,\tau_0), \tag{24}$$

where h_i is the second member of (18).

Differentiating the preceeding equation with respect to t,

$$b_i'(t) + \lambda_i \psi(t) b_i(t) - \delta\lambda_i \int_{\tau_0}^{t} b_i(\tau)(\psi' + \delta\psi)e^{-\delta(t-\tau)}\,d\tau = h_i'(t,\tau_0). \tag{25}$$

Eliminating the integral

$$\int_{\tau_0}^{t} b_i(\tau)(\psi' + \delta\psi)e^{-\delta(t-\tau)}\,d\tau$$

between (24) and (25), and differentiating once again with respect to t, the following differential equation is obtained

$$b_i''(t) + \delta[1 + \lambda_i \psi(t)]b_i'(t) = 0 \tag{26}$$

with the following boundary conditions

$$b_i(\tau_0) = \frac{a_i}{1 + \frac{EI}{k}k_i}, \tag{27}$$

$$b_i'(\tau_0) = \frac{\frac{EI}{k}k_i}{\left(1 + \frac{EI}{k}k_i\right)^2} a_i E \delta \psi(\tau_0). \tag{28}$$

The general solution of eq. (26) is obtained by means of two integrations

$$b_i(t) = b_i(\tau_0) + b_i'(\tau_0) \int_{\tau_0}^{t} e^{-J_i(\xi)} d\xi, \tag{29}$$

where

$$J_i(t) = \delta \int_{\tau_0}^{t} [1 + \lambda_i \psi(\tau)] d\tau.$$

Substituting $\psi(\tau)$ by its equivalent (22), the preceeding integral is transformed into

$$J_i(t) = \delta(1 + \lambda_i \gamma_0)(t - \tau_0) + \ln\left(\frac{t}{\tau_0}\right)^{\lambda_i C \delta},$$

which substituted in (29) gives

$$b_i(t) = b_i(\tau_0) + b_i'(\tau_0)\, e^{\delta(1+\lambda_i\gamma_0)\tau_0} \tau_0^{\lambda_i C\delta} \int_{\tau_0}^{t} e^{-\delta(1+\lambda_i\gamma_0)\tau} \tau^{-\lambda_i C \delta} d\tau. \tag{30}$$

Introducing the incomplete gamma function $\phi(\alpha, t)$ defined by

$$\phi(\alpha, t) = \int_0^t e^{-\tau} \tau^{\alpha-1} d\tau,$$

eq. (30) can be written

$$b_i(t) = b_i(\tau_0) + b_i'(\tau_0)\, e^{r_i\tau_0} \tau_0^{1-\alpha_i} r_i^{-\alpha_i} [\phi(\alpha_i, r_i t) - \phi(\alpha_i, r_i \tau_0)], \tag{31}$$

where

$$\alpha_i = 1 - \lambda_i C\delta; \qquad r_i = \delta(1 + \lambda_i \gamma_0).$$

The integral that appears in eq. (30) is convergent if and only if

a) $1 + \lambda_i \gamma_0 > 0$,
b) $1 - \lambda_i C\delta > 0$.

Condition a) is always satisfied because λ_i and γ_0 are positive. Condition b) is satisfied when

$$E C \delta < 1 + \frac{E I}{k} k_{min},$$

this inequality is generally fulfilled, because $E C \delta$ does not exceed in practical cases 0,5.

Now, if we substitute the value of the $b_i(t)$ coefficients given by eq. (31) in the second expansion (6) and substitute also the boundary values (27) and (28), we shall obtain

$$q(x, t) = q(x, \tau_0) - E\delta\psi(\tau_0) \sum_1^\infty \frac{a_i \frac{E I}{k} k_i}{\left(1 + \frac{E I}{k} k_i\right)^2} H_i(t)\, \varphi_i(x), \tag{32}$$

where

$$H_i(t) = e^{r_i\tau_0} \tau_0^{1-\alpha_i} r_i^{-\alpha_i} [\phi(\alpha_i, r_i t) - \phi(\alpha_i, r_i \tau_0)]$$

and $q(x, \tau_0)$ is the instantaneous contact pressure.

V. Comparison with the Dischinger Formula of Creep

At the beginning of the last section we have seen that, following DISCHINGER, we can write the specific creep by means of the following function

$$\bar{\epsilon}_0(t,\tau) = \theta(t) - \theta(\tau), \tag{33}$$

where $\theta(t)$ represents the creep curve obtained at the initial age. Function $\theta(t)$ can be represented by means of

$$\theta(t) = \gamma_0 (1 - e^{-\delta t}). \tag{34}$$

It follows that the creep function defined in (2) will be

$$f_1(t,\tau) = -\frac{\partial}{\partial \tau} \bar{\epsilon}_0(t,\tau) = \frac{d\theta(\tau)}{d\tau}. \tag{35}$$

Hence the creep function depends only on the variable τ.

Substituting expression (35) in the general integral eq. (18) we obtain

$$b_i(t) + \lambda_i \int_{\tau_0}^{t} b_i(\tau) \frac{d\theta(\tau)}{d\tau} d\tau = -\frac{a_i}{1+\frac{EI}{k}k_i}[1 + E\theta(t)].$$

Differentiating the preceeding equation with respect to the upper limit we shall obtain the following differential equation

$$b_i'(t) + \lambda_i b_i(t)\theta'(t) = -\frac{a_i}{1+\frac{EI}{k}k_i} E\theta'(t) \tag{36}$$

the solution of which is immediately obtained as follows

$$b_i(t) = -\left[1 - \frac{\frac{EI}{k}k_i}{1+\frac{EI}{k}k_i} e^{-\lambda_i \theta(t)}\right] a_i.$$

Substituting the preceeding expression in (6) we obtain the following expression for the contact pressure

$$q(x,t) = -\sum_1^{\infty}\left[1 - \frac{\frac{EI}{k}k_i}{1+\frac{EI}{k}k_i} e^{-\lambda_i \theta_i(t)}\right] \alpha_i \varphi_i(x). \tag{37}$$

In order to compare this solution with formula (32), we have solved a particular case represented in Fig. 3. A concrete beam, hinged at the ends and continuously supported on an elastic foundation, is loaded with a uniformly distributed load p_0 at $\tau_0 = 7$ days. It remains constant during three months. Then it is doubled, remaining thereafter constant during the following three months.

The specific creep was assumed to be

$$\bar{\epsilon}_0(t,\tau) = \left(0{,}9 + \frac{4.82}{\tau}\right)[1 - e^{-0.026(t-\tau)}]\cdot 10^{-5}.$$

so at 7 days it will be $\bar{\epsilon}_0(t,7) = 1{,}588\,[1 - e^{-0.026(t-7)}]\cdot 10^{-5}$.

In order to calculate with DISCHINGER's formula, creep at 7 days was assumed to be equal, that is to say, function $\theta(t)$ considered at 7 days will be

$$\theta(t) = 1.588\left[1 - e^{-0.026(t-7)}\right] \cdot 10^{-5}.$$

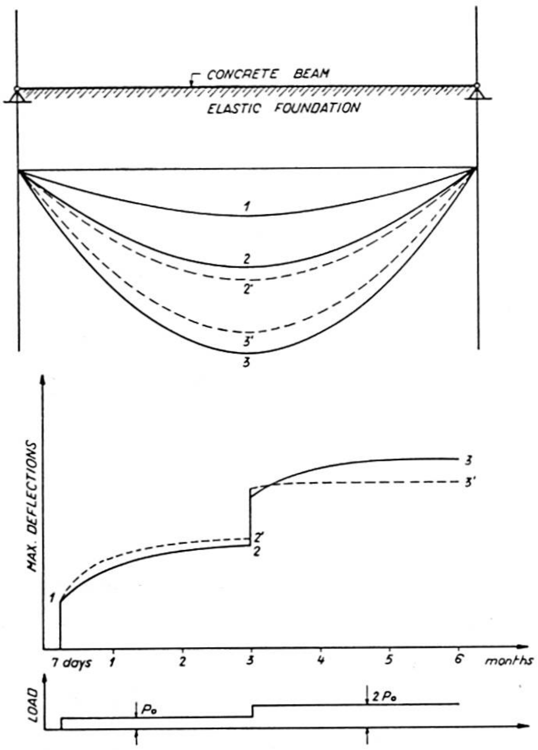

Fig. 3.

VI. Conclusion

In the third section we have seen that — to investigate asimptotic behaviour — the method of the effective modulus can be rigourously applied to structures the creep of which does not vary with the age of the materials. This conclusion [3]) is not only important for the more and more extended use of modern plastics, but also for the aged concrete, in which the specific creep, after a certain period of time — practically 4 or 5 months — tends to repeat at any age of loading the same creep curve. This observation is important when the structure is formed by pre-cast parts, long-time stored before use.

For young concrete, we think that the criterion used to represent the specific creep in Section IV permits us to evaluate more accurately creep effects than the DISCHINGER formula, which can only represent in one way the diminution of creep intensity due to age.

[3]) This conclusion is valid for any non-homogenous structure — if the creep of each component material is invariable — (see the paper of the author "Sul comportamento asintotico di corpi viscoelastici a ereditarietà invariabile" in the "Atti dell'Accademia delle Scienze di Torino", Nov. 1960, Vol. 95).

List of Notations

M	Bending moment [kg cm].
μ	Curvature of the beam.
y	Deflection of the beam [cm].
E	Elastic modulus of concrete [kg cm^{-2}].
I	Inertia moment of the cross section of the beam [cm^4].
t	Time.
τ	Age of the viscoelastic materials.
$\bar{\epsilon}_0(t,\tau)$	Specific creep of concrete [cm^2 kg^{-1}].
$f_1(t,\tau)$	Creep function of concrete [cm^2 kg^{-1}/day].
$f_2(t,\tau)$	Creep function of the support [cm^2 kg^{-1}/day].
k	Coefficient of elastic reaction of the support [kg cm^{-2}].
k_i	Eigenvalues [cm^{-4}].
$p(x,t)$	External loads acting on the beam [kg cm^{-1}].
$-q(x,t)$	Reaction of the support [kg cm^{-1}].
E^*	Effective modulus [kg cm^{-2}].
k^*	Effective coefficient of reaction of the support [kg cm^{-2}].
γ_1	Asimptotic creep strain in aged concrete produced by a stress equal to 1 kg cm^{-2} [cm^2 kg^{-1}].
γ_2	Asimptotic creep deflection in the support produced by a distributed load equal to 1 kg cm^{-1} [cm^2 kg^{-1}].
$\phi(\alpha,t)$	Incomplete Gamma function.

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Summary

In the present paper the redistribution of stresses, due to creep in a foundation beam, is studied, considering not only the creep of the beam but also a possible creep of the foundation. Linear behaviour of creep is assumed, and the general integral-differential equation of deflections is solved for the most important cases of applications. Particularly, it is shown that when creep of materials composing the beam and foundation are independent of the age of loading, the problem is immediately solved by means of the classical (elastic) methods.

Finally, the case of materials, the creep of which varies with age, is studied, and a discussion of the Dischinger criterion of creep of concrete is made.

Résumé

L'auteur étudie la redistribution des contraintes dans une poutre de fondation, les matériaux de la poutre et de sa fondation étant considérés comme visco-élastiques. On a admis un comportement linéaire du fluage, conformément à la théorie des phénomènes héréditaires de VOLTERRA. L'équation intégrale-différentielle régissant les déformations est résolue pour les cas d'application pratique les plus importants. L'étude a montré que, pour le calcul des déformations finales, la méthode du module effectif est parfaitement correcte quand le fluage du matériau n'est pas variable avec le temps.

Finalement, l'auteur étudie le cas du fluage variable avec l'âge du matériau et il discute le critère de fluage du béton proposé par DISCHINGER.

Zusammenfassung

Der vorliegende Artikel befaßt sich mit den durch das Kriechen bedingten Kräfteumlagerungen eines Fundationsträgers, wobei nicht nur das Kriechen des Trägers, sondern auch dasjenige des Untergrundes berücksichtigt werden. Zur Lösung der die Deformationen beherrschenden, allgemeinen Integral-Differential-Gleichung wurde ein linearer Zusammenhang zwischen Beanspruchung und Kriechen angenommen, wobei die Resultate für die wichtigsten Anwendungsfälle angegeben werden. Haben Träger und Untergrund ein vom Alter des Materials im Moment der Belastung unabhängiges Kriechmaß, so kann das Problem sofort mit Hilfe der klassischen (elastischen) Theorie gelöst werden. Zum Schluß wird der Fall des vom Alter des Materials abhängigen Kriechens untersucht sowie das Dischingersche Betonkriechkriterium diskutiert.