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Autor(en): **Li, Shu-T'ien**

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Control of Prestress in Steel Structures

Le calcul des charpentes métalliques précontraintes

Berechnung von vorgespannten Stahlkonstruktionen

SHU-T'IEN LI

Ph. D., F. ASCE, M. IABSE, Professor of Civil Engineering, South Dakota School of Mines and Technology, Rapid City, S. Dak., U.S.A.

Introduction

Since World War II, the increasing availability of high-tensile steel wires, high-tensile steel rods, and high-strength steel plates for cover plates and welded flanges of girders, has given impetus to prestressed steel structures. They may now be proportioned for higher performance and better economy with equal factor of safety as the classical steel structures.

Take the United States, for instance; besides the introduction of ASTM designation A 36 structural steel with 60,000-psi tensile strength and 36,000-psi yield point for the fabrication of classical steel structures, the following commonly available constructional high-strength steels are in current use: high-strength steels of ASTM designations A 242, A 440, and A 441, all with 70,000-psi tensile strength and 50,000-psi yield point; and heat-treated constructional alloy steel with 115,000-psi tensile strength and 100,000-psi yield point. Plates and rods made of these high-strength steels can be very expediently used in steel construction for prestressing beams, stringers, and girders.

In the category of high-tensile steel wires, a comparatively common grade of 238,000—268,000-psi tensile strength and 140,000-psi allowable versus an allowable of 22,000-psi for A 36 steel, costs only about 3 times as high as the latter. Still higher-strength wires of up to 380,000-psi tensile strength, $28(10)^6$-psi modulus of elasticity, and 230,000-psi allowable is also obtainable. Cables made of these high-strength-steel wire strands may be used in various configurations as a medium in prestressing component members of steel structures.

Back in 1958, the author emphasized the importance of the promising future of prestressing steel structures as a means of evolving competitive steel designs in connection with the case for all-welded bridges[1]). This paper will not deal with other aspects of prestressing structures but rather limit its scope

[1]) LI, SHU-T'IEN: "The Case for All-Welded Bridges." Railway Track and Structures, Vol. 54, No. 6, June 1958, pp. 28—31; Abstracted in Bulletin, Chinese Association for the Advancement of Science (CAAS), Vol. VI, No. 6, December 1958, Summary of Essays B 2, pp. 6—7.

to a refreshing approach to the basic problem of stress control analysis in the application of prestress to framed structures with particular reference to securing appropriate factor of safety and attaining better economy.

Recognizing conditions of structural internal equilibrium and available fabrication and erection techniques as well as inherent economy involved, basic criteria for prestress control in framed structures may be stated as follows:

1. Whereas both tension and compression members may theoretically be respectively precompressed and pretensioned, it is more expedient in practice to precompress tension members either directly or indirectly and pretension, or reduce compression in, compression members indirectly.

2. As the desideratum of applying prestress to steel structures is to secure better economy than, but with the same factor of safety as, classical steel structures, the better economy has to be shown justified by the saving in structural steel more than offsetting the additional cost of cables and labor required.

3. By the same factor of safety as classical steel structures is meant the factor of safety as usually prescribed by the applicable design specifications and codes, and, in addition, at least the same factor of safety under all conditions of service life of the structure from permanent load alone to the application of maximum design live load. This latter imposition arises from the fact that in prestressed steel structures both magnitudes and range of stresses become important considerations as compared with, usually, the maximum magnitudes alone in classical steel structures.

4. During fabrication and erection and before the transfer of full dead load, any compression member may be pretensioned to below some predetermined margin within the proportional limit of the steel in the basic structure, and any tension member may be precompressed to the lower of either a) below the same margin if it is short enough without danger of buckling, or b) below a similar margin within the critical buckling stress if it is susceptible to buckling below the proportional limit due to inherent slenderness ratio and end restraint conditions.

5. Whenever a steel structure whose failure has no probability of involving the safety of human life and whose external loads will be applied gradually from an initial, highly-prestressed condition to a permanently loaded condition having normal factor of safety, the aforesaid margin during prestressing may even be reduced to, say, 10 per cent below the proportional limit of the steel, or 10 per cent below the critical buckling stress, whichever is the smaller.

6. In any complicated case, the decision of optimum, consistent criteria for prestress should be made after due analysis of appropriate safety, attainable economy, and foreseeable probability.

7. Tension chord and web members may be compressed by tensioning cables as the medium for applying prestress, either locally, or collectively, or in

assembly. As the tension members are compressed, either the compression members are reduced in stress or changed to tension members. This follows directly from structural internal equilibrium, and leads in practice to the crucial problem of logical control of prestress in groups of key tension members of the basic structure.

8. Safety, economy, prestress, and deformation can always be controlled by analyzing the unprestressed basic structure, the prestressed structure before and after transfer of dead loads, the prestressed structure under maximum design live loads of all kinds, to secure the same factor of safety as in classical steel structures at crucial moments, to keep deformations within acceptable limits, and to use such prestresses in the key group of tension members that will give the least total cost for the entire structure.

9. As factor of safety is usually stipulated or implied in applicable design specifications and codes, and allowable deformations are, in general, governed by the particular service function of the structure, the crucial problem, in the last analysis, reduces to optimum prestress for maximum economy (or least cost) rather than maximum prestress possible.

A Simplified Approach to the Problem

Let a simplified approach to the problem of prestress control be stated in its fundamental way thus: A key tension member in a very simple framed structure, or one of a series of main tension members forming the lower chord of a simply supported truss, the upper chord of the cantilever arms of a cantilever truss, or the lower chord of a braced bowstring girder, etc., be substituted by a prestressed design. The member under consideration is to resist an externally applied load varying from zero to F.

To provide the same safety factor in the prestressed version as in the conventional design, and to limit the deformation in the former to within allowable bounds, if the conventional unprestressed counterpart requires a cross-section A, the prestressed version would require a much reduced cross-section A_r plus, of course, a prestressing cable of cross-section A_c. The problem boils down, therefore, to the evaluation of the most appropriate A_c and A_r to satisfy stipulated safety factor, acceptable elastic deformation, and allowable unit stresses.

Four allowable steel unit stresses will enter into the problem:

F_T = Allowable tensile stress in conventional structural steel members;

F_a = Allowable axial compressive stress in structural steel members of such small slenderness ratio that its strength is in no way limited by critical buckling Euler stress F_e divided by the factor of safety (S. F.); in case $F_e < F_a \cdot$ (S. F.), use allowable $F_e' = F_e/$(S. F.);

F_t = Allowable tensile stress in prestressed structural steel members to be determined from F_a, yield point stress F_y, and stipulated safety factor S.F.; and

F_c = Allowable tensile stress in the prestressing cable.

A consideration of these allowable unit stresses in the necessary equilibrium and deformation equations will yield the most appropriate A_t and A_c.

Allowable Tensile Stress in Prestressed Members for Maintaining a Stipulated Safety Factor

Standard design specifications and codes usually stipulate a safety factor (S.F.) as the ratio of yield-point stress F_y of the structural steel in use to the allowable tensile stress F_T in conventional structural steel members.

In prestressed design, the ratio defining factor of safety must be construed as the ratio of range of yield point stress in tension plus allowable precompressive stress to the range of allowable tensile stress plus allowable precompressive stress. Thus, to maintain the same safety factor in the prestressed version, the chosen allowable tensile stress F_t must be such that

$$F_t + F_a\,(\text{or } F_e') = \frac{F_y + F_a\,(\text{or } F_e')}{\text{S.F.}}. \qquad (1)$$

In the case of ASTM A 36 steel, $F_y = 36{,}000$ psi, $F_a = F_T = 22{,}000$ psi, S.F. $= \frac{36}{22} = 1.64$, Eq. (1) gives an allowable $F_t = 13{,}440$ psi.

Simultaneous Solution of Cable and Member Areas

Now, let the initial prestressing force P be gradually applied through the cable to put the cross-section A_r of the tension member of the basic structure under precompression, and the cable be tensioned only to a working value f_c less than F_c.

Having completed prestressing, when the external load F is applied to the assembly, it will be shared by both the structural steel member of A_r and the cable of A_c under their natural elastic adjustment and collaboration such that

$$F_c A_c + F_t A_r = F. \qquad (2)$$

The same equation may be obtained in a different way by recognizing that the cable has been tensioned to the initial prestressing force P plus the increment ΔP due to the application of F, and that the precompressed structural steel tension member has expended its precompression and become tensioned to the difference between F and $(P + \Delta P)$; thus

$$\begin{bmatrix} F_c A_c \\ F_t A_r \end{bmatrix} = \begin{bmatrix} P \\ -P \end{bmatrix} + \begin{bmatrix} \Delta P \\ F - \Delta P \end{bmatrix}. \tag{3}$$

It is seen that the addition of both Eqs. (3) will give Eq. (2) identically.

To formulate a second independent equation relating A_c with A_r, resort is made to equating the initial tensioning force exerted by the cable with the precompression in the structural steel tension member, which yields

$$f_c A_c = P = F_a A_r. \tag{4}$$

Just what f_c should be, it has to be ascertained from the overall identical strain relation in the cable and in the prestressed structural steel member resulting from the application of the external load F, during which the range of stress in the cable is $(F_c - f_c)$ and in the prestressed structural steel member $(F_a + F_t)$; thus

$$\frac{F_c - f_c}{E_c} = \frac{F_a + F_t}{E}, \tag{5}$$

where E_c and E are respectively the modulus of elasticity of the cable and of the structural steel. Calling the modular ratio

$$n = \frac{E}{E_c}, \tag{6}$$

Eq. (5) becomes

$$f_c = F_c - \frac{F_a + F_t}{n}, \tag{7}$$

Substituting f_c as expressed by Eq. (7) into Eq. (4), it will form with Eq. (2) a set of two simultaneous linear equations in A_c and A_r; thus

$$\begin{bmatrix} F_c & F_t \\ F_c - \dfrac{F_a + F_t}{n} & -F_a \end{bmatrix} \begin{bmatrix} A_c \\ A_r \end{bmatrix} = \begin{bmatrix} F \\ 0 \end{bmatrix} \tag{8}$$

whose solution by Cramer's rule or matrix inversion yields:

$$\begin{bmatrix} A_c \\ A_r \end{bmatrix} = K \begin{bmatrix} F_a \\ F_c - \dfrac{F_a + F_t}{n} \end{bmatrix}, \tag{9}$$

where

$$K = \frac{n F}{(F_a + F_t)(n F_c - F_t)}. \tag{10}$$

In cases where the critical buckling Euler stress F_e is lower than F_a multiplied by the safety factor, a reduced F_e' should be used instead of F_a in Eqs. (9) and (10), such that

$$F_e' = \frac{F_e}{\text{S.F.}}. \tag{11}$$

When $E = 30\,(10)^6$ psi, $E_c = 28\,(10)^6$ psi, $n = \frac{30}{28} = 1.071$, $F_a = 22{,}000$ psi, $F_t = 13{,}440$ psi, $F_c = 140{,}000$ psi, Eq. (10) gives

$$K = \frac{F}{4517\,(10)^6} \text{ in in.}^4/\text{lb.} \tag{12}$$

and Eq. (9) yields

$$\begin{bmatrix} A_c \\ A_r \end{bmatrix} = \frac{F}{(10)^6} \begin{bmatrix} 4.87 \\ 23.67 \end{bmatrix} \text{ in in.}^2 \tag{13}$$

Determination of Initial and Incremental Prestresses

Initial and incremental prestresses may now be determined respectively by transposing the first of Eqs. (3) and by equating the identical incremental strains in the cable and in the structural steel member; thus writing this second relation first,

$$\begin{bmatrix} \dfrac{\Delta P}{A_c E_c} \\ P \end{bmatrix} = \begin{bmatrix} \dfrac{F - \Delta P}{A_r E} \\ F_c A_c - \Delta P \end{bmatrix} \tag{14}$$

whence the solution of the first equation first will immediately give that of the second, resulting in

$$\begin{bmatrix} \Delta P \\ P \end{bmatrix} = \begin{bmatrix} \dfrac{F}{1 + n\left(\dfrac{A_r}{A_c}\right)} \\ F_c A_c - \Delta P \end{bmatrix}. \tag{15}$$

If it is desired to compute P and ΔP before A_c and A_r are known, Eq. (15) may be expressed in terms of allowable stresses by substituting the values of A_c and A_r given by Eqs. (9), resulting in

$$\begin{bmatrix} \Delta P \\ P \end{bmatrix} = \begin{bmatrix} \dfrac{F F_a}{n F_c - F_i} \\ K F_a F_c - \Delta P \end{bmatrix}. \tag{16}$$

For the case of $n = 1.071$, $K = \frac{F}{4517\,(10)^6}$, $F_a = 22{,}000$ psi, $F_c = 140{,}000$ psi, $F_i = 13{,}440$ psi, Eq. (16) becomes

$$\begin{bmatrix} \Delta P \\ P \end{bmatrix} = F \begin{bmatrix} 0.1612 \\ 0.6819 \end{bmatrix} \tag{17}$$

in the same units as F.

Section Areas of Other Members

If other web members and chord compression members be prestressed directly, their A_r may be determined analogously. If they are only prestressed indirectly, their A_r may always be found from equilibrium conditions and allowable unit stresses, taking due consideration of indirect prestresses and each member's deformation compatibility with joint displacements.

Control of Elongation and Deflection

The elongation ratio R_Δ of the precompressed tension member to its uncompressed counterpart must remain within reasonably allowable limits so that the deflection of the prestressed structure will not become excessive or otherwise objectionable for the particular service function intended.

Using Eq. (5), R_Δ may be expressed as

$$R_\Delta = \frac{F_a + F_t}{F_T} = \frac{(F_c - f_c)\,E}{F_T\,E_c}. \tag{18}$$

In the case of $F_a = F_T = 22{,}000$ psi, Eq. (18) reduces to

$$R_\Delta = 1 + \frac{F_t}{22{,}000} \tag{19}$$

giving $R_\Delta = 1.61$ for $F_t = 13{,}440$ psi which provides the same safety factor as in conventional design.

In a given case, the deflection of the structure should be computed unless it is known that similar elongations in similar prestressed structures do not cause excessive or objectionable deflections. Should the deflection show excessively large for a particular service function, the prestressed design would have to be revised using a lower F_t.

Comparative Economics

The justification and extent of applying prestress to a given framed structure depends on an ultimate analysis of comparative economics of the prestressed version versus a conventional design for the same function with the same safety factor. This comparison may be readily made by translating section areas or unit stresses into weight and cost ratios.

Let R_w and R_c represent respectively the weight ratio and the cost ratio of the entire prestressed design to its entire conventional counterpart, and

$$r_c = \frac{c_c}{c} \tag{20}$$

where c_c and c denote respectively the estimated cost in place of unit weight of high-strength wires and structural steel. If L be the length of any member, and there be m members directly prestressed among all the $(m+n)$ members of the entire structure, then in case where the lengths of cables are the same as the directly prestressed members,

$$\begin{bmatrix} R_w \\ R_c \end{bmatrix} = \frac{1}{\sum_1^{m+n} A\,L} \begin{bmatrix} \sum_1^m \;(A_c + A_r)\,L + \sum_1^n A_r\,L \\ \sum_1^m (r_c\,A_c + A_r)\,L + \sum_1^n A_r\,L \end{bmatrix} \tag{21}$$

or in the case where the lengths of cables are different from those of the directly prestressed members,

$$\begin{bmatrix} R_w \\ R_c \end{bmatrix} = \frac{1}{\sum_1^{m+n} A\,L} \begin{bmatrix} \sum A_c\,L + \sum_1^m A_r\,L + \sum_1^n A_r\,L \\ r_c \sum A_c\,L + \sum_1^m A_r\,L + \sum_1^n A_r\,L \end{bmatrix}. \tag{22}$$

For a singled out comparison of unit length of the precompressed tension member with the uncompressed conventional tension member, let R_{w_1} and R_{c_1} be respectively the weight ratio and the cost ratio, then in terms of allowable unit stresses, modular ratio, and unit cost ratio r_c, Eq. (9) may be transformed to the following form:

$$\begin{bmatrix} R_{w_1} \\ R_{c_1} \end{bmatrix} = \frac{F_T}{(F_a + F_t)\,(n\,F_c - F_t)} \begin{bmatrix} n\,(F_c + \quad F_a) - (F_t + F_a) \\ n\,(F_c + r_c\,F_a) - (F_t + F_a) \end{bmatrix}. \tag{23}$$

In the case of $n = 1.071$, $F_a = F_T = 22{,}000$ psi, $F_c = 140{,}000$ psi, $r_c = 3$, Eqs. (23) become

$$\begin{bmatrix} R_{w_1} \\ R_{c_1} \end{bmatrix} = \frac{22{,}000}{(22{,}000 + F_t)\,(149{,}900 - F_t)} \begin{bmatrix} 151{,}500 - F_t \\ 198{,}600 - F_t \end{bmatrix}. \tag{24}$$

If we take $n = E : E_c = 1$, Eqs. (24) reduce to the simpler form:

$$\begin{bmatrix} R_{w_1} \\ R_{c_1} \end{bmatrix} = \frac{22{,}000}{22{,}000 + F_t} \begin{bmatrix} 1 \\ 1 + \dfrac{44{,}000}{140{,}000 - F_t} \end{bmatrix}. \tag{25}$$

For the same safety factor as in conventional steel design, that is, $F_t = 13{,}440$ psi, Eqs. (25) vs. Eqs. (24) give

$$\begin{bmatrix} R_{w_1} \\ R_{c_1} \end{bmatrix} = \begin{bmatrix} 0.62 \\ 0.84 \end{bmatrix} \text{ vs. } \begin{bmatrix} 0.63- \\ 0.84 \end{bmatrix}$$

showing that the simplified form is sufficiently accurate for practical purposes. The decimal results may be directly read as a saving of 38 per cent in weight and a saving of 16 per cent in cost for the prestressed tension member alone versus its counterpart in the conventional design. The total saving for the entire structure, however, would be less on account of less saving in the indirectly prestressed members taken into consideration by Eqs. (21) and (22).

By putting the left-hand side of the second of Eqs. (24) equal to "one", and solving for F_t, we find that the condition of equal cost in prestressed tension members alone as in conventional design is reached when F_t in the prestressed structural steel tension member is reduced to 7,530 psi.

Unit-length cost ratios R_{c_1} and savings in per cent by the second of Eqs. (24), safety factors by Eq. (1), and R_{c_1} by an alternate formula, are computed for different values of F_t as shown in Table 1 on the basis of using ASTM A 36 structural steel and high-strength wire cables having 140,000 psi allowable.

Table 1. Unit-Length Cost Ratio, Percentage Saving, and Safety Factor

F_t psi	Unit-length Cost Ratio R_{c_1}	Saving in %	Safety Factor S. F.	R_{c_1} by Alternate Formula
15,000	0.81–	19	1.57	0.80–
13,440	0.84	16	1.64 *)	0.84
12,000	0.87	13	1.71	0.87
11,000	0.90	10	1.76	0.90
10,000	0.93–	7	1.81	0.92+
9,000	0.95	5	1.87	0.95
7,530	1.00	0	1.96	1.00

*) At $F_t = 13{,}440$ psi, the same safety factor is maintained in prestressed as in unprestressed design.

From an examination of Table 1, the following observations are justified:

1. The allowable tensile stress F_t in the precompressed tension member for maintaining the same safety factor $36/22 = 1.64$ for ASTM A 36 structural steel is $13{,}440/22{,}000 = 0.61$ or 61 per cent lower than the 22,000 psi allowable in AISC Design Specifications.

2. Any increase in F_t, though accompanied by an increase in cost saving, is not recommended on the basis of producing a prestressed steel structure of not less safety factor versus its unprestressed conventional counterpart.

3. Any decrease in F_t, while accompanied by a decrease in cost saving, is enhanced by faster increases in the safety factor.

4. At $R_{c_1} = 1.00$, and $F_t = 7{,}530$ psi, the saving in cost by applying prestress reduces to zero, but the safety factor increases by $1.96/1.64 = 1.20$, or by 20 per cent, giving justification for prestressing in providing more potential resistance in the structure, when saving in cost is not the sole primary consideration.

5. In practice, the overall cost saving in the entire structure would be somewhat less than shown in Table 1 due to reduced cost saving in the indirectly prestressed members as taken into consideration in Eqs. (21) and (22).

6. As a practical economic minimum, where deflection considerations require a lower F_t, the latter should not go below 9,000 psi at which the 5 per cent cost saving in directly prestressed members would be almost averaged down to "nihil" by the indirectly prestressed members and the additional factor of safety gained would not warrant the trouble of prestressing unless an increase of factor of safety alone be the primary consideration.

An Alternative for Computing Unit-length Cost Ratios

Two kinds of dimensionless numbers appear in Table 1, the unit-length cost ratios R_{c_1} and the safety factors S.F. The closeness of S.F. to twice R_{c_1}

naturally suggests that an alternative way for computing the latter presents itself. The closest agreement could be obtained by using a more realistic value of F_y for A 36 steel in Eq. (1) for computing S.F., recognizing that the 36,000-psi yield point is not a statistic value but rather the minimum specified by specifications, or the minimum guaranteed by steel producers. As the lower yield point for A 36 steel is generally even higher than 38,000 psi, it is conservative to use $F_y = 37{,}000$ psi for estimating cost ratios and savings. The last column in Table 1 was computed as half of S.F. by Eq. (1), using this alternate value of F_y, checking almost exactly with the second column computed by the second of Eqs. (24).

To adapt the safety factor relation of Eq. (1) as an alternative way for computing unit-length cost ratio, we need simply rewrite Eq. (1) in the form of

$$R_{C_1} = \frac{1}{2}\,\frac{F_y + 1{,}000 + F_a\,(\text{or}\,f_e')}{F_t + F_a\,(\text{or}\,F_e')}. \tag{26}$$

This alternate formula for R_{c_1} is convincingly accurate as already shown in the almost identical agreement for the whole range of values in columns 2 and 5 of Table 1. It bears out one of the author's corollaries [2]) of Buckingham's Pi Theorem. That corollary states: "Any dimensionless quantity may be expressed as a function of any other dimensionless quantity."

Summary

This paper enunciates criteria for prestress control in framed structures; determines allowable tensile stress in prestressed members for maintaining the same safety factor as in classical steel designs; formulates simultaneous solution of cable and member sections; evaluates initial and incremental prestresses; outlines the calculation of section areas of indirectly prestressed members; gives criterion for control of elongation and deflection; discusses comparative economics with concluding observations; and finally provides an alternative for computing unit-length cost ratio.

Résumé

L'auteur rappelle les principes du calcul des charpentes métalliques précontraintes; il détermine les contraintes admissibles dans les charpentes précontraintes conduisant à un coefficient de sécurité égal à celui des charpentes non précontraintes. Il propose des règles pour le dimensionnement des câbles

[2]) LI, SHU-T'IEN: "Synthesizing Hydraulic Formulae by Dimensional Matrix Analysis." Trans., Chinese Association for the Advancement of Science (CAAS), Vol. 3, No. 1, November 1962, pp. 10—15.

et des éléments et détermine la précontrainte initiale et l'augmentation des efforts. Il montre comment calculer la section des éléments précontraints indirectement et définit un critère pour le contrôle des déformations. Le point de vue économique est finalement abordé et il est proposé une méthode pour le calcul du coût des éléments précontraints rapporté à l'unité de longueur.

Zusammenfassung

Die Arbeit behandelt die Grundlagen der Berechnung von vorgespannten Stahlkonstruktionen. Es wird die zulässige Spannung in vorgespannten Konstruktionselementen unter Beibehaltung des für nicht vorgespannte Konstruktionen gültigen Sicherheitsfaktors ermittelt und Regeln für die Bemessung der Stahlseile und Konstruktionselemente bei anfänglicher und zusätzlicher Vorspannung gegeben. Die Berechnung der erforderlichen Querschnittsflächen von indirekt vorgespannten Elementen wird dargestellt und ein Kriterium für die Ermittlung der Formänderungen gegeben. Die Arbeit schließt mit Wirtschaftlichkeitsbetrachtungen und zeigt einen Weg zur Berechnung der Kosten des vorgespannten Konstruktionselementes bezogen auf die Längeneinheit.

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