Beams and columns braced by thin-walled steel diaphragms

DISCUSSION PRÉPARÉE / VORBEREITETE DISKUSSION / PREPARED DISCUSSION

Beams and Columns Braced by Thin-Walled Steel Diaphragms

Poutres et piliers renforcés par des tôles transversales minces

Durch dünnwandige Stahlquerscheiben versteifte Träger und Stützen

S.J. ERRERA and T.V.S.R. APPARAO
Department of Structural Engineering
Cornell University
Ithaca, New York

INTRODUCTION

Thin-walled corrugated sheets or ribbed steel panels are often used as side wall sheathing, roof decking or floor decking of steel framed buildings. An illustration of such a building is shown in Fig. 1, and some examples of the cross sectional shapes of available steel panels are shown in Fig. 2. These panels carry
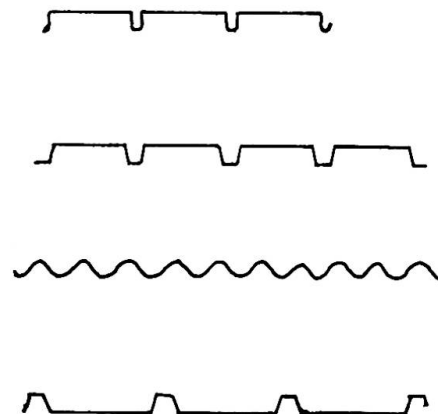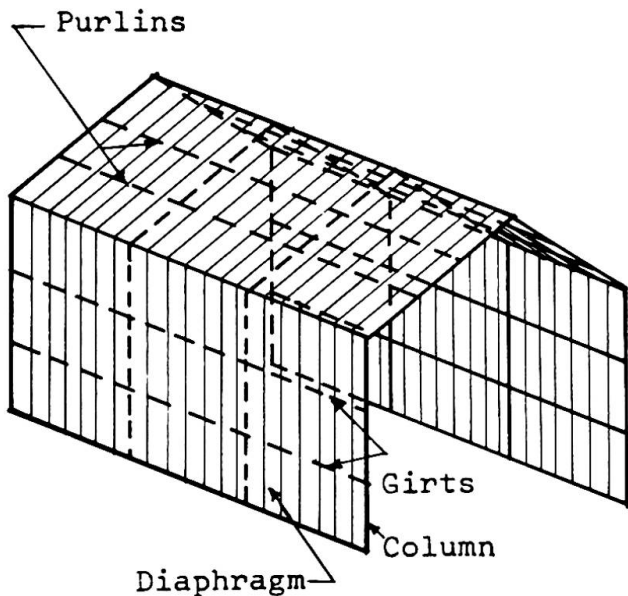

Fig. 1 A Steel Framed Building   Fig. 2 Panel Configurations

loads normal to their planes (such as wind, snow or floor loads) by virtue of their bending strength in that plane, and transmit these loads to the building frame. In addition, as mentioned in

Professor Winter's report on Theme IIa, because of their in-plane shear rigidity, these interconnected panels act as shear-resistant diaphragms, and can provide restraint to the members of the steel frame against buckling in the plane of the diaphragm.

Fig. 3 shows a pair of beams or columns with attached dia-

Stable
Configuration
(a)
ELEVATION (for columns)
PLAN VIEW (for beams)

Failure
Configuration
(b)

(c) Diaphragms
COLUMNS

(d) Diaphragm
COLUMNS

(e) Diaphragm
BEAMS

——— original position

---- deflected position
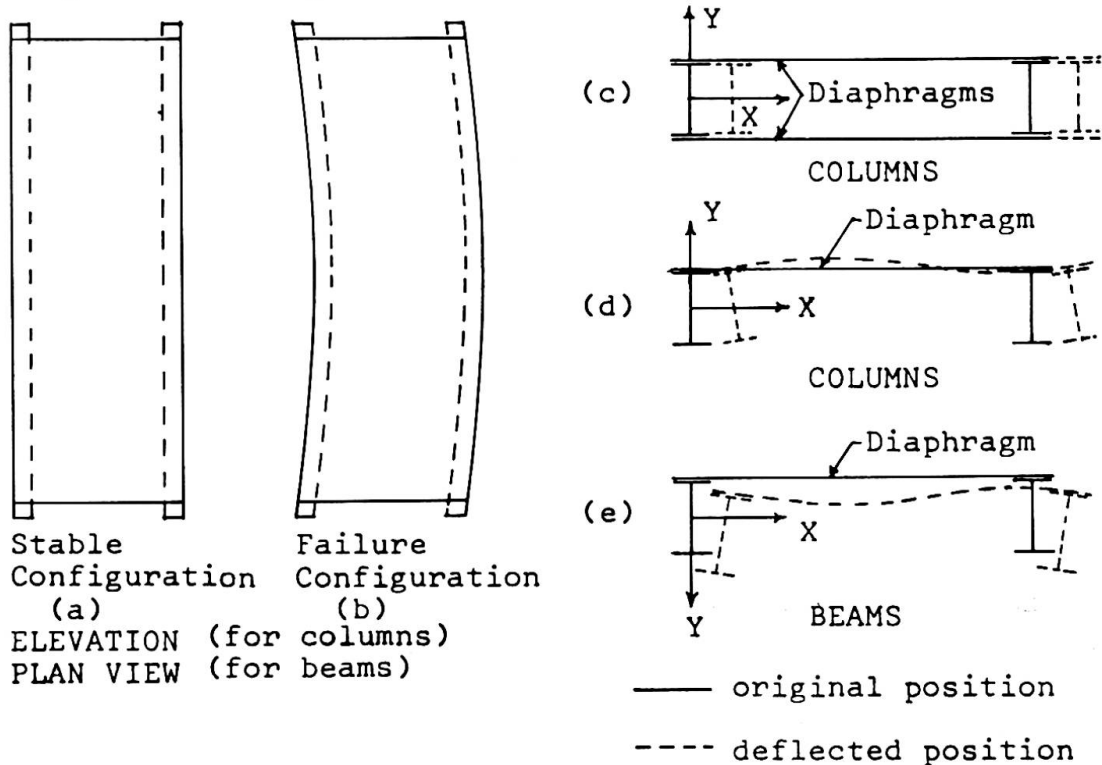
Fig. 3   Diaphragm-Braced Columns or Beams

phragm (a) with no load, and (b) in an assumed buckled shape. A diaphragm that is attached to two or more parallel members and constrained to deform with the members as they deflect laterally (Fig. 3b) is in a state of shear. Properly connected diaphragms are quite effective in resisting shear, and therefore can be very efficient in bracing beams, joists or purlins, against lateral torsional-flexural buckling. Similarly, buckling loads or failure loads of columns can be increased by either (1) bracing the columns along their length with diaphragms or (2) bracing the columns with girts which in turn are braced by a shear diaphragm, as shown in Figs. 1 and 4, and hereafter called diaphragm-girt bracing.

Diaphragm   Girt

——— original position
- - - - deflected position

Fig. 4   Columns with Diaphragm-Girt Bracing

This report presents a summary of the results of an investigation at Cornell University on the effectiveness of thin-walled steel diaphragms as bracing for columns and beams. Comparisons of theoretically computed buckling loads and experimental failure loads are presented. Theoretically predicted deflections considering initial imperfections of columns, and based on small deflection theory, are compared with experimental deflections for a typical case.

CHARACTERISTICS OF THE DIAPHRAGM

In the present discussion, the two important characteristics of the diaphragm are its shear rigidity and its shear strength. No reliable theory has yet been developed to predict the rigidity and strength of a diaphragm; however, a standard procedure for testing a diaphragm in shear and for computing the rigidity and strength from its load-deflection relationship has been established.[1] (Superscripts indicate Reference Numbers).

For light gage steel diaphragms the effective shear modulus may be denoted as G_{eff},[2] expressed in force per unit of cross-sectional area. The shear rigidity Q is then expressed as

$$Q = A_d \, G_{eff} \qquad\qquad (1)$$

where A_d is the cross-sectional area of the diaphragm contributing to the support of one member.

THEORY

<u>General</u> - Theory of diaphragm-braced members has been developed to cover the following:

1. the critical moment for diaphragm-braced I-,[2] channel[3] and Z-section[3] beams subjected to uniform moment

2. the buckling load for axially-loaded I-section columns[4],[5] directly braced by diaphragms on one or both flanges

3. the buckling load for axially loaded I-section columns with diaphragm-girt bracing[3]

4. load-deflection relationships[3] for diaphragm-braced members with initial imperfections

5. failure loads[3] of diaphragm-braced members with imperfections, based on stipulated failure criteria.

Two analysis methods will be illustrated by particular cases; one based on equilibrium, the other based on an energy method.

<u>Diaphragm-Braced Beams</u> (Equilibrium Method) - Diaphragm-braced I-
and channel section beams under uniform moment in the YZ plane
(Fig. 3e) bend in the Y direction before they buckle laterally.
Due to initial imperfections, real, imperfect beams deflect later-
ally and twist even at moments below the buckling moments. To
determine the precritical deflections considering these initial
imperfections, equilibrium equations of a diaphragm-braced I or
channel beam may be written as:

$$EI_x v^{iv} = 0 \tag{2}$$

$$EI_y u^{iv} - Q(u''+e\beta'') + M_x(\beta''+\beta_o'') = 0 \tag{3}$$

$$GK\beta'' - E\Gamma\beta^{iv} + Qe(u''+e\beta'') - F\beta - M_x(u''+u_o'') = 0 \tag{4}$$

where EI_x is the bending rigidity about the x-axis

EI_y is the bending rigidity about the y-axis

$E\Gamma$ is the warping rigidity

GK is the torsional rigidity

F   is a parameter of the cross-bending rigidity of the
diaphragm[2]

e   is the distance from the center of gravity of the
beam section to the plane of the diaphragm

M_x is the moment about the X-axis

u_o and β_o are the initial deflections in the X-direction
and about the Z-axis respectively

v   is the deflection in the Y direction

and   u and β are the additional deflections corresponding to
u_o and β_o respectively.

Knowing the initial deflections, the above equations 2, 3 and
4 can be solved to obtain load-deflection relationships of the
diaphragm-braced beam. Further, if the initial imperfections are
set equal to zero, the non-trivial solution of the resulting homo-
geneous equations gives the lateral torsional-flexural buckling
moment of the diaphragm-braced beam. Following the above proce-
dure, the buckling moment M_{cr}, the amplitude of the additional
lateral deflection C of the centroidal axis in the X-direction,
and the amplitude of the additional twist D of a diaphragm-braced
I- or channel beam are given by:

$$M_{cr} = \sqrt{\{EI_y(\frac{n\pi}{L})^2 + Q\}\{E\Gamma(\frac{n\pi}{L})^2 + GK+Qe^2\}} + Qe \tag{5}$$

$$C = \frac{M\delta_\beta\{E\Gamma(\frac{n\pi}{L})^2 + GK + Qe^2\} + M\delta_u\{M-Qe\}}{\{EI_y(\frac{n\pi}{L})^2 + Q\}\{E\Gamma(\frac{n\pi}{L})^2 + GK + Qe^2\}-(M-Qe)^2} \tag{6}$$

$$D = \frac{M\delta_u\{EI_y(\frac{n\pi}{L})^2 + Q\} + M\delta_\beta(M-Qe)}{\{EI_y(\frac{n\pi}{L})^2 + Q\}\{E\Gamma(\frac{n\pi}{L})^2 + GK + Qe^2\}-(M-Qe)^2} \tag{7}$$

where L   is the total length of the member

n = 1 or 2 depending on whether the ends of the beam are "simply supported" or "fixed" against lateral deflection

δ_u and δ_β are the amplitudes of the initial lateral deflection and twist of the beam respectively (initial deflections are assumed to be affine to the buckled shape of an ideal beam)

and   M   is the moment under consideration.

The cross-bending rigidity of the diaphragm is neglected in Eqs. 5, 6 and 7.

Diaphragm-Braced Columns (Energy Method) - The change in total energy U from the compressed stable position to the compressed and deflected unstable position of the portion of the system related to one column of a diaphragm-braced column assembly is formulated below. The total energy U consists of strain energy V of the column, potential energy U_w of the axial load, and energy B_s due to shear in the diaphragm. Thus the total energy is given by

$$U = \frac{1}{2} \int_0^L \{EI_y u''^2 + GK\beta'^2 + E\Gamma\beta''^2 - P(u_t'^2-u_o'^2)-P\frac{I_P}{A}(\beta_t'^2-\beta_o'^2) + Q\ (u'^2+e^2\beta'^2)\}dz \tag{8}$$

where $u_t = u_o + u$, and $\beta_t = \beta_o + \beta$. The energy due to the cross-bending rigidity of the diaphragm is neglected. Deflections of a diaphragm-braced column at precritical loads are obtained by solving the equations resulting from minimization of U using the Rayleigh-Ritz technique.

If the diaphragm bracing is symmetric (Fig. 3c) the terms with derivatives of β in Eq. 8 will vanish, and the load-deflection relationship and buckling load P_{cr} of a column are given by:

$$\{EI_y(\frac{n\pi}{L})^2 - P + Q\}\ C = PE_{10} \tag{9}$$

$$P_{cr} = EI_y(\frac{n\pi}{L})^2 + Q \qquad\qquad (10)$$

where E_{10} is the amplitude of initial lateral deflection of the centroidal axis and P is the axial load on the column.

If columns are braced on one flange only (Fig. 3d), the load-deflection relationship for the column is given by:

$$\begin{bmatrix} EI_y(\frac{n\pi}{L})^2 + Q-P & Qe \\ Qe & E\Gamma(\frac{n\pi}{L})^2 + GK + Qe^2 \end{bmatrix} \begin{Bmatrix} C \\ D \end{Bmatrix} = P \begin{Bmatrix} E_{10} \\ \frac{I_p}{A} F_o \end{Bmatrix} \qquad (11)$$

where F_o is the amplitude of initial twist of the column. The buckling load P_{cr} of the column is obtained by letting the initial deflection equal zero and solving the resulting homogeneous equations for the non-trivial solutions; the smaller value of P from the solutions gives P_{cr}. Eqs. 9 and 11 are obtained by assuming either sine or cosine functions for the deflection pattern in the first mode, depending on whether the ends of the column are "hinged" or "fixed" respectively.

Columns with Diaphragm-Girt Bracing - The method of solution to obtain buckling loads and deflections at precritical loads for this case is similar to that employed in the case of diaphragm-braced columns, and is described in detail in Reference 3. For a column with j intermediate girts there are (j+2) possible modes of buckling, including sidesway. The actual mode of buckling depends on the diaphragm rigidity Q, the bending stiffness of the girts, m, the eccentricity e of bracing, and the geometry of the column assembly. For example, for a column with one intermediate girt and with "hinged" ends (i.e. flexurally hinged, and warping unrestrained) possible modes of buckling are shown in Fig. 5 and possible types of elastic behavior of the column are shown in Fig. 6. In Fig. 6 each curve in each graph represents a particular failure mode. The governing critical load and corresponding critical mode are given by the solid curves.

Inelastic Theory - In all cases the equations obtained in the elastic range can be modified by replacing the modulus of elasticity E by a suitable reduced modulus E_r, and the shear modulus G by a corresponding reduced modulus G_r to describe the behavior in the inelastic range.[3]
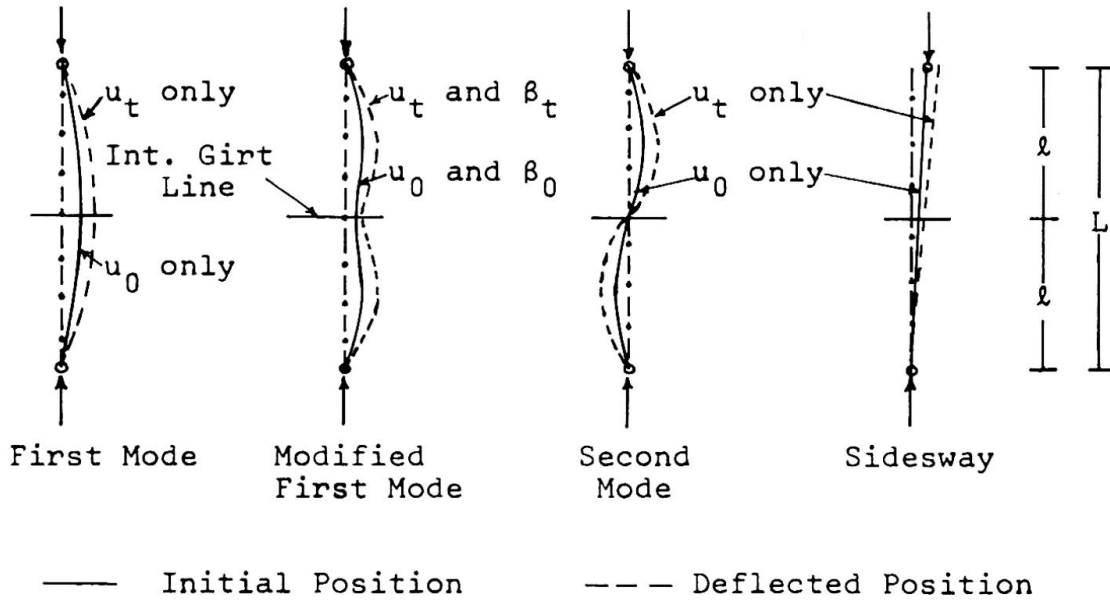
Fig. 5 Buckling Pattern of an Imperfect Column with
        One Intermediate Girt

Legend: M.F.M. - Modified First Mode, S.M. - Second Mode,
         S. - Sidesway, as shown in Fig. 5; $P_e = \dfrac{\pi^2 EI_y}{\ell^2}$
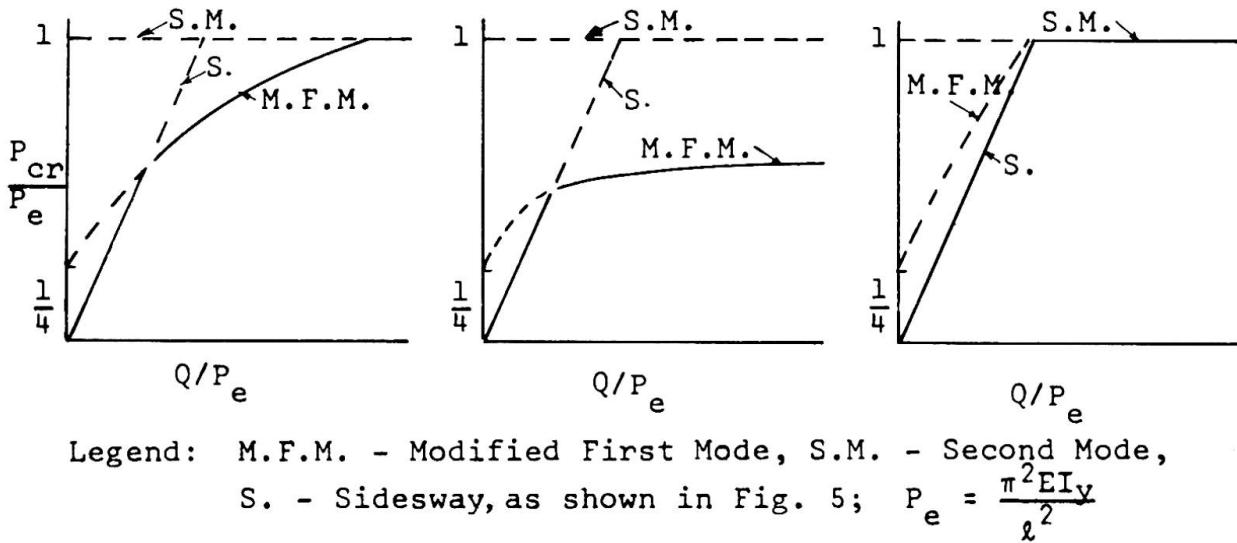
Fig. 6 Possible Types of Behavior of an Elastic Column with
        one Intermediate Girt

THEORETICAL FAILURE LOADS

Actual failure loads of beams and columns are usually
smaller than the theoretical buckling loads because of initial
imperfections and consequent deflections at precritical loads.
The deflections of diaphragm-braced beam and column assemblies
can be computed using the methods described above; therefore,
theoretical failure loads can be computed if appropriate failure

criteria are established for the beams, columns, diaphragm and girts. For example, it may be assumed that a column with continuous diaphragm bracing fails when

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} \geq 1 \tag{12}$$

where f_a is the computed axial stress, P/A

$\quad\quad f_b$ is the computed bending stress at the critical section, including the effect of additional deflection under axial load

$\quad\quad F_a$ is the buckling stress

and $\quad F_y$ is the yield stress.

Then, knowing the deflections of a column at a given load, one can compute the axial and bending stresses in the column, and compare the results with Eq. 12 to determine if column failure will occur at that load. Similarly, the computed deflections can be used to determine the shear strain in the diaphragm, which can be compared with the shear strain that would cause failure of the diaphragm. Computation of theoretical failure loads of beams and columns is illustrated in Ref. 3.

EXPERIMENTAL RESULTS

General - To corroborate the theoretical results, tests were conducted on 35 diaphragm-braced beam and column assemblies all using 50 ksi nominal yield strength steel. Several types of light gage steel diaphragms were used; these were fastened to the members by power driven pins or sheet metal screws. In each case, G_{eff} for the diaphragm was determined experimentally, and the shear rigidity Q was calculated from Eq. 1. To obtain a conservative estimate of the theoretical deflections at pre-failure loads, the amplitudes of the initial deflections used in the calculations were the largest measured values irrespective of their location.

Diaphragm-Braced Beams - Beam tests included pairs of 8JR6.5 and 10B17 I-sections, 8[6.2 sections, and an assembly of four 8JR6.5 beams. Beam spacing ranged from 1'-3" to 3'-8", (38 cm to 112 cm) and lengths ranged from 20' to 30' (6.1 m to 9.15 m) with ends fixed against lateral bending. A comparison of the theoretical
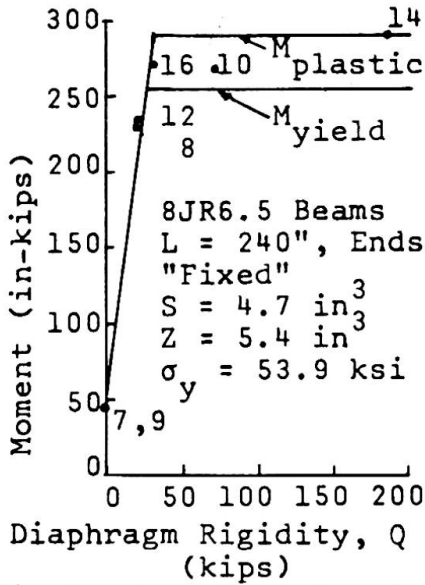
Fig.7 Beam Test Results and Predicted Values

8JR6.5 Beams
L = 240", Ends "Fixed"
$S = 4.7$ in^3
$Z = 5.4$ in^3
$\sigma_y = 53.9$ ksi

buckling loads with test failure loads for 8JR6.5 assemblies is shown in Fig. 7. The moments sustained by the diaphragm-braced beams were always more than 80% of predicted buckling values for ideal members, and as high as six times the buckling moments of corresponding unbraced beams.

Diaphragm-Braced Columns - Tests were conducted on assemblies of pairs of 4I7.7 columns braced by diaphragms on one or both flanges. Spacing of columns ranged from about 14" to 17" (36 cm to 43 cm), and slenderness ratios ranged from 280 to 50. A comparison of theoretically predicted buckling loads with the test failure loads is shown in Fig. 8. Failure loads were 80% to 98% of predicted buckling loads for ideal members and as high as ten times the buckling loads of unbraced columns.
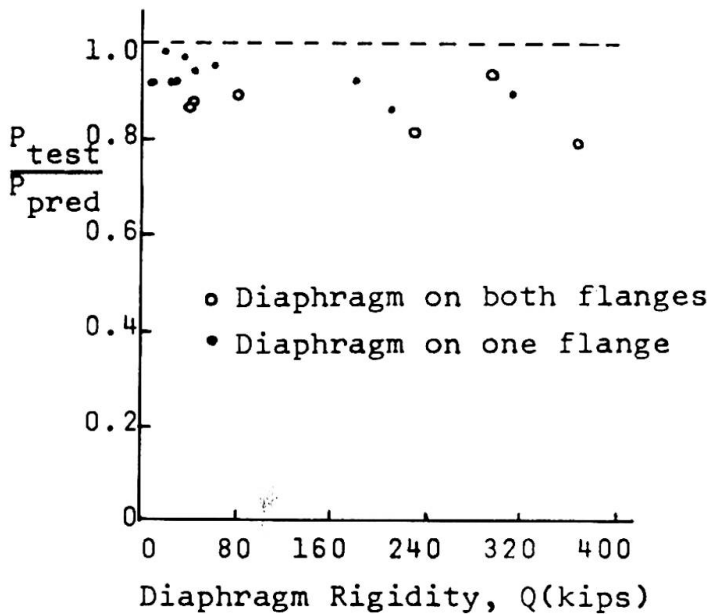

Fig. 8. Comparison of Theory and Column Test Results

o Diaphragm on both flanges
• Diaphragm on one flange

Column Test GT-3
8 JR6.5 Column
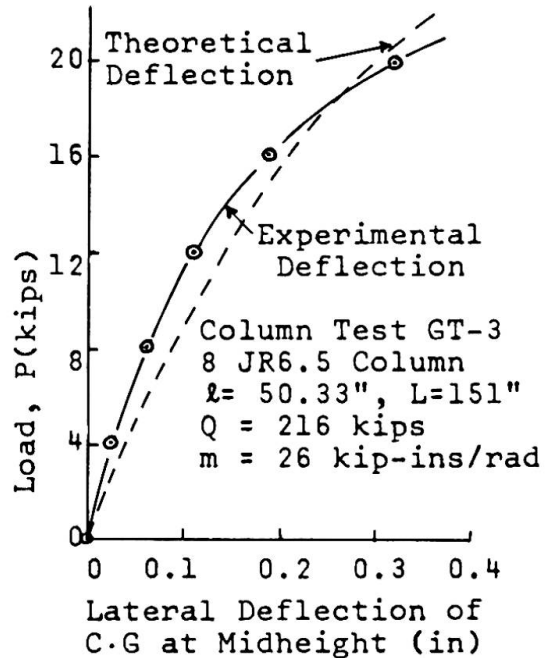$\ell = 50.33"$, L=151"
Q = 216 kips
m = 26 kip-ins/rad

Fig. 9 Load-Deflection Curves

Columns with Diaphragm-Girt Bracing - Tests were made on three assemblies of axially-loaded pairs of 8JR6.5 I-section columns braced by two intermediate girts and a 26 gage standard corrugated steel diaphragm. Columns were spaced 6' (1.8 m) apart and were

12'-7" (3.8 m) high. Theoretical buckling loads and experimental
failure loads for the three assemblies are given in Table 1, and

TABLE 1. Comparison of Test Failure Loads
with Predicted Buckling Loads of
Columns with Diaphragm-Girt Bracing

Test	Twist Restraint, m(kip-in/ rad.)	Failure Mode	Distance, e (in)	Predicted Critical Load (kips)	Max Test Load (kips)	Test/Pred.
GT-1	0	Modified First Mode (Tor-Flex)	6	21.4	17.7	0.84
GT-2	7750	Third Mode (Flexural)	10	39.9	37.3	0.94
GT-3	13	Modified First Mode (Tor-Flex)	6	29.6	25.5	0.86

the load-deflection relationships for a particular assembly are
shown in Fig. 9. Failure loads were 84% to 94% of predicted
buckling loads, and as high as nine times the buckling load of
an unbraced column. Deflections predicted by the theory give a
conservative estimate of the experimental deflections (Fig. 9),
except at very high loads where secondary effects of large deflec-
tions began to prevail. A photograph of assembly GT-2 after test
appears in Fig. 10. It is evident that failure was by buckling
of the column over one-third its total length.

FURTHER STUDIES

Recent efforts have been directed toward assessing the prac-
tical implications and applications of diaphragm bracing. The
dimensions and member sizes of the column-girt-diaphragm assembly
of a typical steel building with metal sheathing is shown in Fig.
11. As ordinarily fastened, the diaphragm bracing has a Q-value
of approximately 1000 kips. Calculations show that this combina-
tion of girts and panels is adequate to fully brace the columns,
where "full bracing" is equivalent to the bracing that would be
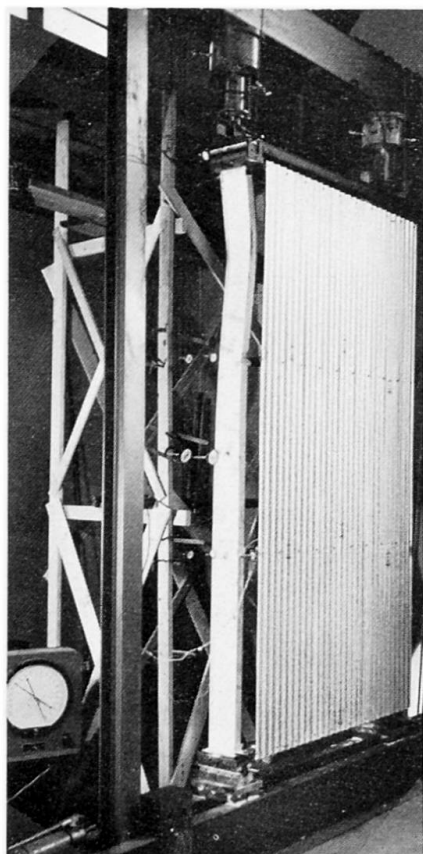provided by an infinitely rigid diaphragm. In other cases, minor

Fig. 11   Column-Girt-Diaphragm Assembly
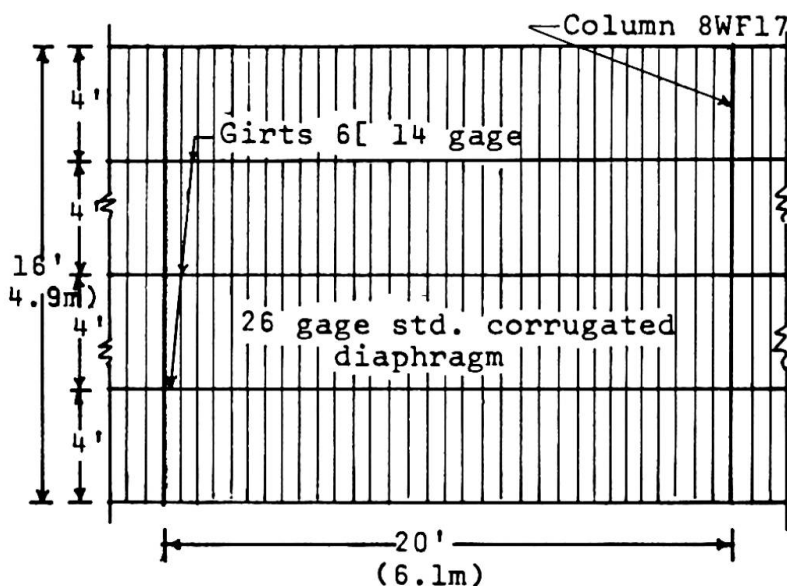of a Typical Steel Building

Fig. 10   Column-Girt-
Diaphragm Assembly
After Test GT-2

modifications in construction practice may be required to achieve
full bracing. Other studies have shown similar results for most
practical sizes of beams braced by diaphragms in this type of con-
struction. In such cases design loads may be increased, or other
types of lateral bracing can be safely omitted.

CONCLUSIONS

Theory and test results both indicate that properly connected
diaphragms can be very effective as bracing for beams and columns.
In tests of actual assemblies, 80% to 100% of the theoretical
critical load was obtained, with failure loads up to 10 times the
critical load for the same member without diaphragm bracing.

This information permits the more economical design of: wall
columns connected directly to steel sheathing; wall columns con-
nected to girts which in turn are connected to steel sheathing;
roof trusses connected directly or through purlins to roof deck-
ing; and floor beams and joists connected to light steel flooring.
In all these cases buckling in the plane of the panels can be pre-
vented by utilizing the shear rigidity of the ribbed thin dia-
phragm which, as wall, roof or floor, must be present in any
event and, therefore, is available at no extra cost.

ACKNOWLEDGMENTS

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SUMMARY

Thin-walled corrugated sheets or ribbed steel panels often are used as side wall sheating, roof decking or floor decking of steel framed buildings. These interconnected panels act as shear-resistant diaphragms, and can restrain the members of the steel frame against buckling in the plane of the diaphragm. Theory and confirming test results are presented for columns and beams braced directly by diaphragms, and also for columns braced by girts which in turn are braced by diaphragms.

RÉSUMÉ

Souvent des tôles ondulées ou profilées servent au revêtement des fassades ou à la couverture des toits et planchers des portiques à étages multiples en acier. Ces tôles, solidement liées entre elles, forment une dalle stabilisante, retenant la construction et pouvant empêcher un flambement dans leur plan. La théorie et les résultats expérimentaux sont présentés ici pour les deux cas possibles, les tôles stabilisant directement poutres et piliers, ou stabilisant seulement les poutres, lesquelles retiennent les piliers à leur tour.

ZUSAMMENFASSUNG

Häufig werden Well- und Rippenblech als Fassadenbekleidung, Dach- und Fussbodeneindeckung von Stahlrahmenkonstruktionen verwendet. Diese Bleche wirken als stabilisierende Querscheiben und können das Knicken der Stahlrahmen in ihrer Ebene verhindern. Theorie und Testresultate werden dargestellt sowohl für direkte Verstärkung von Stützen und Balken als auch für verstärkte Träger, die ihrerseits dann auch die Stützen halten.