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Anchorage of Beam Reinforcement Shortened According to the Moment Distribution Curve

L'ancrage des armatures arretées en fonction de la courbe des moments

Verankerung der dem Momentenverlauf entsprechend abgestuften Armierung

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1. Introduction

This paper studies the anchorage of beam reinforcement of deformed bars (corrugated Swedish reinforcement steel), when the reinforcement is cut off according to the moment distribution curve. The influence of shear cracks and web reinforcement on this anchorage has been taken into consideration.

When the reinforcement is cut off to cover the moment distribution curve, it is normally required that a cut-off bar should be fully anchored outside that beam section where the bar has to begin to act according to the moment curve. This is a case of anchorage where the stress in the through reinforcement must be transferred to the cut-off reinforcement by means of the surrounding concrete, and it is *not* directly comparable with the classical anchorage case studied through the traditional pull-out test. Furthermore, the traditional methods of treating problems concerning bond and anchorage are generally based on conditions valid for smooth bars, where the bond acts as a pure glue effect with a sudden bond failure at a fixed and relatively low ultimate strength value. The bond of deformed bars, however, depends largely on the direct contact pressure between the bar ridges and the surrounding concrete, and the bond is so good that in many cases the ultimate strength is not reached, and this is the case at the ends of a cut-off reinforcement bar. In such a case it is necessary to study in detail the stress transfer between concrete and reinforcement when the displacement between these components takes place.

For the study of this and analogous problems a hypothesis has been put forward by GRANHOLM, originally in his works about composite wooden constructions [1], where he assumes that there is pure proportionality between the bond stress τ_b and the corresponding displacement φ between the section parts

$$\tau_b = K\varphi, \tag{1}$$

where K is called *modulus of displacement*. Used for analysing the slip between concrete and reinforcement, this assumption can, of course, not be quite correct, for the displacement actually takes place as a shear deformation in the concrete layer surrounding the reinforcement bars. Besides, the deformation in this layer will rapidly increase and become of non-elastic character. Thus the relation between bond stress and deformation will not be straight-lined, a fact which is also well-known from pull-out tests. Such tests of the type shown in fig. 1 have been made with a very short bond length, which will give a fairly level bond stress distribution, and the test results give a direct idea of the stress-displacement curve.

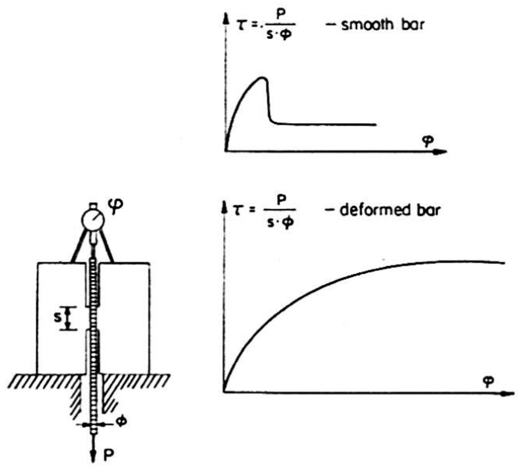


Fig. 1. Pull-out test specimen with short bond length and typical stress-deformation curves. The upper curve is valid for smooth bars, the lower one for deformed bars.

The investigation briefly presented here will show, through a comparison of test results and theoretical analysis based on the simple hypothesis above, that, in spite of its imperfection, this hypothesis can be used for treating bond and anchorage problems. The investigation is not yet finished.

2. Theoretical Treatment

Let us to begin with study a basic bond problem, the tensile test on a prism with through reinforcement as shown in fig. 2. This case is valid for instance for the analysis of crack formation [2].

The assumption (1) can be expressed in components of displacement ξ and η for steel and concrete respectively (see fig. 2)

$$\tau_b = K\,\varphi = K\,(\xi - \eta). \tag{2}$$

These components can also be used to express the strains ϵ_r and ϵ_c and the stresses σ_r and σ_c in the two materials

$$\epsilon_r = \frac{d\xi}{dx} = \frac{\sigma_r}{E_r}; \qquad \epsilon_c = \frac{d\eta}{dx} = \frac{\sigma_c}{E_c}, \tag{3}$$

where Hooke's law is presupposed to apply and the shrinkage of concrete is left out of account. The connection between bond stress τ_b and normal stress σ_a gives as usual

$$\tau_b = \frac{A_r}{p} \frac{d\sigma_r}{dx} = \frac{\Phi}{4} \frac{d\sigma_r}{dx} \tag{4}$$

and the equilibrium condition gives

$$\sigma_r^0 A_r = \sigma_r A_r + \sigma_c A_c. \tag{5}$$

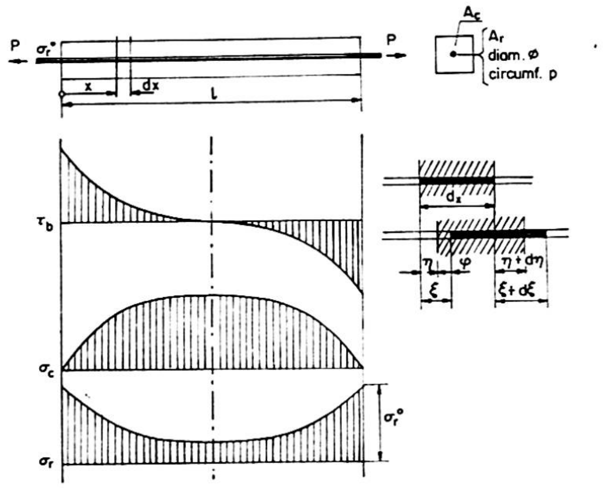


Fig. 2. Tensile prism with a through reinforcement bar, analysed according to the K-value theory. Typical stress distribution curves are given.

Combining the equations (2)—(5) we get the differential equation

$$\frac{d^2 \sigma_c}{dx^2} - \kappa^2 \sigma_c = -\kappa^2 \frac{\mu}{1+n\mu} \sigma_r^0 \tag{6}$$

with the notations

$$\kappa^2 = \frac{\Phi}{4} \frac{K}{E_r} (1+n\mu); \qquad n = \frac{E_r}{E_c}; \qquad \mu = \frac{A_r}{A_c}. \tag{6a}$$

The solution of this equation for the limit conditions here valid is

$$\sigma_c = \frac{\mu}{1+n\mu} \sigma_r^0 \left[1 - \frac{\cosh \kappa\left(\frac{l}{2} - x\right)}{\cosh \kappa \frac{l}{2}}\right]$$

$$\sigma_r = \sigma_r^0 - \frac{1}{\mu} \sigma_c \tag{7}$$

and the corresponding solution for τ_b from equation (4) (see fig. 2).

It may be pointed out that the same differential equation is valid for other cases of anchorage, e. g. the anchorage of the end of a reinforcement bar in a concrete prism. This case can be applied to the study of anchorage of reinforcement at the end of a beam.

A more complicated case directly related to the primary aim of this investigation is shown in fig. 3. Here the problem is to find how the stress is transferred from the through bars to the bars cut off between the ends of the prism. These ends can be said to represent cracks which arise in the neighbourhood of the cut-off bar ends in the tension zone of a beam. Between them there may be a more or less short-distanced crack formation. The concrete stress

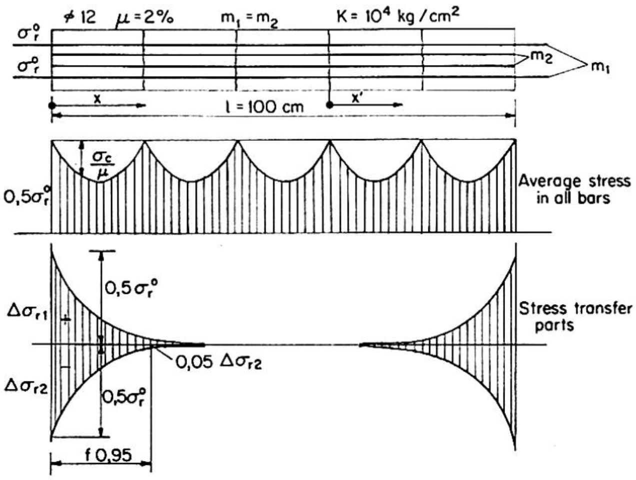


Fig. 3. Tensile prism with through bars (number m_1) and cut-off bars (number m_2), analysed according to the K-value theory. The reinforcement stress curves are calculated for a prism of the length $l = 100$ cm with equally spaced cracks, reinforcement percentage $\mu = 2{,}0\,\%$, $\varnothing$ 12 mm, bars with $m_1 = m_2$ and a K-value 10^4 kg/cm². The upper curve represents the average stress in all bars; the "waves" are caused by the bond stress transfer to the concrete between the cracks (last two terms of Eq. (10)). The lower curves represent the stress transfer from the through bars (upper branch) to the cut-off bars (lower branch, first terms of Eq. (10)). The distance $f_{0.95}$ represents the length along which the stress transfer is carried through up to 95 %.

can be calculated in exactly the same way as in the basic problem above and it will be found that the stress distribution between any two cracks can be written with an expression analogous to (4)

$$\sigma_c = \frac{\mu}{1+n\,\mu}\,\frac{m_1}{m_1+m_2}\,\sigma_r^0\left[1-\frac{\cosh\kappa\left(\frac{s}{2}-x'\right)}{\cosh\kappa\frac{s}{2}}\right], \tag{8}$$

where m_1 is the number of through bars and m_2 the number of cut-off bars, and x' is the distance from a crack position $s=0$. The reinforcement stresses σ_{r1} and σ_{r2} respectively can be obtained from the two differential equations

$$\frac{d\sigma_{r1,2}}{dx^2} - \kappa^2\,\sigma_{r1,2} = -\kappa_0^2\,n\,\sigma_c, \tag{9}$$

where the notation

$$\kappa_0^2 = \frac{K}{E_r}\,\frac{4}{\Phi} \tag{9a}$$

is used and σ_c is inserted according to equation (8). The solutions can be written in the simple form

$$\sigma_{r\,1,2} = \Delta\,\sigma_{r\,1,2} + \frac{m_{1,2}}{m_1 + m_2}\,\sigma_r^0 - \frac{\sigma_c}{\mu}, \tag{10}$$

where $\Delta\,\sigma_{r\,1}$ and $\Delta\,\sigma_{r\,2}$ represent the stress change in the through and cut-off reinforcement bars respectively due to the stress transfer from the through bars to the cut-off bars. From the limit conditions on both sides of a crack we can find that these stress functions are entirely continuous through the crack and thus quite independant of the number or location of the cracks. If the entire length of the cut-off bars is fairly great, these stress functions can be written under the simplified form

$$\Delta\,\sigma_{r\,1} = \frac{m_1}{m_1 + m_2}\,\sigma_r^0\,e^{-\kappa_0 x}; \qquad \Delta\,\sigma_{r\,2} = -\frac{m_2}{m_1 + m_2}\,\sigma_r^0\,e^{-\kappa_0 x}. \tag{11}$$

These results have been applied on a numerical example in fig. 3. The quantity (9a) indicates the rapidity of the stress transfer and the process of anchorage. Theoretically the anchorage can never reach 100%, but if we consider for instance an anchorage of 95% as in practical cases complete the necessary anchorage length $f_{0.95}$ can be calculated from equation (11) and will be

$$f_{0.95} = \frac{3{,}0}{\kappa_0}. \tag{12}$$

As pointed out in the introduction the relation (1) between bond stress and displacement is far from linear. It is possible to obtain an idea of the

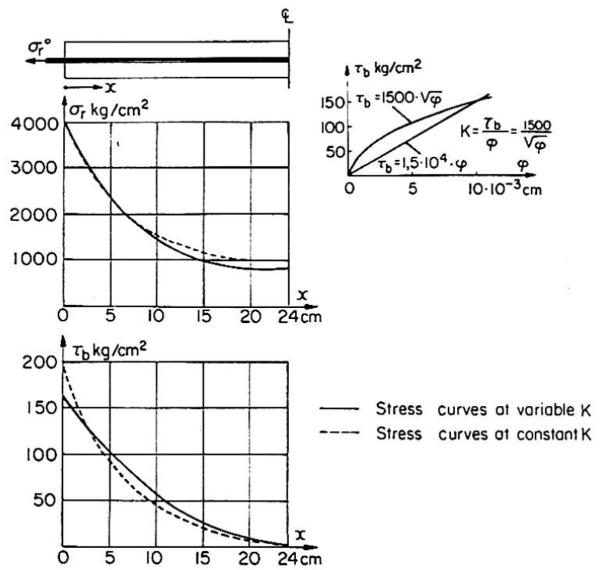


Fig. 4. Stress distribution in the reinforcement in the basic case according to Figure 2 calculated for a parabolic relation curve $\tau_b - \varphi$ with the aid of iterative difference calculus and compared with the stress curve valid for a constant K-value estimated from the τ_b and φ values in the prism end.

influence of this on the stress distribution with the aid of iterative difference calculus. For such a calculation, based on an arbitrary relation between τ_b and φ, the equations (2), (3) and (5) are re-written as

$$\frac{d\varphi}{dx} = \frac{1}{E_r}\sigma_r(1+n\mu) - n\mu\sigma_r^0 \tag{13}$$

and applied together with the equation (4). As is obvious from the example illustrated in fig. 4 the influence of the non-linearity is, however, comparatively small.

3. Tests on Tensile Test Specimens

To verify these theoretical results tests have been made on test specimens which agree with the theoretical models discussed above. Test specimens were made as tensile forced prisms with through or end-anchored reinforcement as well as prisms such as the theoretical model in fig. 3 with cut-off reinforcement. In order to measure the stress distribution the cast-in bars were provided with slots all along the bar in which resistive strain gauges of very small dimensions were glued very close to each other. The test results are in fairly good agreement with the theoretical results. Fig. 5 gives an example of the test results and the corresponding theoretical curves for two of the specimens with cut-off bars.

Thus, the test results also show, that the simple assumption (1) of proportionality between bond stress and displacement could be used for a theoretical study of bond and anchorage problems.

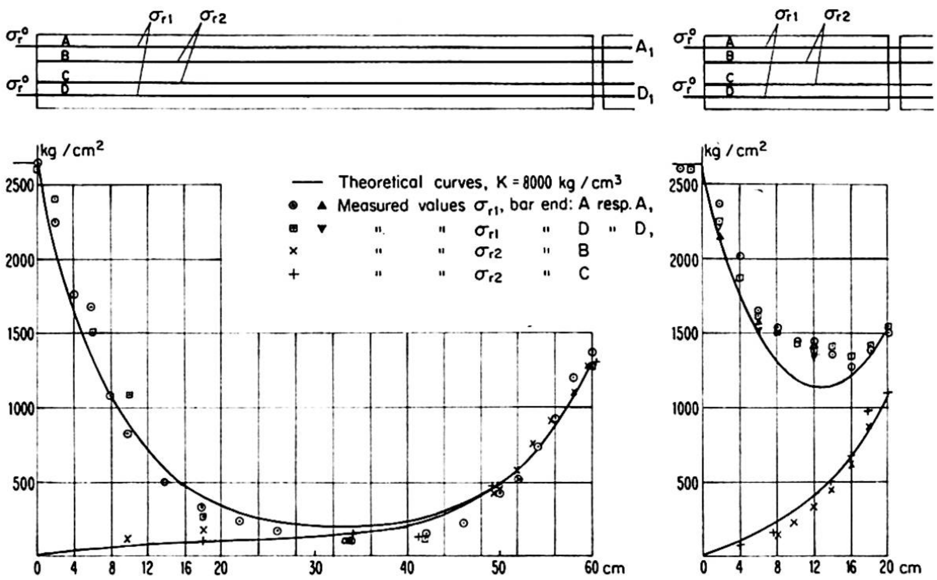


Fig. 5. Test results of stress measurements on two tensile prism specimens with through bars A and D and cut-off bars B and C. The prisms were provided with an artificial crack in the middle. For purposes of comparison the theoretical curves have been calculated for a constant $K = 8000$ kg/cm² and are shown in the diagram.

4. Application to Beams

Finally, the results from pure tensile cases will be applied to the case of anchorage of cut-off reinforcement bars in beams. Theoretical considerations show that the same formula (11) for stress transfer between through and cut-off bars can be used, as shown in fig. 6. In the example given in that figure

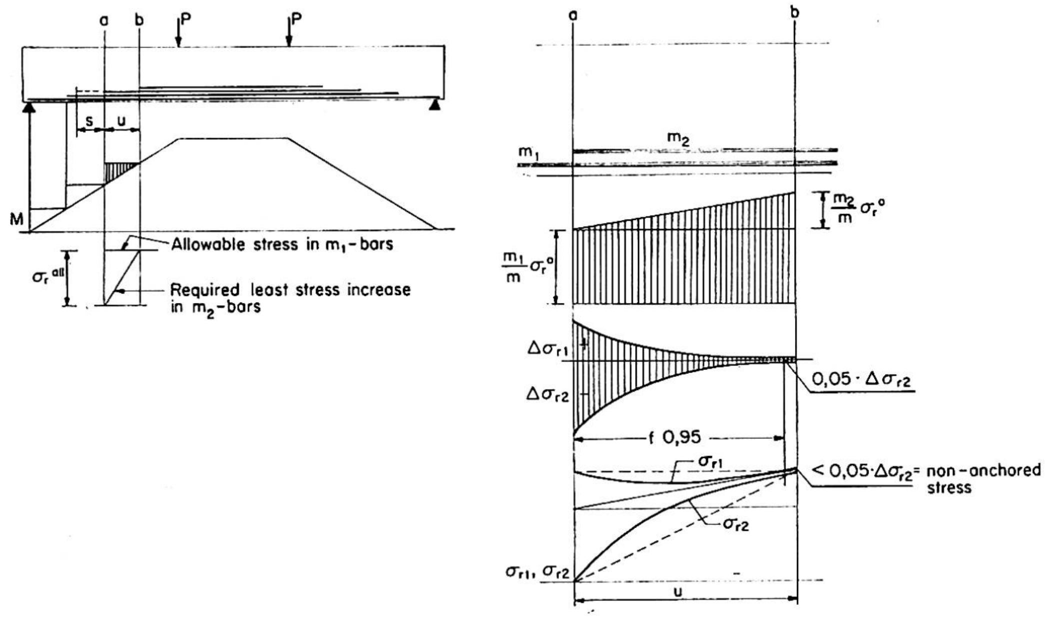


Fig. 6. Assumed stress transfer in beam reinforcement from through bars m_1 to bars m_2 cut off according to the moment curve. The stress variation caused by the bond stress transfer to the concrete between the cracks has not been considered, otherwise the stress curves in the detailed part of the figure agree with the ones shown in Figure 3.

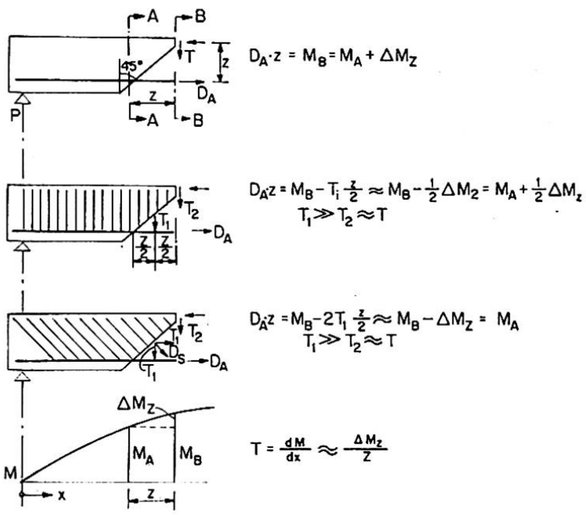
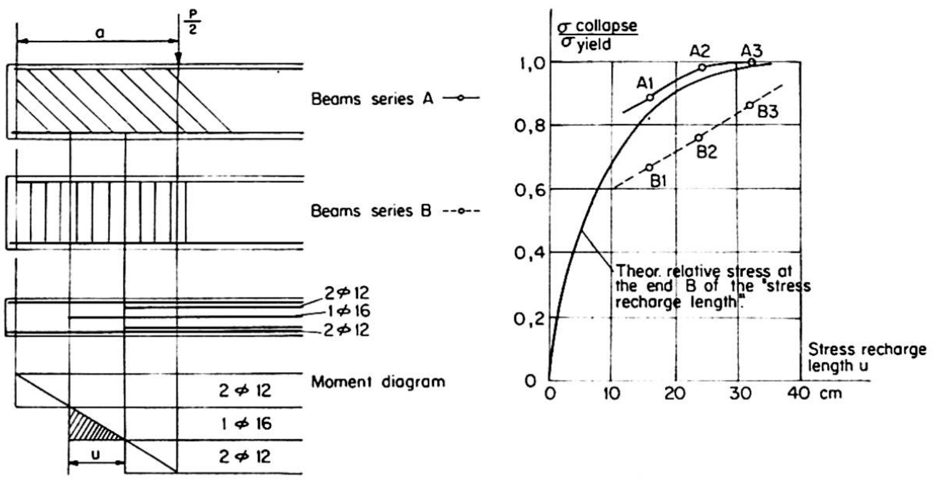


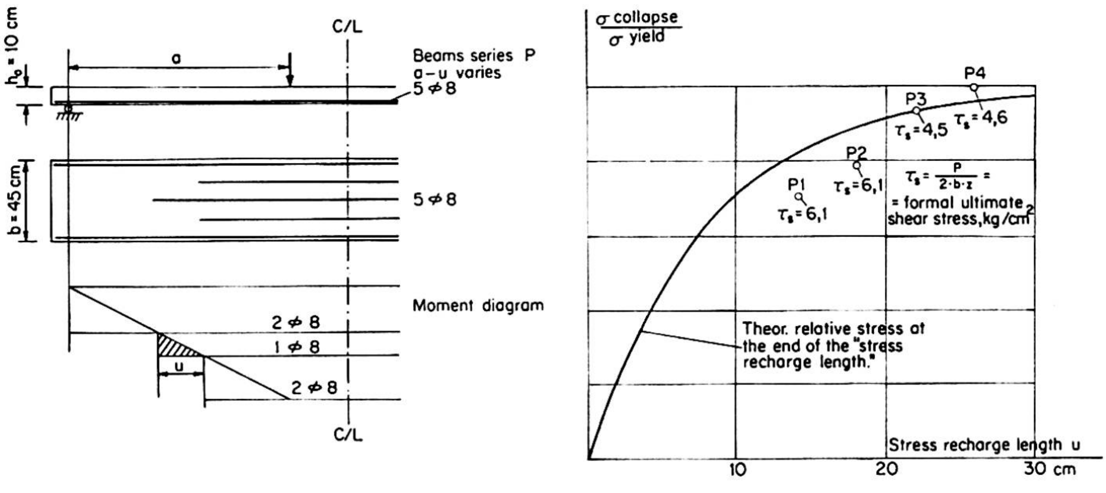
Fig. 7. The effect of the dislocation of the moment centre caused by inclined shear cracks, the "z effect". In a beam without web reinforcement the tension reinforcement force in the section $A-A$ corresponds to the moment in section $B-B$, so the cut-off point must be dislocated the corresponding distance. In beams with stirrups this moment increase is less.

a cut-off bar which, in accordance with the moment distribution curve, begins to act at a cross-section a, need not have reached full stress before the cross-section b. In such cases the length $a-b$ may be used as "*stress recharge length*", u, which will give the cut-off bar full stress anchorage at point a, if the distance $a-b$ is long enough compared with the anchorage length f according to fig. 3. No anchorage whatsoever is then necessary outside the theoretical point a.

In judging this it is, however, necessary to observe the effect of the dislocation of the moment centre in the beam due to the rise of diagonal shear tensile cracks illustrated in fig. 7. This so called *z effect* will give an increase of the necessary anchorage length of the same magnitude as the inner moment



a) Beams with vertical or diagonal stirrups.



b) Slab strips without shear reinforcement. The formal shear stress τ_s at collapse is shown underneath the result points.

Fig. 8. Stress in the beam reinforcement calculated from the collapse load in relation to the stress recharge length u and compared with the theoretical transferred stress at the inner end of the stress recharge length of a bar cut off directly according to the moment curve.

arm z. If the beam has web reinforcement of vertical stirrups, this effect will be reduced to about half, and if the web reinforcement is made of bent-up bars or inclined stirrups, the effect will be practically completely reduced.

These theoretical considerations are applied to results from test series of beams with diagonal and vertical stirrups as well as without web reinforcement. In all beams the reinforcement is cut off directly according to the moment curve. In fig. 8a the results from one series of beams with vertical stirrups and one series with diagonal stirrups have been assembled and the stresses in the reinforcement at collapse are compared with the theoretical stress at the inner end of the stress recharge length. The agreement seems to be satisfactory for the diagonally web reinforced beams, but for the beams with vertical stirrups it is obvious, quite in accordance with the theoretical considerations, that the z effect requires an extension of the anchorage length. Fig. 8b shows some of the results from tests on slab strips without web reinforcement. The slabs with the longest stress recharge length agree with the test results and the theoretical curve, but for the slabs with short stress recharge length the agreement is not so good, obviously due to the fact that in these cases with a shorter shear span the shear stresses are higher and produce inclined tensile cracks giving z effect.

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Summary

This paper studies the anchorage of cut-off reinforcement bars and the stress transfer from cut-off to through bars in concrete prisms and beams with reinforcement of deformed bars. The theoretical treatment is based on the hypothesis that the bond stress is directly proportional to the displacement between steel and concrete. To verify the theory, tests on tensile specimens were made, in which the steel stress was carefully measured with strain gauges glued in slots along the bars. The results were applied to the beam case where the reinforcement is cut-off according to the moment curve, and the influence of shear crack and web reinforcement was also considered.

Résumé

Cette communication a pour objet l'étude de l'ancrage des armatures et de la transmission des contraintes des armatures interrompues à celles qui sont continues dans des primes en béton et des poutres comportant des armatures à empreintes. L'analyse théorique procède de l'hypothèse selon laquelle la contrainte d'adhérence est directement proportionnelle au déplacement relatif de l'acier et du béton. On a éprouvé la validité de la théorie en soumettant des éprouvettes de traction à des essais qui ont permis, grâce à des jauges ohmiques collées dans des fentes le long des armatures, de mesurer avec précision les contraintes développées dans l'acier. Les résultats sont appliqués au cas d'une poutre comportant des armatures arrêtées conformément à la courbe des moments, avec prise en compte également des fissures par cisaillement et de l'armature de cisaillement.

Zusammenfassung

Dieser Beitrag untersucht das Verhalten von Verankerungen bei abgestuften Stahleinlagen und die Spannungsübertragung von abgestuften auf durchlaufende Stahleinlagen von mit Rippenstahl bewehrten Betonprismen und Betonträgern. Die theoretischen Untersuchungen fußten auf der Annahme direkter Proportionalität zwischen der Haftspannung und der relativen Verschiebung zwischen Stahlarmierung und Beton. Durch Versuche an Zugproben, wobei die Spannungen in der Armierung durch aufgeklebte Spannungsmesser sorgfältig ermittelt wurden, konnte die Richtigkeit dieser Annahme bestätigt werden. Die erhaltenen Ergebnisse wurden für den Biegeträger mit entsprechend dem Momentenverlauf abgestuften Stahleinlagen übertragen, wobei auch der Einfluß von Schubrissen und der Stegarmierung berücksichtigt wurde.