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Objekttyp: **Article**

Zeitschrift: **IABSE congress report = Rapport du congrès AIPC = IVBH Kongressbericht**

Band (Jahr): **8 (1968)**

PDF erstellt am: **13.09.2024**

Persistenter Link: https://doi.org/10.5169/seals-8762

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Computation of the Postbuckling Strength of Thin-Walled Sections

Estimation de la tension après flambement dans des sections à parois minces

Berechnung der überkritischen Knickspannungen dünnwandiger Querschnitte

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INTRODUCTION

In some design methods for the assessment of local buckling strengths of thin-walled columns it is assumed that the maximum carrying-capacity of a column is the sum of the maximum loads of the separate component plates, with each plate having simple boundary conditions along its edges. The aim of this paper is to assess the accuracy of this design assumption for a range of structural shapes. It is shown first that the maximum carrying-capacity of a thin-walled column of any simple cross-section composed of component flat plates is a function of the initial elastic buckling stress, σ_{cr}, of the column and the compressive yield stress, σ_Y, of the material; this property, which is confirmed by a wide range of experimental studies of mild-steel and aluminium-alloy columns of many different cross-sectional forms, is used as a basis for estimating the accuracy of the simple design assumption of separate component plate strengths.

STRENGTH OF SINGLE PLATES

It is well known that the initial elastic buckling stress of a long rectangular plate, uniformly-compressed in the longitudinal direction, may be written

$$\sigma_{cr} = \frac{k\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2, \qquad (1)$$

where

k is a constant depending on the boundary conditions on the longitudinal edges and on the length/breadth ratio,

E is Young's modulus,

ν is Poisson's ratio,

b is the breadth of the plate,

t is the uniform thickness.

When the longitudinal edges are simply-supported, a long compressed plate buckles approximately into square panels, and as the length of the plate increases the constant k approaches the value 4.

Elastic local buckling leads to a re-distribution of compressive stress over the loaded edges, if these edges are loaded through rigid platens. The maximum load-carrying capacity is reached when the yield stress is induced at the supported edges and in highly-strained regions at the centres of the buckles. It has been suggested (ref.1) that at the maximum load-carrying capacity of the plate the average compressive stress, σ_{max}, is a function of the initial elastic buckling stress, σ_{cr}, and the compressive yield stress, σ_Y, in the form

$$\sigma_{max}/\sigma_Y = F(\sigma_{cr}/\sigma_Y), \qquad (2)$$

where $F(\sigma_{cr}/\sigma_Y)$ is a function of (σ_{cr}/σ_Y) to be determined by experiment for plates of a given material. Tests on single flat plates in compression confirm a dependence of (σ_{max}/σ_Y) on (σ_{cr}/σ_Y).

STRENGTH OF COMPRESSED SECTIONS

In ref.1 it was shown that this dependence is also true of some compressed thin-walled sections composed of thin plates. In the case of a thin-walled section σ_{max} is the maximum average compressive stress, σ_{cr} is the initial elastic local-buckling stress of the section as a whole, and σ_Y is the yield stress. A study of tests on channel and lipped-channel sections of different materials suggests that

$$\sigma_{max}/\sigma_Y = A(\sigma_{cr}/\sigma_Y)^{1/3}, \qquad (3)$$

where $A = 0.736$ for mild-steel sections and $A = 0.863$ for aluminium-alloy sections.

Studies of other types of sections and of single plates with different types of edge-support suggest that the range of validity of equation (3) can be extended. Winter (ref.2), for example, has outlined the results of compression tests on mild-steel flanges with one longitudinal edge supported and the other completely free; the range of test results described by Winter is shown in Fig.1, and the relation

$$\sigma_{max}/\sigma_Y = 0.736\,(\sigma_{cr}/\sigma_Y)^{\frac{1}{3}}, \tag{4}$$

fits these tests results reasonably well. Winter (ref.3) has also described the results of many experiments on mild-steel plates, with two supported longitudinal edges, by the empirical relation

$$\sigma_{max}/\sigma_Y = (\sigma_{cr}/\sigma_Y)^{\frac{1}{2}}\left[1 - 0.25\,(\sigma_{cr}/\sigma_Y)^{\frac{1}{2}}\right], \tag{5}$$

which can be shown (see Fig.1) to give similar results to equation (4). Kenedi and others (ref.4) have compared equation (4) with the results of tests on some 200 mild-steel sections of different shapes; the limits of scatter of these tests are shown in Fig.2; equation (4) defines the average

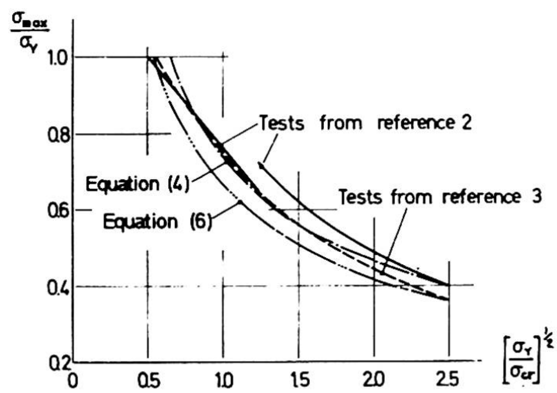


Fig.1 Average collapse stresses of mild-steel plates and thin-walled sections.

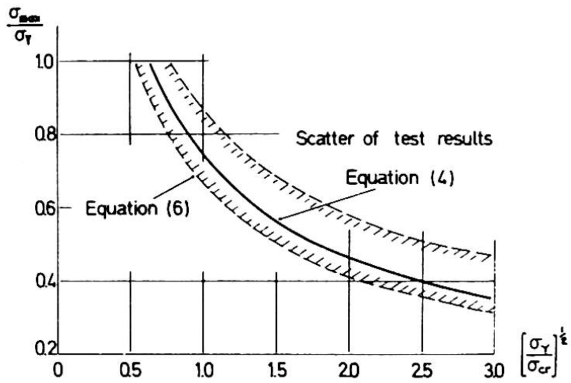


Fig.2 Collapse tests on mild-steel sections of different shapes (ref.4).

maximum stress with reasonable accuracy, while a conservative form of this relation, approximating the lower scatter boundary, (see Figs.1 and 2), can be written as

$$\sigma_{max}/\sigma_Y = 0.66\,(\sigma_{cr}/\sigma_Y)^{\frac{1}{3}}. \tag{6}$$

That equation (4) can be extended to mild-steel plates with different types of longitudinal edge supports suggests that this expression can be used for a wide range of thin-walled mild-steel sections.

Similar conclusions can be made about aluminium-alloy plates and sections; the appropriate form of equation (3) is

$$\sigma_{max}/\sigma_Y = 0.863\,(\sigma_{cr}/\sigma_Y)^{\frac{1}{3}}, \tag{7}$$

and we will discuss the relevance of this relation to both plates and structural sections. The results of tests on single plates by Schumann (ref.12), Gerard (ref.13) and Stüssi (ref.14) are shown in Fig.3. The boundary conditions of a simply-supported plate are reproduced in the local buckling of a square tube of uniform thickness; such tubes have been used to study the buckling strengths of single plates; the results of tests of this sort by Needham (ref.7), Heimerl (ref.10), Bijlaard (ref.11) and Schumann (ref.12) are shown in Fig.4. The conditions of one longitudinal

Fig.3 Results of tests on single compressed plates of aluminium-alloy materials.

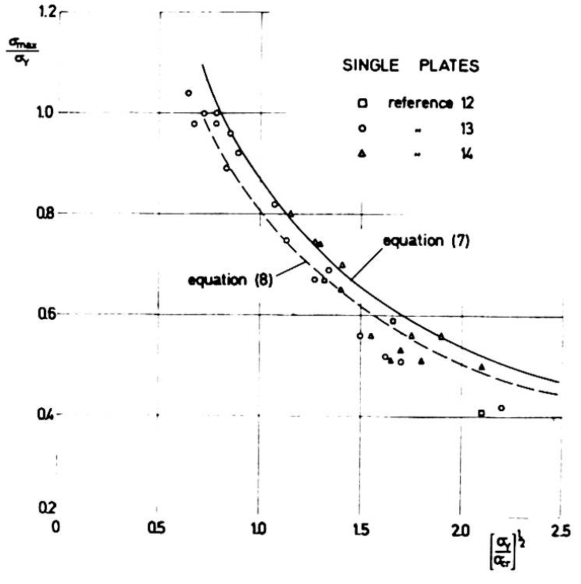
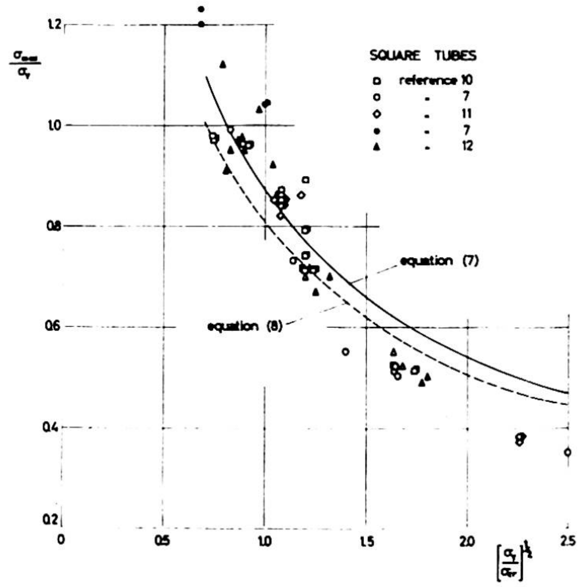


Fig.4 Results of tests on compressed square tubes of aluminium-alloy materials.



edge of a plate simply-supported and the other free are reproduced in equal angle-sections, cruciform sections and certain T-sections; the results of some tests by Needham(ref.7), Heimerl (ref.9) and Gerard (ref.13) on such sections are shown in Fig.5; component plates in these sections are said to be "unstiffened"elements.

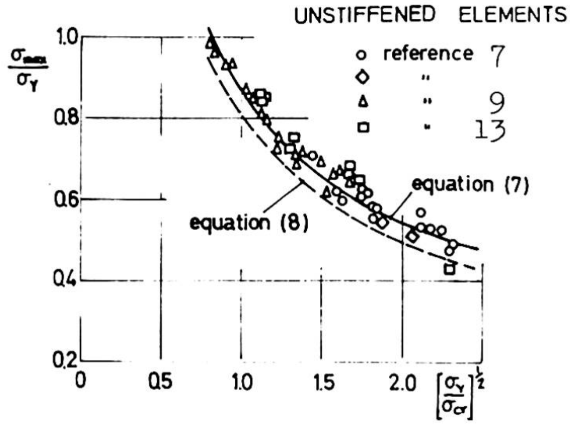


Fig.5 Results of tests on unstiffened plate elements of aluminium-alloy materials.

Fig.6 Results of tests on channel sections of aluminium-alloy.

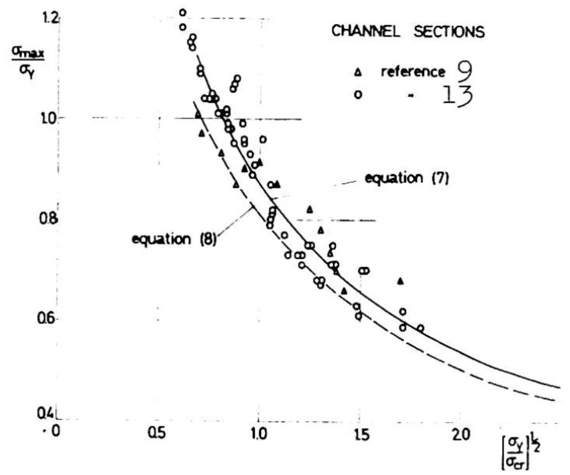
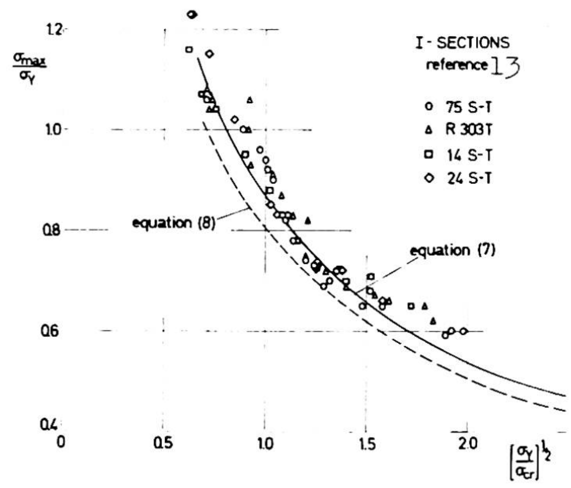


Fig.7 Results of tests on I-sections of aluminium-alloy materials.



The results of tests on channel sections by Heimerl (ref.9) and Gerard (ref.13) are shown in Fig.6, on I-sections by Gerard (ref.13) in Fig.7 and on a variety of open sections reported by Needham (ref.7) and Gerard (ref.13) are shown in Fig.8. Again, the test results in the case of sections are described reasonably accurately by equation (7).

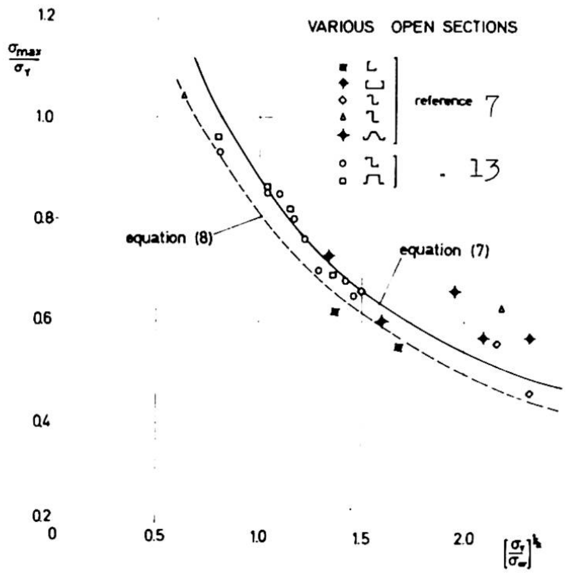


Fig.8 Results of tests on various open sections of aluminium-alloy materials.

A conservative estimate of strength, approximating to the lower boundaries of scatter of Figs.3 to 8, takes the form

$$\sigma_{max}/\sigma_Y = 0.80(\sigma_{cr}/\sigma_Y)^{\frac{1}{3}}. \qquad (8)$$

It seems that for a given material, and over a wide range of structural shapes, the maximum average compressive stress, σ_{max}, of a composite section is a function of the initial elastic local buckling stress, σ_{cr}, and of the yield stress, σ_Y. Of the independent variables, σ_{cr} and σ_Y, only σ_{cr} is dependent on the geometry of the section. For a section composed of a number of component flat plates,

$$\sigma_{cr} = \frac{k_o \pi^2 E}{12(1-\nu^2)} \left(\frac{t}{b}\right)_o^2, \qquad (9)$$

where k_o is a constant depending on the interaction of the component plates and where $(b/t)_o$ is the thinness ratio of a representative component plate. On substituting this value of σ_{cr} into equation (3), we have

$$\sigma_{max} = B k_o^{1/3} (t/b)_o^{2/3} , \qquad (10)$$

where

$$B = A\sigma_y^{2/3} \left[\pi^2 E / 12(1-\nu^2)\right]^{1/3} . \qquad (11)$$

If B and k_o are known, equation (10) is a direct relationship between σ_{max} and $(b/t)_o$. Values of $k_o^{1/3}$ for a number of sections of uniform thickness, t, are shown in Fig.9

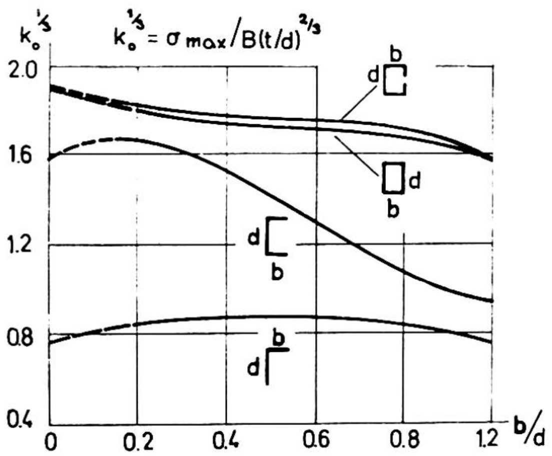


Fig.9 Values of the constant k_o for a number of shapes of cross-section of uniform thickness.

ACCURACY OF SIMPLE DESIGN METHODS

An approximate method of estimating the maximum average compressive stress of a thin-walled steel section is given in Addendum No.1 to British Standard 449 (ref.5). In this method it is assumed that the component plates of a compressed thin-walled section buckle independently of each other; the maximum stress is calculated by summing the maximum average stresses of the individual component plates, weighted according to their areas. A similar approximate method is used in the design manual of the American Iron and Steel Institute (ref.6) for sections composed of fully-supported component plates. We will now discuss the accuracy of these design methods in the light of the test results presented and surveyed in the previous section.

Consider a thin-walled section composed of a number of plates, each of breadth b_i and thickness t_i. A component plate of the section can be attached to other plates either on one longitudinal edge only or on both longitudinal edges. If the component plate is supported on both longitudinal edges, then the longitudinal edges are assumed simply-supported, and the buckling constant, k_i, for that plate is equal to 4. If one edge only is supported, then k_i is equal to 0.425. With one or other of these values of k_i, the individual collapse loads of the plates are evaluated. The total compressive load at collapse is then

$$P_{max} = \sum_i B k_i^{\frac{1}{3}} \left(\frac{t}{b}\right)_i^{\frac{2}{3}} (bt)_i \, . \tag{12}$$

The maximum average compressive stress is, therefore,

$$\sigma^*_{max} = \frac{\sum_i B k_i^{\frac{1}{3}} (t/b)_i^{\frac{2}{3}} (bt)_i}{\sum_i (bt)_i} \, . \tag{13}$$

If t_i is constant throughout the cross-section, and if b_c is a reference breadth, then

$$\sigma^*_{max} = B\left(\frac{t}{b_c}\right)^{\frac{2}{3}} \frac{\sum_i k_i^{\frac{1}{3}} (b_i/b_o)^{\frac{1}{3}}}{\sum_i (b_i/b_o)} \, . \tag{14}$$

Thus

$$\frac{\sigma^*_{max}}{\sigma_y} = A\left(\frac{\pi^2 E}{12(1-\nu^2)\sigma_y}\right)^{\frac{1}{3}} \left(\frac{t}{b_o}\right)^{\frac{2}{3}} \frac{\sum_i (b_i/b_o)^{1/3}}{\sum_i (b_i/b_o)} \, . \tag{15}$$

The values of $(\sigma^*_{max}/\sigma_y)$ are now compared with those given by equation (3) for the composite section, and we have

$$\frac{(\sigma^*_{max}/\sigma_y)}{(\sigma_{max}/\sigma_y)} = \frac{\sum_i k_i^{\frac{1}{3}} (b_i/b_o)^{\frac{1}{3}}}{k_o^{\frac{1}{3}} \sum_i (b_i/b_o)} \, , \tag{16}$$

since from equation (9)

$$\sigma_{cr} = \frac{k_o \pi^2 E}{12(1-\nu^2)} \left(\frac{t}{b_o}\right)^2 .$$

Equation (16) is useful for the comparison of the approximate methods employed by the British and American design codes with the empirical relation based on full section tests. The ratio $(\sigma^*_{max}/\sigma_{max})$ serves as a measure of the accuracy of the approximate method; this ratio depends only indirectly on the value of σ_y because of the limitation that the maximum average stress of each individual plate must not exceed the yield stress. The difference between σ^*_{max} and σ_{max} depends, therefore, only on the shape of the section and not on the plate thickness and the material.

Fig.10 Comparison of σ_{max} and σ^*_{max} for channel sections.

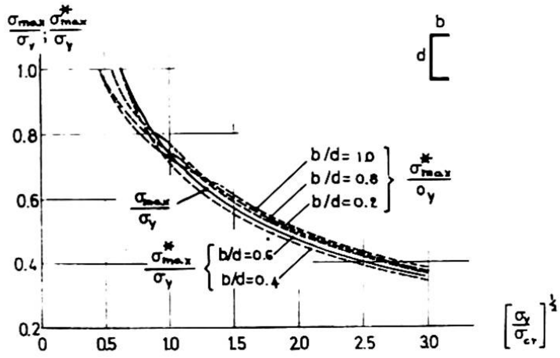
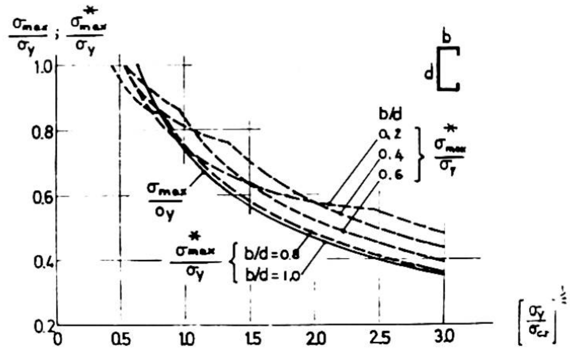


Fig.11 Comparison of σ_{max} and σ^*_{max} for lipped channel sections.



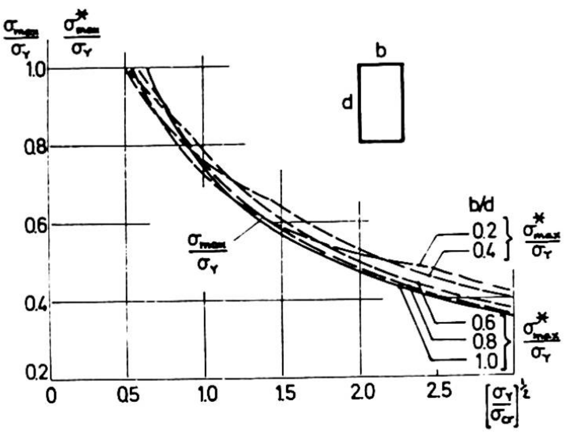


Fig.12 Comparison of σ_{max} and σ^*_{max} for rectangular tubes.

Figs.10, 11 and 12 show the comparison of stresses computed by the approximate method, equation (15), with the experimentally based empirical equation (4). It can be seen that the difference between the broken lines, representing the stresses according to the approximate method, and the full lines, corresponding to equation (4), are indeed constant, subject to the limitation of yielding of each plate. For channel and rectangular tube sections the simple design method gives reasonably accurate collapse strengths, as shown in Figs.10 and 12, but for lipped channel sections the approximate method tends to over-estimate the strength of the section considerably in some cases. If this approximate method is used, it should be appreciated, therefore, that the estimates of collapse strengths of some sections may err on the unsafe side by as much as 20%.

CONCLUSIONS

This study of the local buckling of columns shows that for some sections the assumption that the strength of a section is the sum of the separate strengths of the component plates, assuming these are simply-supported or free on the longitudinal edges, may be in error on the unsafe side by as much as 20% for some sections. The paper also shows regimes where this simple design assumption is more accurate.

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SUMMARY

The paper concerns the accuracy of some current methods of design of thin-walled columns which fail by local buckling when under uniform compression. It is suggested that simple design methods using linear addition of component plate strengths may be on the unsafe side by as much as 20% for some sections.

RÉSUMÉ

Cette étude du flambage local de piliers montre que la supposition courante, à savoir que l'effort dans une section est la somme des efforts dans les différentes parois dans le cas où celles-ci sont supportées simplement ou libres sur les bords longitudinaux, peut être erronnée dans le mauvais sens, et cela jusqu' à 20 % dans certaines sections. Cette rédaction montre aussi des cas où cette simple supposition de dimensionnement et plus rigoureuse.

ZUSAMMENFASSUNG

Diese Studie des lokalen Knickens an Stützen zeigt, dass für einige Querschnitte die Annahme, die Spannung eines Querschnittes sei die Summe der Spannungen der Teilplatten, angenommen dass deren Längsränder frei oder dass sie einfach aufgelegt sind, einen Fehler für bestimmte Querschnitte zeigt, der mit 20 % auf der unsicheren Seite liegt. Dieser Beitrag zeigt zudem Fälle, wo die einfache Annahme genauer ist.

Questions raised by Dr. P.S. Bulson:

In their paper, Reiss and Chilver have indicated discrepancies when the simple design method is applied to lipped channels. Could I ask whether, in their calculations for lipped channels,

(a) the elastic critical stress, σ_{cr} , was calculated by exact theory, or measured experimentally, and

(b) the lip was treated as a component plate offering simple support to the flange; and, if so, whether K_i for the flange was taken as 4, and K_i for the lip as 0.425?

Would the authors also comment, please, on the assumption often made that lips of a given minimum size offer simple support to flanges in the post-buckling region? Tests suggest that after initial buckling the longitudinal junction between lip and plate does not remain straight.

Authors' replies:

(a) The elastic critical stress, σ_{cr} , was calculated by exact local buckling theory,

(b) the junction of lip and flange was treated as simply-supported and assumed to remain straight during buckling; the lip was assumed to be unaffected by buckling, either flexurally or torsionally.

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