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Strength of Concrete in Finished Structures and Its Effect on Safety

La résistance du béton dans les ouvrages et son influence sur la sécurité

Der Einfluß der Betonfestigkeit auf die Sicherheit der Bauwerke

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1. Introduction

Concrete control on the site comprises, among other things, making and testing of standard test specimens. The tests are based on the assumption that the strength of these test specimens, which are usually small in number, represents the strength of the concrete in the whole structure. Several objections can be raised against the correctness of this assumption.

In recent times, the problem of more accurate estimation of the strength of the concrete in structures has met with increasing attention.

The European Concrete Committee (Comité Européen du Béton, CEB) has proposed that the design of concrete structures should be based on a characteristic strength. This strength is calculated by means of statistical methods from values observed in tests on standard specimens, but includes, in addition, a coefficient of safety which shall represent the difference in strength between the material in the structure and that in the standard test specimens.

Furthermore, it is of interest to know how the safety of structures is influenced by variations in the strength of the concrete as compared with variations in other factors, e.g. in the characteristics of the reinforcement and in the dimensions of the cross section.

2. Estimation of Strength of Concrete in Structures from Data Available in Literature

The quality or the strength of the concrete in finished structures has so far been studied to a limited extent only.

In general, the strength of the concrete in a finished structure converted into cube strength may be supposed to be different from that which has been determined on standard test specimens at the same age. To a very rough approximation, the mean value of the strength of the concrete in a structure may be assumed to be about 10 to 20 per cent lower than the strength of standard test specimens, but both smaller and greater differences can be met with.

Calculations based on a large number of tests to failure of columns subjected to concentric loads have shown that the strength of the concrete in columns is on an average equal to about 68 per cent of the cube strength, and that it may sometimes be as low as about 56 per cent of this strength.

3. Investigation of Strength of Concrete in Finished Structures

3.1. Strength of Concrete in Columns

In an investigation which has recently been completed at the *Swedish Cement and Concrete Research Institute*, Stockholm, the variation in the strength of the concrete has been studied in tests on 37 square columns.

After curing, the columns were cut into cubes, and test cylinders were cored from these cubes by drilling in the longitudinal direction of the column.

The values of strength observed in these tests were submitted to a statistical analysis. Some results of this analysis are stated in what follows. The strength of the concrete in the columns exhibited a greater dispersion than in the cube and cylinder specimens, which were cured in conformity with the relevant standard specifications or in the same manner as the columns themselves.

In a region extending about 60 cm from the top of the column, the strength of the concrete was lower than in the lower portion of the column.

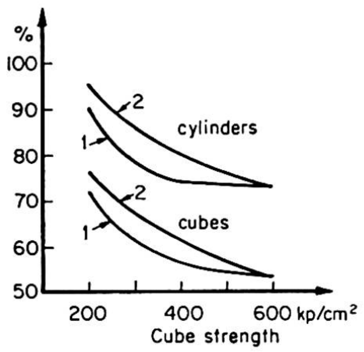


Fig. 1. Cylinder strength of the concrete in the upper portions of columns, in per cent of the strength of cylinders and cubes cured in accordance with standard specifications, Curves 1, or cured in the same way as the columns, Curves 2.

Fig. 1 shows the lower limit, the 10-per-cent fractile, of the cylinder strength of the concrete in the upper portions of the columns, expressed in per cent of the strength of test specimens cured in accordance with the relevant standard specifications or in the same way as the columns.

The strength of the concrete in the upper portions of the columns, expressed in per cent of that of the control test specimens, decreased as the cube strength became greater. This was independent of whether the concrete had a plastic consistence and was fairly cohesive or whether it had a fluid consistence and was liable to separation.

Fig. 2 represents the lower limit, the 10-per-cent fractile, of the cylinder

Table 1. Strength of Concrete in Finished Structures

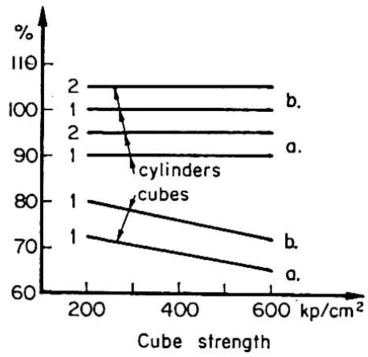
Type of structure	$\frac{\sigma^{structure}_{cube,\,28}}{\sigma_{cube}}$							$\frac{\sigma^{structure}_{cube,\,28}}{K}$						
	Number of structures	Number of test specimens taken	Mean value m	Standard deviation s	Maximum value	Minimum value	10-per-cent fractile, $m = 1.28\,s$	Number of structures	Number of test specimens taken	Mean value m	Standard deviation s	Maximum value	Minimum value	10-per-cent fractile, $m = 1.28\,s$
Slabs	13	31	1.12	0.30	1.64	0.63	0.74	33	82	0.94	0.23	1.33	0.60	0.65
Walls	10	22	1.02	0.27	1.35	0.65	0.67	25	56	1.04	0.24	1.63	0.70	0.73
Columns, supports	12	32	0.91	0.15	1.32	0.74	0.72	12	22	1.03	0.15	1.35	0.86	0.84
Footings	11	53	0.91	0.20	1.34	0.70	0.65	16	77	1.05	0.24	1.50	0.61	0.74
Beams	3	10	0.92	0.02	0.95	0.91		1	2	1.01	—	—	—	
Factory-made elements	2	31	0.80	—	0.90	0.69		3	31	0.73	0.11	0.83	0.62	
Dams	6	34	1.15	0.20	1.43	0.94		—	—	—	—	—	—	
Floors	—	—	—	—	—	—		2	10	0.73	—	0.80	0.65	
Total Mean value	57	213					0.70	92	280					0.74

strength of the concrete in the columns below their upper portions, expressed in per cent of the strength of test specimens cured in accordance with the relevant standard specifications or in the same way as the columns.

It is seen that the weakest part in respect of the compressive strength of the concrete was the upper portion of the column.

The lower strength of the concrete in the upper portions of the columns corresponds to that of beams cast in a horizontal position.

Fig. 2. Cylinder strength of the concrete in columns, in per cent of the strength of cylinders and cubes cured in accordance with standard specifications, Curves 1, or cured in the same way as the columns, Curves 2. Curves *a* refer to a concrete which had a fluid consistence and was liable to separation. Curves *b* relate to a concrete which had a plastic consistence and was fairly cohesive.



3.2. Data Collected on Building Sites

Table 1 reproduces the data which have been collected on building sites, and which concern the strength of the concrete in finished structures, 112 in all. The strength of the concrete in these structures was compared with the strength of standard test specimens and with the strength specified for the grade of concrete in question. As a rule, the strength of the concrete in the structure, $\sigma_{cube,\,28}^{structure}$, was lower than the strength of the standard test specimens, σ_{cube}, compared at the same age, 28 days. In relatively many cases, the strength of the concrete in the structures reached only about 65 to 75 per cent of the strength of the standard test specimens, and was sometimes about 25 per cent lower than the strength K specified for the grade of concrete.

4. Effects of Deviations in Various Factors on Safety Against Failure

In discussing the effects of the deviations in various factors on the safety against failure, it is convenient to utilise the relation between the factor of safety and the risk of failure, and to base the comparison on the effects produced by these deviations on the risk of failure. If the safety against failure is expressed implicitly in terms of permissible stresses or coefficients of safety, then it is more difficult to form an idea of the actual safety.

A comparison of the effects of the deviations on safety can be made with the help of Fig. 3, which is applicable when the load-carrying capacity, S, and the load effect[1]), Q, are distributed in accordance with the logarithmic

[1]) The load effect is an effect produced by loads on structures, e. g. a bending moment, a normal force, etc.

normal distribution. The general trend of this graph would also be the same if it were based on the normal distribution or on some distribution of extreme values. The only difference would be that the same standard deviations in the load-carrying capacity, s_S, and in the load effect, s_Q, would influence the risk of failure in a higher degree in the case of the normal distribution, and in a still higher degree in the case of the distribution of extreme values.

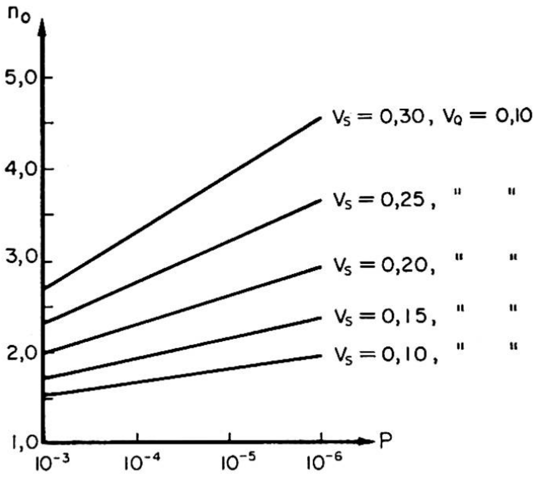


Fig. 3. Relation between the risk of failure, P, and the factor of safety, n_0, for $V_Q = s_Q/\bar{Q} = 0.10$ and $V_S = S_S/\bar{S} = 0.10$, 0.15, 0.20, 0.25, and 0.30 in the case where S and Q are distributed in conformity with the logarithmic normal distribution.

$n_0 = \bar{S}/\bar{Q}$, where $\bar{S}$ = mean value of S and $\bar{Q}$ = mean value of Q.

The standard deviations in the ultimate load have been deduced in [1] for eccentrically loaded columns in the case where the primary failure occurs in the concrete and in the case where the reinforcement is the decisive factor determining failure, as well as for over-reinforced and under-reinforced beams, and have been expressed as functions of the strength of the concrete, the characteristics of the reinforcement, and the dimensions of the cross section. If we study the functional relations between the factor of safety or the risk of failure, on the one hand, and the different quantities by which it is influenced, on the other hand, then we find that it is not possible to take into account each of these quantities separately. Therefore, the standard deviations or uncertainties in all these quantities must be taken into consideration at the same time.

Figs. 4a, 4b, and 4c show how the coefficient of variation, $V_N = \frac{s_N}{\bar{N}}$, in the ultimate load varies in the case of eccentrically loaded columns, where whole cross section is subjected to compression, when the standard deviation in the strength of the concrete, $s_{\sigma_b^l}$, the standard deviation in the depth of the cross section, s_{h_t}, and the standard deviation in the eccentricity of the load, s_η, are variable, while the standard deviations in the other quantities to be considered at the same time are constant.

As is seen from Fig. 4a, if the column is acted upon by a concentric load, then, in Case 5, the coefficient of variation in the ultimate load, V_N, increases only from about 0.15 to 0.19, when the standard deviation in the strength of

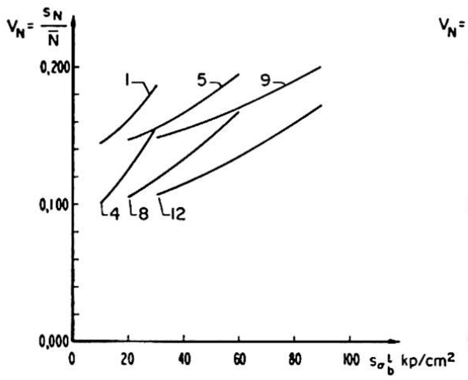


Fig. 4a. Relation between the coefficient of variation, $V_N = s_N/\bar{N}$, in the ultimate load and the standard deviation in the strength of the concrete $s_{\sigma_b^l}$. The standard deviations in the other relevant quantities are assumed to be constant.

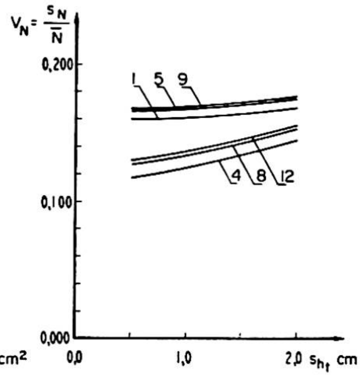


Fig. 4b. Relation between the coefficient of variation, $V_N = s_N/\bar{N}$, in the ultimate load and the standard deviation in the depth of the cross section s_{h_t}. The standard deviations in the other relevant quantities are assumed to be constant.

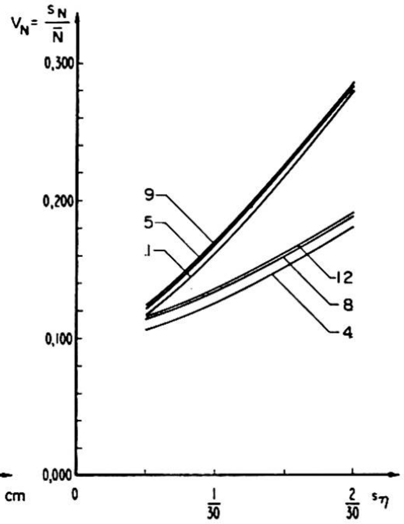


Fig. 4c. Relation between the coefficient of variation, $V_N = s_N/\bar{N}$, in the ultimate load and the standard deviation in the eccentricity of the load s_η. The standard deviations in the other relevant quantities are assumed to be constant.

1. $\sigma_b^l = 200$ kp per cm², $\eta = 0$ 4. $\sigma_b^l = 200$ kp per cm², $\eta = \frac{5}{30}$
5. $\sigma_b^l = 400$ kp per cm², $\eta = 0$ 8. $\sigma_b^l = 400$ kp per cm², $\eta = \frac{5}{30}$
9. $\sigma_b^l = 600$ kp per cm², $\eta = 0$ 12. $\sigma_b^l = 600$ kp per cm², $\eta = \frac{5}{30}$

the concrete increases from about 20 kp per cm² to 60 kp per cm². As may be found from Fig. 3, this change has but a very slight effect on the safety against failure. The reason is that the predominant amount is contributed to V_N by the standard deviation in the eccentricity, s_η, when the eccentricity is small. On the other hand, if, for instance, the eccentricity of the load is $\eta = \frac{5}{30}$, then the same change in the standard deviation in the strength of the concrete causes the coefficient of variation, V_N, to change from about 0.11 to about 0.17. As may be found from Fig. 3, if the risk of failure is to remain unchanged, e.g. $P = 10^{-6}$, then the factor of safety should be increased from about 2.0 to about 2.5. However, this change in the factor of safety is not great.

The effect produced by the variation in the standard deviation in the depth of the cross section, s_{h_t}, on the safety against failure can be estimated in an analogous manner with the help of Fig. 4b.

The effect of the variation in the standard deviation in the eccentricity, s_η, on the safety against failure can likewise be estimated in a similar way by the aid of Fig. 4c. As is seen from this graph, if, in the case of a concentric load, the standard deviation in the position of the point of load application changes from $s_\eta = \frac{1}{30}$ to $s_\eta = \frac{2}{30}$, then the coefficient of variation in the ultimate load changes from about $V_N = 0.15$ to about $V_N = 0.29$. As may be found from Fig. 3, if the risk of failure is to remain unchanged, e.g. $P = 10^{-6}$, then the factor of safety, n_0, should be increased from about 2.4 to about 4.4. This change in the factor of safety is considerable. On the other hand, if the eccentricity of the load is, say, $\eta = \frac{5}{30}$, then the same increase of the standard deviation in the eccentricity, s_η, causes the coefficient of variation, V_N, to increase from about 0.12 to about 0.18. Accordingly, the factor of safety should be increased from about 2.0 to about 2.7.

As has been mentioned in the above, the decrease in the mean strength of the concrete in the upper portion of the column was of the order of some 10 to 20 per cent when the cube strength was 400 kp per cm², and some 25 to 30 per cent when the cube strength was 600 kp per cm². We shall now estimate the effect produced by a change of 25 per cent in σ_b^l on the safety against failure in the case where σ_b^l should have been 400 kp per cm² in a column, 30 by 30 cm in cross section, reinforced with 4 Swedish standard Type Ks 40 ribbed bars, 16 mm in diameter. When the load is concentric, $\eta = 0$, the ultimate load N decreases from about 394 metric tons to about 304 metric tons, and when the load is eccentric, say, $\eta = \frac{5}{30}$, the ultimate load diminishes from about 241 metric tons to about 187 metric tons. Consequently, the coefficient of variation increases in the first case from $V_N = 0.17$ to about $V_N = 0.21$, and in the second case from $V_N = 0.13$ to about $V_N = 0.17$. Accordingly, if the risk of failure is to remain unchanged, then the factor of safety, should be increased in the first case from 2.5 to about 3.0, and in the second case from 2.3 to 2.5.

As can be found from similar calculations, if the eccentricity of the load increases, and the cross section becomes under-reinforced, then the effects produced by the standard deviations in the eccentricity and in the strength of the concrete on the safety against failure become unimportant. The effect of the standard deviation in the dimensions of the cross section still manifests itself. The standard deviation in the yield point stress of the reinforcement produces an effect as the eccentricity increases.

The safety against failure of over-reinforced beams is influenced in a high degree by the standard deviations in the dimensions of the cross section and in the strength of the concrete. On the other hand, the influence of a change in the shape of the stress block of the concrete is of no importance in this connection.

The safety against failure of under-reinforced beams is predominantly affected by the standard deviations in the dimensions of the cross section and in the yield point stress of the reinforcement, whereas the effect of the standard deviation in the strength of the concrete matters little or nothing.

Thus, the standard deviation in the dimensions of the cross section has a great effect on the safety against failure in all cases dealt with in the above, while the effects of the standard deviations in the concrete or in the yield point stress of the reinforcement, vary from one structure to another.

5. Effect of Strength of Concrete on Ultimate Moments of Over-Reinforced Beams

In the calculation of ultimate moments, the limit stress of the concrete at failure, σ_b^l, is brought into relation with the strength of standard test specimens, and according to a proposal of the European Concrete Committee, $\sigma_b^l = \sigma_{cyl}$, i.e. the cylinder strength.

By comparing the observed and calculated values of the ultimate moments of over-reinforced beams which have been obtained from tests made by various researchers, it is found that the standard deviation in the ratio of the observed value to the calculated value is small in each strength range. However, it is to be noted that the magnitude of these values exhibits a distinct trend, namely, the above-mentioned ratio decreases as the strength of the concrete increases.

The results of the investigation described in [1] can also be used to estimate the strength of the concrete in the compression zone of over-reinforced beams as compared with the strength of test specimens, see Fig. 5.

If the ultimate moments of over-reinforced beams are calculated so as to take account of the fact that the actual strength of the concrete in the com-

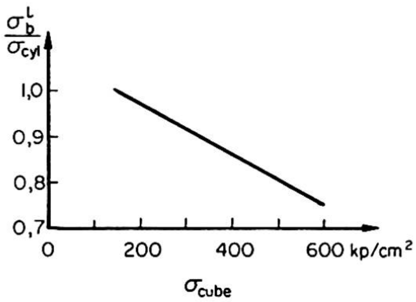


Fig. 5. Relation between the compressive strength of the concrete in beams, σ_b^l, and the strength of cylinder test specimens, σ_{cyl}, and cube test specimens, σ_{cube}.

Table 2. Ultimate Moments of Over Reinforced Beams. Comparison of Calculated and Observed Values

Number of beams	Range of strength, kp per cm²	Observed value / Calculated value	
		$\sigma_b^l = \sigma_{cyl}$	σ_b^l from Fig. 5
24	200 to 300	1.05	1.09
9	300 to 400	0.96	1.04
3	>400	0.93	1.07

pression zone of the beam differs from the strength of standard test specimens, then the observed and calculated values of the ultimate moment will be more closely in agreement, see Table 2.

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1. N. Petersons: "Strength of Concrete in Finished Structures." Reprint 26, Swedish Cement and Concrete Research Institute, Stockholm 1964.

Summary

Tests have been made in order to study the dispersion in the strength of the concrete in vertically cast columns and in a few horizontally cast beams. The strength of the concrete in these structural members was compared with the strength of cube and cylinder specimens which had been cast at the same time.

An analysis of the results obtained from tests made on building sites has shown the relation between the strength of the concrete in various structures and the strength of test specimens cast at the same time.

Finally, this paper discusses the effects produced on the safety against failure of structures by the standard deviations in the strength of the concrete as compared with the effects of the standard deviations in the characteristics of the reinforcement, in the dimensions of the cross section, and in the eccentricity of the load.

Résumé

L'auteur décrit des essais effectués pour étudier la dispersion de la résistance du béton dans des colonnes coulées verticalement et dans quelques poutres coulées horizontalement. La résistance du béton dans ces éléments de construction a été comparée à celle des cubes et des cylindres d'essai qui ont été coulés en même temps.

Une analyse des résultats d'essais faits sur des chantiers a permis d'établir une relation entre la résistance du béton dans divers ouvrages et la résistance des éprouvettes coulées en même temps.

Enfin, l'auteur discute l'influence de l'écart quadratique moyen de la résistance du béton sur la sécurité à la rupture des constructions et il compare cette influence à celle des écarts quadratiques moyens des caractéristiques des armatures, des dimensions de la section transversale et de l'excentricité de la charge.

Zusammenfassung

Anhand von Versuchen wird die Streuung der Betonfestigkeiten in vertikal gegossenen Säulen und horizontal gegossenen Balken untersucht. Die hierin bestimmten Festigkeiten werden mit denen gleichzeitig in Würfel- und Prismenform gegossener Probekörper verglichen.

Ferner wird auf Grund von Versuchen eine Beziehung zwischen den Festigkeiten von auf der Baustelle hergestellten Betonelementen und von gleichzeitig gegossenen Probekörpern hergeleitet.

Schließlich wird versucht, die Bruchsicherheit eines Tragwerks aus der Streuung der Betonfestigkeiten und in Abhängigkeit von Bewehrungsgehalt, Querschnittsabmessungen und Lastexzentrizität herzuleiten.