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The Effect of Creep in Compressed Plates

Phénomènes d'écoulement dans les dalles comprimées

Kriecherscheinungen in gedrückten Scheiben

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Introduction

The phenomenon of creep, which consists in the property of a solid to change its shape with time under constant loading, has gained considerable significance during the past decade and a half. Even though such structural materials as for example concrete, possess this undesirable feature, it is only with the advent of structures and machinery operating at elevated temperatures, that aluminum alloys and even steel began to exhibit the property of creep. As a result, the concept of useful life-time of a structure had to be introduced and became one of the chief characteristic design quantities to be determined by the structural analyst.

The concept of life-time of a structural element is based on experimental evidence that under constant loads the deformations increase at a certain rate with time and become eventually so large that the structure has to be considered useless. In case of plates or columns subjected to compressive loads, it has been observed that creep causes instability in a sense similar to classical buckling, which occurs after a definite lapse of time subsequent to the application of constant loads.

A variety of viewpoints may be adopted to determine analytically this critical time. For example, expressions for the deflection of a compressed element as a function of time can be found, and the critical time can be defined as the time at which this deflection becomes infinite. Another approach could consist in assuming an initial imperfection that would increase with time. The critical time (or life-time) could then be defined as the time at which a certain allowable quantity, such as stress, strain or deflection, is reached. A very comprehensive and illuminating survey of the theories of creep buckling was presented by HOFF [1].

Still another point of view was adopted in a more recent study by this author [2]. It was assumed that the plate is perfectly flat and that the constant compressive stresses will cause a time-dependent decrease of Young's (or shear) modulus, which in turn is proportional to the flexural rigidity. This decrease is determined by establishing the plate equations on the basis of stress-strain relations derived from spring-dashpot models, as is customary in visco-elasticity. The critical time was defined as the instant at which the flexural rigidity reaches a value that would make a corresponding elastic plate unstable under the given loading.

It was shown that for a certain aluminum alloy and a certain temperature level, the theoretically predicted life-times compared favorably with experimental data. Because of the scarcity of information on material properties and life-times a comparison on a broader scale could not be carried out.

The purpose of the present contribution is to suggest an improved version of the approach described above. Whereas in the preceding study [2] the plate was considered isotropic with regard to all its properties, and thus paralleled in this respect the assumptions of the few other plate creep buckling investigations [3, 4], the assumption of different creep behavior in the two principal directions of a uniaxially compressed plate is made in what follows. Regarding this anisotropy, precisely the same point of view is taken as in solving the problem of elasto-plastic buckling of a rectangular plate (see, for example [5, 6], and also [7]), under the same loading.

It is believed that the present version represents a theoretically more satisfactory variant of the approach adopted in [2]. Furthermore, a comparison with experimental results reported in [8] shows a better overall agreement between theory and experiment.

Basic Equations

A flat rectangular plate with edge lengths a and b and thickness h is referred to the system of coordinates shown in fig. 1 and is loaded along the edges $x=0$, a by a uniformly distributed (compressive) force per unit of length F. All four edges are simply supported.

The stress equation of equilibrium is

$$\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} = F\frac{\partial^2 w}{\partial x^2}, \tag{1}$$

where w is the deflection and the plate moments M_x, M_y, M_{xy} are defined in the usual way.

In the present linearized analysis the force F is not related to w. To express equilibrium in terms of one single unknown (deflection w), the stress-strain relations of the material and the geometrical strain-displacement relations have to be specified. If the plate is assumed to be orthotropic and the strain-displacement relations are taken to be those of the linear classical plate theory, the well-known stability equation of an orthotropic plate is obtained from (1) in the form

$$D_x \frac{\partial^4 w}{\partial x^4} + 2\, D_{xy} \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} + F \frac{\partial^2 w}{\partial x^2} = 0\,, \tag{2}$$

where D_x and D_y are the two flexural rigidities, while D_{xy} is the twisting rigidity.

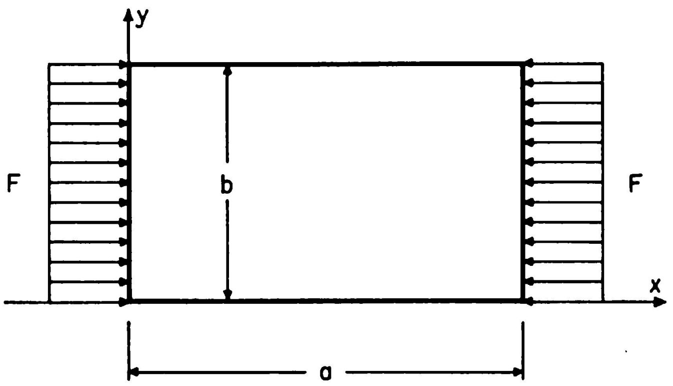

Fig. 1.

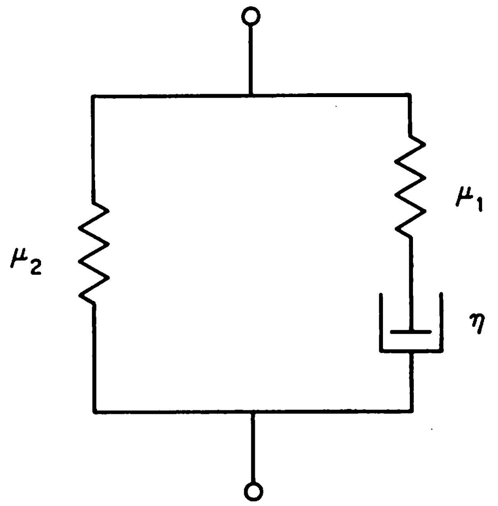

Fig. 2.

The assumption of the plate being orthotropic is introduced for similar reasons as in the case of buckling of plates in the elasto-plastic range, as is discussed below.

To describe its creep behavior, the material is considered to be linearly visco-elastic and its response in shear to be governed by a three-parameter model (see fig. 2). This model of a so-called "standard linear solid" consists of 2 springs with constants μ_1 and μ_2 and a dashpot with constant η. The instantaneous response is governed by

$$\mu_1 + \mu_2 = G_u\,, \tag{3}$$

which may be called the unrelaxed shear modulus; after a very long time the spring μ_1 will become ineffective and

$$\mu_2 = G_r \tag{4}$$

is called the relaxed shear modulus. The quantity

$$\tau_\epsilon = \eta/\mu_1 \tag{5}$$

is known as the time of relaxation of stress under constant strain, while

$$\tau_\sigma = \eta/\mu_1 + \eta/\mu_2 \tag{6}$$

is the time of relaxation of strain under constant stress.

It will be assumed now that since the plate is subjected to a uniaxial state of stress, it creeps only in the direction of stress, while no creep takes place in the direction perpendicular to it. This implies that the flexural modulus D_x decreases with time, while the flexural modulus D_y does not change as long as the plate remains flat. As regards the twisting modulus D_{xy}, it may be assumed that it also decreases, but slower than D_x. The above assumptions are completely analogous to the ones made in elasto-plastic analysis of plate buckling, [5, 6].

To establish the rate of decrease of D_x we recall that the flexural rigidity D is related to the shear modulus G by the expression

$$D = \frac{G h^3}{6(1-\nu)}. \tag{7}$$

Since the (constant) force F produces constant stresses in the perfectly flat plate, the stress-strain relation in shear, on the basis of the model of fig. 2, takes on the form

$$\tau_{xy} = G_r\left(1 + \tau_\sigma \frac{\partial}{\partial t}\right)\gamma_{xy}. \tag{8}$$

During the interval of time $0 < t < \infty$, the shear modulus $G(t)$ will decrease from G_u to G_r. Since at $t = 0$, $\tau_{xy} = G_u \gamma_{xy}$ we obtain from the above equation

$$\frac{G(t)}{G_u} = \frac{G_r}{G_u} \cdot \frac{e^{t/\tau_\sigma}}{e^{t/\tau_\sigma} + G_r/G_u - 1} \tag{9}$$

and further

$$\frac{D_x(t)}{D_u} = \frac{G(t)}{G_u} = \beta. \tag{10}$$

Similarly, as in elasto-plastic analysis of plate buckling we can assume

$$\frac{D_{xy}(t)}{D_u} = \beta^*, \qquad \frac{D_y}{D_u} = 1, \tag{11}$$

where β^* is such that

$$\beta < \beta^* < 1 \tag{12}$$

and the buckling eq. (2) takes on the form

$$D_u\left[\beta\frac{\partial^4 w}{\partial x^4} + 2\beta^*\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right] + F\frac{\partial^2 w}{\partial x^2} = 0. \tag{13}$$

This is the basic equation which can be used to determine the critical or lifetime of the plate.

Determination of Life-Time

Regarding the response of the plate subjected at a time $t = 0$ to a constant force F the following observations can be made: If the force F is larger or equal to an upper buckling force F_u which would cause an isotropic plate with stiffness D_u to buckle, then instability occurs, just as in the purely elastic plate, instantaneously, and no creep effects are exhibited. F_u is obtained from the basic eq. (13) by letting $\beta=\beta^*=1$ and results in the classical expression

$$F_u = \frac{D_u \pi^2}{b^2}\left(\frac{b}{a} m + \frac{a}{b m}\right)^2 . \tag{14}$$

If, on the other hand, the force F is smaller than the lower buckling force F_l, then no buckling will occur even after infinite time. F_l is obtained from the basic equation by taking the value for β reached after infinite time, namely $\beta = G_r/G_u$. This results in

$$F_l = \frac{D_u \pi^2}{b^2}\left(\frac{G_r b^2 m^2}{G_u a^2} + 2\beta^* + \frac{a^2}{m^2 b^2}\right). \tag{15}$$

If the force F has a value between F_u and F_l, then buckling will ensue after a finite period of time, which is referred to as the lifetime or critical time t_{cr}. Actually, it is more straightforward to find the buckling load corresponding to a given lifetime. All one has to do is to determine first β and β^* for this particular time, from eq. (10), and the buckling load, from eq. (13), will then be given by

$$F_{cr} = \frac{\pi^2 D_u}{b^2}\left(\beta \frac{b^2 m^2}{a^2} + 2\beta^* + \frac{a^2}{m^2 b^2}\right). \tag{16}$$

Regarding the determination of β^*, we could assume, by analogy to the suggestion of Bleich (see [5]),

$$\beta^* = \sqrt{\beta} \tag{17}$$

or a more refined relation, by analogy to the suggestion of Kollbrunner (see [6])

$$\beta^* = \frac{\beta + \sqrt{\beta}}{2}. \tag{18}$$

It appears that both of the above expressions give nearly the same results in creep analysis.

Numerical Example

In order to predict a numerical value for the critical time, three material constants corresponding to the three elements of the model must be known, in addition to Poisson's ratio. These constants may be obtained from a standard creep test.

In reference [2] it was shown how the constants for a 2024-T 3 aluminum sheet at 450° F subjected to a compressive constant stress of 26,000 pounds per square inch may be obtained from a curve which gives total strain against time. With Poisson's ratio of $\nu = 0.3$, the following values were obtained

$$
\begin{aligned}
G_u &= 2.5 \times 10^6 psi \\
G_u/G_r &= 4.5 \\
\tau_\sigma &= 30 \text{ hours}
\end{aligned}
\tag{19}
$$

The numerical value of the critical load (or stress), corresponding to any given time, can readily be calculated from eq. (16). To compare with experimental data of reference [8] (see also reference [2]), $b\,m/a$ can assumed to be equal to unity.

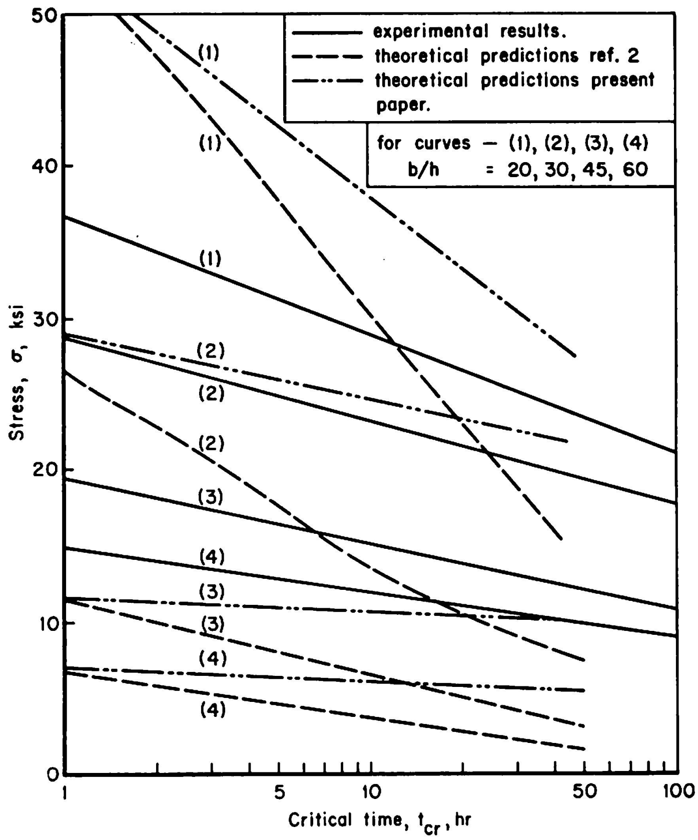

Fig. 3.

The critical stress σ, plotted against the critical time, is given in fig. 3. It is seen that the assumption of the plate being anisotropic in its creep behavior furnishes a somewhat better overall correlation with experimental data, than the earlier study [2], where the plate was assumed to be isotropic.

In conclusion it should be emphasized that it might be difficult to furnish a rigorous justification for the approach to creep buckling employed in this paper. However, the procedure adopted here appears to yield theoretical predictions which are reasonably close to experimental results; this is true not only for the actual numerical values for a given set of relevant parameters, but also with regard to the proper dependence of the critical times upon these parameters. Due to a large scatter of basic creep data, such as relaxation times, an analysis of creep buckling, at least for the present, cannot hope to achieve more than to provide an order of magnitude of critical times. From this point of view the method of analysis suggested in this paper has the advantage that it conceptually reduces the time-dependent phenomenon of creep buckling to the more familiar time-independent buckling in the elasto-plastic range. Due to an extreme temperature sensitivity of material properties in creep (some examples are quoted in [2]), which excludes a priori a great accuracy in creep buckling analysis, this engineering-type approach to a very complex problem appears to be warranted. Much theoretical and experimental work however remains to be done to provide a firmer basis for a deeper understanding of time-dependent structural response.

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Summary

It has been repeatedly observed that if a plate (or a column), made of a material which creeps, is subjected to compressive loads, instability may occur, provided the loads are applied over a sufficiently long period of time. Using a simple viscoelastic model to describe the material properties, this phenomenon of creep buckling of plates is reduced conceptually to the case of elasto-plastic buckling. Linearized equations are used throughout and a simple procedure to determine the critical time, at which the plate buckles, is indicated. The theoretical results compare favorably with some available experimental data.

Résumé

Il a été déjà constaté à plusieurs reprises qu'une dalle comprimée (ou un appui comprimé) constituée par un matériau susceptible de fluage peut être instable lorsque les contraintes s'exercent pendant un temps suffisamment long. Ce phénomène de voilement par fluage des dalles a été ramené au voilement élasto-plastique à l'aide d'un modèle visco-élastique. L'auteur emploie partout des équations linéarisées et indique une méthode simple pour la détermination du temps critique après lequel intervient le voilement. Les résultats théoriques concordent assez bien avec quelques résultats expérimentaux connus.

Zusammenfassung

Es wurde schon mehrere Male festgestellt, daß eine gedrückte Scheibe (oder Stütze) aus kriechfähigem Material unstabil werden kann, wenn die Kräfte über eine genügend lange Zeit wirken. Mit Hilfe eines viskoelastischen Modells, das den Charakteristiken des Baustoffes entspricht, wird dieses Phänomen des Kriech-Beulens von Platten auf das elastoplastische Beulen zurückgeführt. Es werden durchwegs linearisierte Gleichungen angewendet, und es wird eine einfache Methode angegeben zur Bestimmung der kritischen Zeit, bei der das Beulen eintritt. Die theoretischen Ergebnisse stimmen ziemlich gut mit einigen vorhandenen Versuchsresultaten überein.