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Plastic and Elastic Designs Compared

Comparaison du calcul plastique et du calcul élastique

Vergleich zwischen plastischer und elastischer Berechnungsweise

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Elastic and plastic designs are different in their approaches. Basically they are both sound, but in actual execution plastic design is inferior to the elastic because it is insufficiently developed and is influenced strongly by the variations in unpredictable properties of material and complex behaviour of the structure when it comes near failure. Weaknesses of the elastic design are in comparison minor.

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Plastic design although a relatively new development, has found numerous advocates in the English speaking world. In November 1961 it was introduced into the specifications of the American Institute of Steel Construction, thus attaining a status equal to that of the conventional elastic method. Projection of the new method into the field of practical use and determined claims as to its superior rationality and economy put on order its critical examination and close comparison with the elastic design.

The inception of the new method may be traced to criticism of certain aspects of the conventional elastic method. Thus it has been suggested that the use of the same allowable working stress, a proposition on which the elastic design is based, is not reasonable for I beams bent about the major axis on the one hand and the solid rectangular beams, or the same I beams bent about the minor axis, on the other, because in the latter case only a small fraction of the area is subjected to a high stress; again it has been stated that statically indeterminate beams are in general farther removed from failure than the determinate ones designed to the same allowable stress.

The writer admits the justice of this criticism and feels that it can be met by proper adjustment of the allowable stresses, which by the way has been already partially done in certain areas. This admission however is far removed from the primary tenet of plastic theory, the acceptance of failure condition as the criterion of design. Failure is a logical basis of design only if it can be properly pinpointed in magnitude and location, — this however, apart from some simple cases, seems impossible, as becomes apparent from the following discussion.

1. Yield Stress

In the early development of plastic theory failure of a structure, such as a statically indeterminate rigid frame, was identified with formation of a requisite number of plastic hinges making the structure geometrically deformable. The values of bending moments at plastic hinges were considered constant and independent of angle changes. This supposition will now be examined closely.

Plastic moment is proportional to the yield stress, the value of which is normally assumed constant. However, L. S. BEEDLE [1] found that the yield stress of beams made of the commonly used ASTM-A 7 steel, nominally 33 kips/sq. in., actually varied between 25 and 48 kips/sq. in. These figures refer to complete sections of beams. Variation between the individual parts of flanges and webs is undoubtedly even greater. With yield stress varying in such wide limits, moments at plastic hinges become unknowable.

It may be argued that proper physical tests will eliminate material with yield stress below some specified nominal value like 33 kips/sq. in., but it is scarcely possible to exclude simultaneously the material stronger than normal, and such material is almost equally objectionable, because its presence at the location of a plastic hinge on the end of a member may lead to a premature failure of the connection designed on the basis of the nominal yield stress. With unpredictable value of yield stress plastic design may be likened to measurement of length with a scale whose divisions are grossly in error.

Unlike its novel counterpart, elastic design is not dependent on yield stress, although physical tests must guarantee a certain minimum value of it. What is important in elastic design is proportionality between stress and strain, and this is normally maintained throughout the range of the working loads. Deflections may sometimes be also significant. They are governed by the modulus of elasticity and the value of the latter is almost invariant for structural steel.

2. Design of Beam-columns

Barring lateral-torsional buckling, the capacity moment that may be carried by a beam equals the plastic moment. On the other hand the capacity moments on the ends of a column are much smaller than plastic value and are affected in a complicated way, by the thrust and the slenderness ratio. Moreover, these moments correspond to some definite angles of rotation on the ends of columns and should these angles be increased, as may be demanded by the consistency of deformations, the end moments will decrease below the capacity values [2]. The behaviour of beam-columns is thus different from beams, as well as more difficult and uncertain to analyze.

Design of structures whether elastic or plastic is normally a check design. This means that assumption of appropriate sections of members is made and is followed by determination of their moments, shears and thrusts caused by the working or factored loads. The operation is concluded by checking the sections on the basis of the computed load functions.

In elastic theory determination of thrusts and moments is subject to a well established rigourous procedure involving nothing intrinsically difficult in principle, even though it may at times be supplanted for reasons of expediency, by appropriate simplified operations. In plastic theory, at least when the structure involves columns, a similar rational analysis is impossible, because the moment-angle change relations under elasto-plastic conditions on the verge of collapse are unavailable. The two methods used for this purpose in plastic analysis: [3, 4] determination of statically consistent sets of thrusts and moments, with no regard for the consistency of deformations, and the method involving moment distribution by elastic distribution factors, must both be considered as crude approximations. The mechanism method based on constancy of moments at plastic hinges is, of course, incorrect with regard to columns.

Once the moments and thrusts have been determined the adequacy of the members must be checked by available methods. In this phase the elastic and plastic designs are more comparable. Verification of sections, especially of columns, is based in both methods on the use of empirical formulae or graphs. In plastic design this procedure however is more uncertain because the column interaction curves [1, 5] specifying safe combinations of moments and thrusts are based on the capacity moment, which, as has been explained earlier, may be reduced by excessive angle changes on the ends of columns. It must however be admitted that the empiricism of the column formulae used in the elastic design represents one of the weaknesses of the latter.

3. Lateral Instability

The problem of lateral instability is very complex even in the elastic range. Apart from single members with well defined conditions of restraint, the problem can be solved only approximately and with considerable difficulty by the energy method. When instability failure occurs in the plastic range the working load is taken as a fraction of the load at which the yield stress is first reached.

Instability problem in plastic design is considerably more formidable, especially in relation to columns. The column theory developed by Professor J. F. Baker of Cambridge University and his associates [3] is based on differentiating the plastically loaded columns from the elastically loaded. In the former the column end moments do not depend on possible rotations of the column ends, in the latter — they do. Assuming similar types of end moments on both ends of the column, BAKER distinguishes nine different loading cases. The writer considers this theory incomplete in spite of its complexity, because it does not cover all necessary cases of column behaviour. For example, an outer column of a two-storey (or multi-storey) rectangular frame should be designed as plastically loaded on one end and elastically on the other, — a case not considered by BAKER.

No wonder that the American column theory developed later [5] ignores completely BAKER's approach and visualizes the column as fully restrained from lateral buckling (as well as free from bending about the minor axis) by adequate bracing with the points of support spaced in accordance with some empirical formulae. These limitations restrict greatly the field of applicability of the American method.

Another important distinction between the American and English methods is their treatment of the residual stresses caused by rolling and cooling. In English method these stresses are completely ignored, in American method they are taken into consideration in accordance with a standard pattern involving compression stresses, equal to 30% of the yield stress, at the edges of flanges of wide flange sections.

The difference of the two methods with regard to residual stresses underlines further the basic uncertainties of plastic design. The American approach is undoubtedly more correct as well as more conservative of the two, but the writer is dubious that the effects of rolling are as constant as assumed; furthermore residual stresses are produced not only by rolling but also by cambering, welding and accidental bending and straightening, whose effects are not likely to conform to the assumed pattern. Accepting the premise that residual stresses are significant in relation to buckling, one should concede that their deviation from the assumed standard, which is certain, must also have a significant effect on failure.

Plastic buckling, unlike elastic, is also affected by creep. The subject of creep is not discussed in plastic literature and to what extent it is allowed for in plastic theory is not clear.

4. Live Loading

Apart from largely academic theories of alternating plasticity and incremental collapse application of plastic method has been limited almost exclusively to continuously acting loads. The writer knows of only one paper in which the presence of intermittent loading is discussed [4]. At the same time he feels that the recommendations contained in it with regard to design of columns underestimate greatly the design moments [6].

Diversity of live load placements required for design of different members of a structure results in an inherent difficulty for plastic method, because removál of live loads of failure intensity required for one set of members, leaves some residual stresses which often affect strongly the design stresses for another set of members. Professor BAKER obtains an economic design of columns [6] by simply ignoring the residual stresses produced by an earlier plastification of beams. This procedure appears to the writer unjustified. Considerations of probability also enter the picture in view of the high intensities of the failure loads exceeding the working loads by the load factor normally as high as 1.85.

Restrictions and qualifications of the type implied in BAKER's approach to the action of live loads in plastic design stand in sharp contrast with totally unqualified application of live loads in most unfavourable positions practiced in the elastic design.

5. Strain Hardening

Stresses higher than yield stress are not contemplated and never used in plastic design, yet it has been demonstrated [7] that without strain hardening plastic theory would be invalid irrespective of how long the yielding part of the stress-strain curve may be, as the beam would rupture at the earliest plastic hinge before the moments at the subsequent plastic hinges to be, would develop their full plastic values. However, with material such as structural steel, endowed with strain hardening, strains in the vicinity of plastic hinges would extend a short way beyond yielding and the equalization of moments would take place substantially as claimed (apart from several uncertain aspects discussed above). On the other hand if the material although ductile is devoid of strain hardening, like some high strength aluminum alloys, the length of the beam on which the plastic hinge is due to develop is very short for reasons governed by statics, the maximum unit strain is extremely high, and the beam must fail at the first plastic hinge well in advance of the value of failure load found by plastic theory. Plastic theory then needs both yielding and strain hardening for its justification, and it is only owing to the presence of strain hardening in structural steel that this theory, in spite of its basically incorrect assumptions, gives a fairly accurate value of the failure load (excluding the uncertainties referred to earlier).

6. Comparative Rationality

Design to a definite load factor or a definite coefficient of overload in excess of the working load is claimed to be pre-eminently rational [3] and is cited by the plasticians as a proof of superiority of their method over the elastic method. The writer however fails to see why the working loads, as high as they are usually specified, should ever be exceeded simultaneously by a factor as high as 1.85, the usual value of the load factor. The purpose of the factor of safety as the writer sees it, is not to provide for a great proportional overload which is hardly possible, but to meet a wide variety of unforeseen contingencies, such as weaknesses and deterioration of material, defects of fabrication and construction, errors in design and detailing, unusual and unexpected loads, catastrophic occurrences etc. Such emergencies are met by the elastic and plastic designs in different ways, but in the manner of meeting them one can discern no apparent advantage of one method over the other.

Although denying the claim of plastic design for superiority in principle, one must admit a degree of justice in the criticism of the elastic design for certain arbitrariness. Only the main stresses are expected to be taken into consideration in the elastic design, while a score of others, described as secondary stresses, are simply left out. To these belong different kinds of residual stresses and stresses caused by load concentrations, holes, fitting etc. Designers normally know these stresses by experience although in unusual cases special studies or intuitive judgment may be necessary for acceptance or rejection of some of them.

Another aspect of elastic design which sometimes raises objections is the use of elastic formulae for calculation of the load carrying stresses, although some of these stresses may extend locally beyond the elastic range. The treatment of the main and secondary stresses as described here is however an essential part of the elastic design, as it is practised. This practice is justified by long experience and is allowed for in the values of the working stresses laid down in specifications.

7. Conclusions

Generally speaking, elastic method, although somewhat discretionary is basically simple in principle. Plastic method, as originally visualized, aimed at even greater simplicity identified with formation of kinematic mechanisms, and also at rationality. The simplicity however proved in the end illusory by becoming enmeshed with the uncertain properties of material: yield, residual stresses, creep; and highly complex phases of structure behaviour: inelastic buckling, deformation of beam-columns and live load action. Determination by the plasticians to cope with the difficulties as they had arisen, led to risky assumptions and questionable procedures transforming plastic method into a collection of rules and empirical formulae, whose relation to failure has become obscure if not altogether non-existent.

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Summary

Plastic and elastic methods of design are comparable in so far as they are both sound in principle and both contain some defects.

The weaknesses of plastic design include:

a) Uncertainty with regard to material properties on which plastic design is critically dependent: residual stresses, creep and, especially, the magnitude of the yield stress, which varies in a wide range.
b) Lack of adequate theory for plastic stress analysis of structures involving beam-columns.
c) Similar deficiency with regard to the action of live loads.
d) Empiricism and inadequacy of provisions for lateral instability.

The defects of elastic method are:

a) Omission of secondary stresses.
b) Empiricism of design provisions for buckling.
c) Unjustified uniformity of the basic allowable stress for statically determinate and indeterminate structures.

Of the two, the weaknesses of the plastic design are judged by far more serious.

Résumé

Les méthodes de calcul basées sur la plasticité et l'élasticité sont comparables en ce sens qu'elles reposent toutes deux sur des principes sains et comportent toutes deux des incorrections. Les faiblesses du calcul plastique se rapportent:

a) A la marge d'incertitude quant aux propriétés des matériaux dont le calcul plastique dépend de façon critique: tensions résiduelles, fluage et, tout particulièrement, valeur de la limite élastique, dont la variation est très étendue.
b) A l'absence de principes sûrs pour l'analyse plastique des tensions dans le cas d'ouvrages comportant des éléments comprimés et fléchis.
c) A une même insuffisance en ce qui concerne l'action des surcharges.
d) A l'empirisme et à l'impropriété des règles relatives à l'instabilité latérale.

Les défauts du calcul élastique sont:

a) L'omission des contraintes secondaires.
b) L'empirisme du calcul au flambage.
c) L'adoption illégitime des mêmes contraintes fondamentales admissibles pour les constructions isostatiques et hyperstatiques.

Ce sont les faiblesses du calcul plastique qui, des deux, sont jugées de beaucoup les plus graves.

Zusammenfassung

Die auf der Plastizitäts- bzw. Elastizitätstheorie aufgebauten Berechnungsweisen haben dieses gemeinsam, daß sie beide auf gesunden Prinzipien beruhen und daneben Unzulänglichkeiten aufweisen. Die Schwächen der plastischen Berechnungsweise bestehen:

a) In den Ungewißheiten, denen die Materialeigenschaften, insbesondere die Eigenspannungen, das Kriechen und die starken Streuungen unterworfene Fließgrenze unterliegen.
b) In der Tatsache, daß man über keine sicheren Grundlagen zur Berücksichtigung von Spannungsproblemen zweiter Ordnung verfügt.
c) In einer ähnlichen Unsicherheit in bezug auf die Wirkung der Auflasten.
d) In der Empirie und Unsauberkeit im Erfassen von Problemen seitlicher Instabilität.

Die Mängel der elastischen Berechnungsmethode sind:

a) Die Vernachlässigung der Nebenspannungen.
b) Die Empirie in der Knickberechnung.
c) Die Annahme gleicher zulässiger Spannungen bei statisch bestimmten und unbestimmten Konstruktionen.

Die Schwächen der plastischen Berechnungsweise werden als wesentlich schwerwiegender beurteilt.