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Dynamic Behavior of Large Structures Studied by Means of Models

Etude du comportement de grandes constructions soumises à des efforts dynamiques, au moyen de modèles réduits

Modelluntersuchungen über das dynamische Verhalten großer Bauwerke

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Foreword

1. The "response" of a complex structure to seismic action can practically be obtained only by testing on models. As is known, the models may be of the following types: electric analogical (HUDSON); mechanical, built after having previously schematized the structure into a set of appropriately sized and connected typical elements (JACOBSEN); and, finally, real structural models (ISMES).

The models mentioned last do not require a previous schematization. However, they do have to meet rather burdensome similitude conditions.

In a previous paper [1] we have outlined the conditions governing experimentation on elastic models endowed with viscous damping and on more complete models which can be carried to failure.

2. The studies at ISMES, Bergamo, were conducted in both directions, viz.:

— complete models, carried to failure, for dams[1]);

— elastic models with viscous damping, for geometrically complex structures (such as frame systems). These models are still to be regarded as calculating machines and need to be interpreted in conformity with appropriate schemes in order to relate the results yielded by them to prototypes subjected to actual earthquake vibrations.

In what follows we shall illustrate, by means of a significant example, the way a study on an elastic model is conducted.

Features of the Model

3. The model here dealt with reproduced a 45-story reinforced concrete building. 193 m high above its foundation (Fig. 1).

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[1]) From 1955 until now ISMES studied the dynamic behavior of the following dams: Ambiesta (Italy), Dez (Iran), Kurobe IV (Japan), Soledad (Mexico), Santa Rosa (Mexico), El Novillo (Mexico), Grancarevo (Yugoslavia) and Rapel (Chile).

The load-bearing elements were reproduced in their entirety, with no simplification whatever. The only exception were the floors which in the prototype consist of ribbed slabs whereas in the model they were reproduced by slabs of constant thickness having the same flexural stiffness. By sheer coincidence, the excess of mass of the constant thickness floors compared to the ribbed slabs was exactly the same as the mass of the accidental overload foreseen in the project.

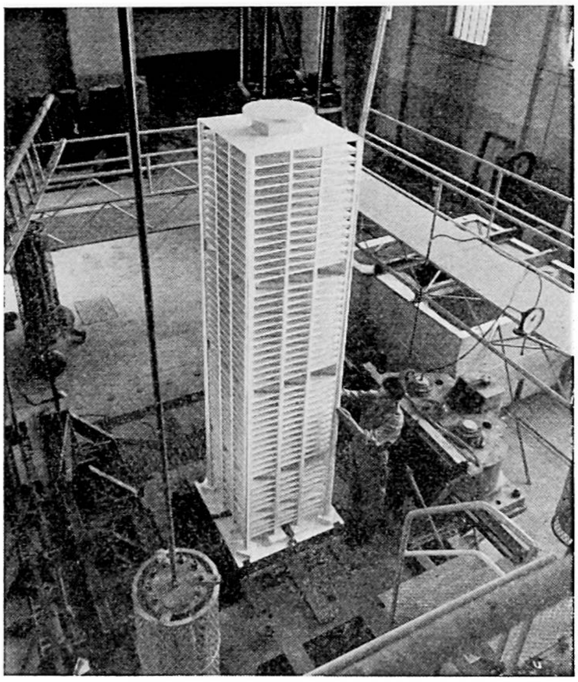

Fig. 1.

4. The model was made of celluloid. The similitude ratios of the basic quantities were as follows

— length $\lambda = L_r/L_m = 52.8$,
— density $\rho = \gamma_r/\gamma_m = 18.8$,
— Young's moduli $\epsilon = E_r/E_m = 13$.

Consequently, the time scale was:

$$\tau = \frac{T_r}{T_m} = \lambda \rho^{1/2} \epsilon^{-1/2} \cong 20 .$$

As to the dimensionless damping coefficient, its determination on the model and the methods of relating to the prototype are described in Section 9.

Testing Procedure

5. The model, complete with foundation and placed on a vibrating table that was moved by a 10 ton vibrodyne, was subjected to nearly stationary horizontal vibrations with a frequency ranging from 0 to 3.75 Hz (on the

full-scale structures) and directed along the normal to one of the faces of the model[2]).

6. The displacements and strain measurements yielded the "response" of the structure for the entire range of the applied vibrations.

The first four vibration modes were identified. Correspondingly, the model displacements and strains were determined, and, using the methods outlined in Section 9, the damping coefficients were evaluated.

For brevity's sake we shall omit giving here the data directly obtained from the tests. In the following Section we shall only bring the values derived from some of them through appropriate elaboration since they are the only ones of interest beyond the specific case here described.

Analysis of the Principal Results

7. Upon obtaining on the model, without resorting to schematization, the "response" of the structure with regard to nearly stationary vibrations, one has to pass to evaluating that "response" as regards the real vibrations which, as is known, are transitory and extremely irregular.

For this purpose we have made use of the well-known "seismic spectrum" technique [2].

Substantially in this technique it is assumed that the effects of a given earthquake may be characterized by a set of curves, called "response curves", which are the loci of the maximum effects (displacements, velocities or accelerations) produced by the earthquake on a group of independent vibrators having a single degree of freedom and a viscous damping capacity.

When starting from these curves, it is not possible to arrive at numerical results for any structure under study, although one can always obtain guide lines and directions not otherwise procurable at present.

On the other hand, fairly correct solutions, also from a quantitative point of view, can be secured from the spectrum technique for structures with well-defined dynamic features.

Should calculation be made use of these features should not be greatly affected by the schematizations required to make the calculation possible.

In our case, by using a model, and with no recourse to any structural schematization, we obtain the "response" of the structure to "civilized" excitations.

At this point, the basic problems are two, and both of them can be solved by the spectrum technique, within the range of its validity. The problems are:

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[2]) The inertia ellipse of the cross-sections of the buildings is a circle. This, to a certain extent, warrants extending the results obtained for one of them to all the vibration directions. Such assumption was also corroborated by appropriate preliminary testing.

a) relating the model earthquake to the real ones[3]). This is feasible by comparing the respective spectra;
b) superimposing the results obtained on the model at the various frequency bands in order to relate them to a single actual earthquake.

This is exactly what we shall do in Sections 8 to 11 as regards defining the real spectra, those of the model, and their mutual relations, and in Sections 12 and 13 which deal with the superimposition of the results pertinent to the single vibration modes of the building.

8. The linear single degree-of-freedom structure has a mass m, stiffness K, damping c, base motion z and relative displacement y.

The equation of motion is:

$$m\ddot{y} + c\dot{y} + Ky = -m\ddot{z}.$$

It is proved that the maximum values of the relative displacement and velocity of the mass m and its absolute acceleration during the earthquake $z(t)$ can be expressed by the following relations:

$$(\ddot{y}+\ddot{z})_{max} \cong \omega_n S_v, \qquad \dot{y}_{max} = S_v, \qquad y_{max} \cong S_v/\omega_n$$

with $S_v = (\sqrt{A^2+B^2})_{max}$,

and $A = \int_0^t \ddot{z}\, e^{-\omega_n \zeta (t-u)} \cos(\omega_n u)\, du, \qquad B = \int_0^t \ddot{z}\, e^{-\omega_n \zeta (t-u)} \sin(\omega_n u)\, du,$

where $\omega_n = \sqrt{\frac{K}{m}} = \frac{2\pi}{T_n}$,

T_n = natural period of vibration,

$\zeta = \frac{\mu}{\omega_n} = \frac{c}{2m\omega_n}$ fraction of critical damping (dimensionless.)

As is known, the California School has calculated the "response" functions S_v for the two horizontal components in the case of the following four earthquakes: El Centro 1938, El Centro 1940, Olympia 1949 and Taft 1952. The appropriately handled values yielded the family of typical curves $S_v(T, \zeta)$ given in Fig. 2 which, on the average, represents all the mentioned earthquakes (to be multiplied by 2.7, 1.9, 1.9 and 1.6 for the above-cited earthquakes respectively).

We, too, shall refer to this family of "real spectra"[4]).

9. As regards the spectra S_v of the model earthquakes, it is expedient to

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[3]) Quite obviously, this does not mean determining the mechanical similitude scale, which has already been evaluated in Section 4, but comparing something which may be likened to a "coefficient of shape" of the two earthquakes.

[4]) It is beyond the scope of the present paper to discuss the right of referring to Californian "spectra" for constructions outside of California. Here, only the method of investigation is dealt with and not the discussion of its numerical aspects.

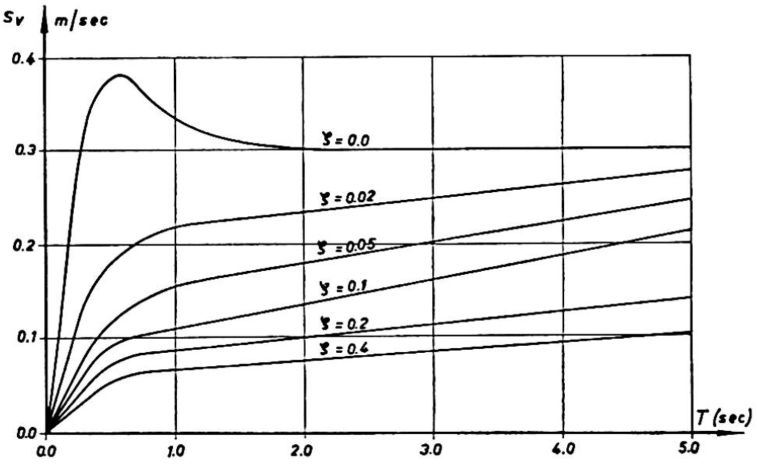

Fig. 2. Average velocity spectrum curves for strong ground motion (cf. [2]).

evaluate them theoretically by previously determining the coefficients of damping which, in turn, are to be directly obtained on the model[5]).

From the resonance curves relating to the strains ascertained in the vicinity of the frequencies 0.23 and 0.72 Hz (relating to the first and second vibrations mode) and by the use of the well-known "$\sqrt{2}$ Method"[6]), the following damping values may be obtained (in fractions of the critical damping):

$$\text{for} \quad f = f_1 = 0.23 \text{ Hz}, \qquad \zeta_1 = 0.059,$$
$$\text{for} \quad f = f_2 = 0.72 \text{ Hz}, \qquad \zeta_2 = 0.020.$$

These values well agree with a widely spread theory, according to which the damping is to be represented by an expression of the type:

$$\zeta = \frac{\mu}{2\pi f}, \tag{1}$$

where $\mu = \mu(f)$ is constant.

[5]) It is preferred to calculate the spectra rather than utilize those which may be obtained experimentally because it is desired to take into account, with the greatest possible precision, the exact damping value of the model which differs from the one of the specimen vibrators owing to the different stress conditions. For the same reason, it is good to directly determine the damping coefficients by experimentation on the model instead of on specimens.

[6]) The "response" obtained from an extensometer or a deflectometer was likened to that of a simple vibrator excited by a centrifugal force, whose equation of motion is:

$$\ddot{x} + 2\,\zeta\,\omega_n \dot{x} + \omega_n^2 x = p\,\omega^2 \sin \omega t.$$

The amplitude under working conditions is:

$$x = p\left(\frac{\omega}{\omega_n}\right)^2 \left\{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(\frac{2\,\zeta\,\omega}{\omega_n}\right)^2\right\}^{-1/2}. \tag{2}$$

In the vicinity of resonance it differs only by the factor ω_n/ω from that of the vibrator excited by forces expressed by $p \sin \omega t$, for which the "$\sqrt{2}$ Method" is rigorous.

It follows that, within the approximation range of interest to us, the use of the more convenient "$\sqrt{2}$ Method" in our case is justified.

Because of the lack of reliable experimental data, the damping coefficients for the third and fourth vibration modes are obtained by extrapolation using expression (1):

$$\text{for}\quad f = f_3 = 1.47\ \text{Hz},\qquad \zeta_3 = 0.010,$$
$$\text{for}\quad f = f_4 = 2.45\ \text{Hz},\qquad \zeta_4 = 0.006.$$

The function $\mu(f)$ thus obtained is given in Fig. 3.

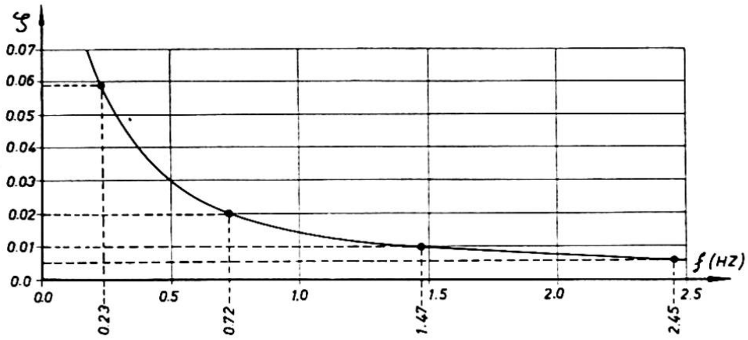

Fig. 3. Damping coefficient ζ of the model.

The calculation of the spectra (i. e., of the maximum velocity and displacement responses pertinent to the simple vibrator), easily feasible for the model earthquakes when starting from equation (2), yields:

$$\dot{y}_{max} = S_v = \pi p / \zeta T_n .$$

Passing to numbers and bearing in mind the previously obtained values, we have:

f (Hz)	ζ	S_v ($m\ s^{-1}$)
$f_1 = 0.23$	0.059	0.612
$f_2 = 0.72$	0.020	5.653
$f_3 = 1.47$	0.010	2.308
$f_4 = 2.45$	0.006	6.412

10. Let us now call "earthquake simulation ratio" the value

$$\overline{\Sigma} = (S_v)_p / (S_v)_m$$

of the ratio between the response spectra of the mean actual earthquakes (without considering the unessential "factor" relative to each single earthquake) and the model earthquakes [7]).

In our case, with the data given above for the first four vibration modes of the structure and the four damping values (supposing these values to be

[7]) It is obvious that assuming (as in our case) stationary model earthquakes of constant amplitude whereas the actual earthquake is far different, the number of the ratios $\overline{\Sigma}$ equals that of the frequencies to be considered.

the same for the model and the prototype as is demanded by the theory of similitude), we have:

f (Hz)	ζ	$\overline{\Sigma}$
f_1 = 0.23	0.059	0.373
f_2 = 0.72	0.020	0.040
f_3 = 1.47	0.010	0.109
f_4 = 2.45	0.006	0.038

11. On the other hand, the coefficient of damping usually taken for concrete structures is $0.05 \leqq \zeta \leqq 0.15$, i. e., by far greater than those assumed so far.

This difference, too, can be taken into account.

When the model is assumed to be operating under partial similitude conditions and the average damping coefficient ζ is taken as unvarying with the frequencies and equal to 0.1, four new "earthquake simulation ratios" can be established[8]).

The interpretation scheme of the spectrum technique seems to fully warrant such a procedure.

Operating as in the preceding case, the following new set of four values is obtained:

f (Hz)	ζ	$\overline{\Sigma}'$
f_1 = 0.23	0.1	0.328
f_2 = 0.72	0.1	0.021
f_3 = 1.47	0.1	0.044
f_4 = 2.45	0.1	0.012

12. Having thus established the ratios $\overline{\Sigma}$ or $\overline{\Sigma}'$, which make it possible to relate to the prototype the measurements made on the model at the various vibration "modes", it is now necessary to assemble them into a "whole" which is the response of the structure to the "totality" of the excitations forming the earthquake. The vibrations of a multiple-freedom system are given by a linear combination of the vibrations relevant to the individual modes. In the case of seismic vibrations according to the spectrum technique, every vibration "mode" has to undergo an "amplification" corresponding to its spectrum $S_v(f)$.

In our case, the model already supplied by direct measurement the responses pertinent to the frequency bands containing single modes, and we already related them to the prototype by means of the ratios $\overline{\Sigma}$ or $\overline{\Sigma}'$.

[8]) Substantially, this indicates that the model was damping the vibrations much less than the prototype is supposed to be capable of doing.

It follows that the total response of the structure to the real earthquakes will be given [2] by the following expression:

$$Y = \sum_{1}^{n}{}_{i}\, y_i(x) \sin(\omega_i t - \alpha_i)$$

and the acceleration response will be:

$$\ddot{z} + \ddot{Y} \cong \sum_{1}^{n}{}_{i}\, \omega_i^2\, y_i(x) \sin(\omega_i t - \beta_i).$$

The assumed phase differences α_i and β_i can be evaluated by different principles, which range from those aiming at obtaining for Y the most probable value to those which merely furnish the absolute maximum value [3].

13. In our case, taking into account some basic considerations concerning the nature of the ground and the consequent importance of the harmonic vibrations [4], we proceeded as follows:

— The experimental deformed lines $y_i(x)$ (Fig. 4) pertinent to the single vibration "modes" (already related to the prototype by the earthquake scales $\bar{\sum}_i$ or $\bar{\sum}'_i$) represent approximately the maximum absolute accelerations.

Multiplying the various $y(x)$ by the mass $m(x)$ distribution, the distributions $F_i(x)$ of the forces of inertia relative to the single modes were obtained.

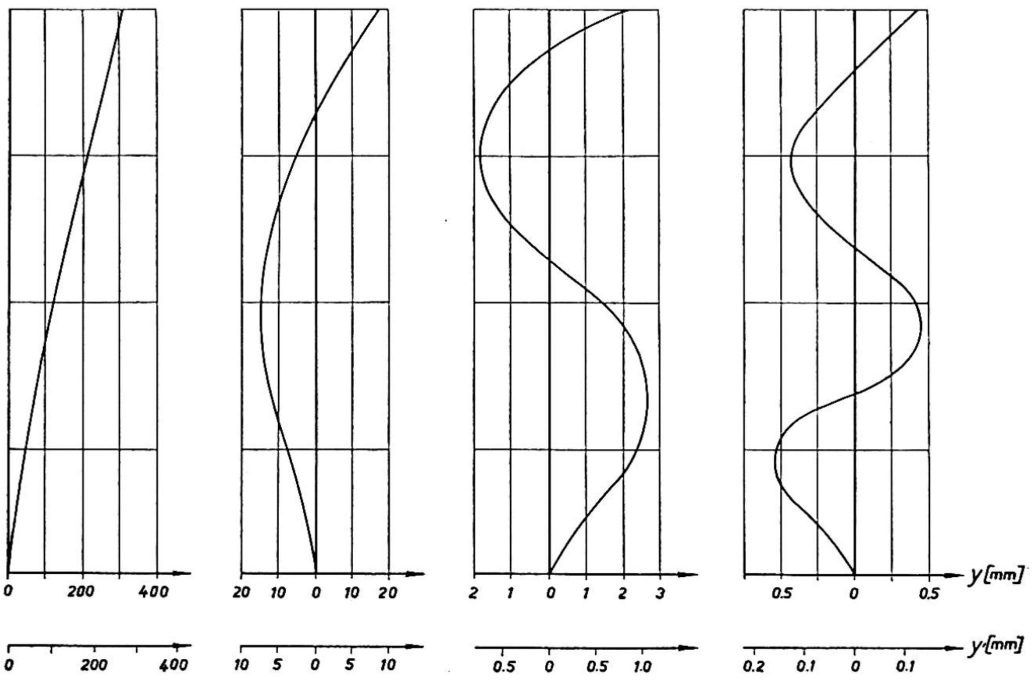

Fig. 4. Deformed lines Y_i and Y'_i for actual earthquakes, corresponding to the first four vibration modes.

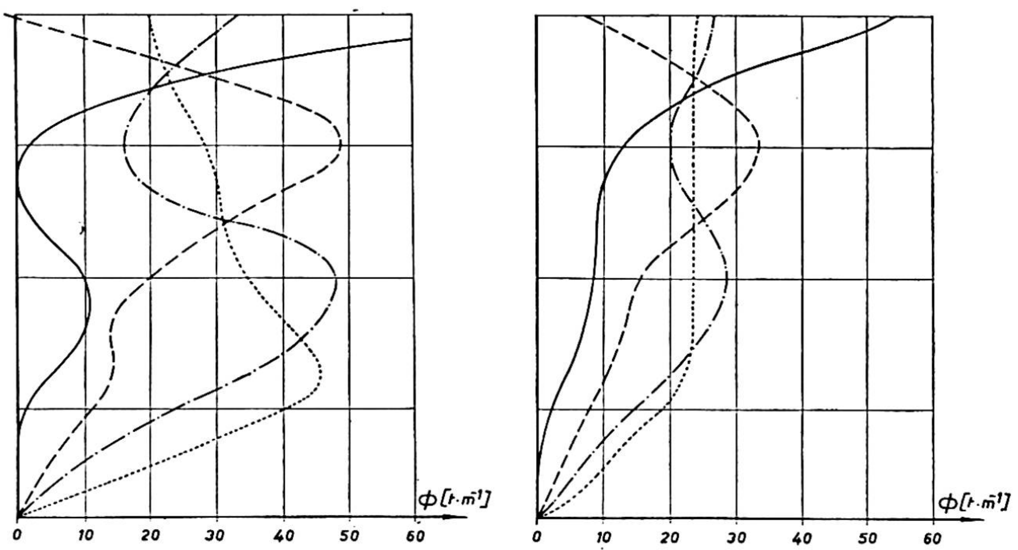

Fig. 5. Some significant diagrams Φ of inertial forces obtained by algebraically summing the forces $Y_i m \omega_i^2$ or $Y'_i m \omega_i^2$.

— It was assumed that all the modes appear at maximum amplitude and phases $\beta_i = 0$ or $\beta_1 = \pi$ simultaneously along the entire structure.

The total inertia forces Φ were then given by variously combining the force systems F_i into groups of four (Fig. 5).

— From the diagrams Φ, the most unfavorable values of the shear and bending moments were obtained.

Conclusions

The above example shows how the dynamic behavior of a highly complex structure may be studied with the aid of an elastic celluloid model, without previous schematization.

Operating as a calculating machine, the model furnished the "response" of the structure to "civilized" vibrations.

An interpretative theory then made it possible to obtain the maximum stresses at the various points of the structure for the real earthquakes.

This, to be sure, is a good result.

However, it must be observed that the coefficient of damping is of primary importance in the behavior of structures (hence, in their both theoretical and modelling schematizations).

Unfortunately, the data about this coefficient, especially for concrete structures, are very scarce, and in the case of high stresses (required in safety problems) they lack almost altogether.

This, in our opinion, is the most serious gap to be removed if earthquake engineering is to be placed on a more solid basis than it is at present.

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Summary

The present paper deals with the guiding criteria used in dynamic tests on multistoried buildings.

The main results obtained for one building subjected to sinusoidal vibrations are given. It is proved that, through the use of the spectrum technique, these results can be applied to earthquake vibrations.

Résumé

La présente communication traite des critères directeurs utilisés pour les essais dynamiques sur des bâtiments à plusieurs étages.

Les principaux résultats obtenus pour un bâtiment soumis à des vibrations sinusoïdales sont présentés. Il est démontré que, par l'emploi de la technique des spectres, ces résultats peuvent être étendus aux vibrations sismiques.

Zusammenfassung

Die vorliegende Arbeit behandelt die Richtlinien für dynamische Versuche an mehrstöckigen Bauwerken.

Es werden die wichtigsten Resultate von Untersuchungen an einem Bauwerk mit sinusförmigen Schwingungen gegeben. Es wird gezeigt, daß durch die Anwendung der «Spektrumtechnik» diese Resultate für Schwingungen von Erdbeben verwendet werden können.