The plastic design of braced multi-storey frames

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DISCUSSION PRÉPARÉE / VORBEREITETE DISKUSSION / PREPARED DISCUSSION

The Plastic Design of Braced Multi-Storey Frames

Calcul plastique de portiques à plusieurs étages renforcés

Plastische Bemessung unverschieblicher Stockwerkrahmen

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INTRODUCTION There are two essential steps in the design of a steel frame which is required to carry given loads. First, a set of structural forces must be determined which is in equilibrium with the applied loads; secondly, individual members must be proportioned to carry those equilibrium forces. These two steps cannot always be separated in the design process, as will be seen, but they are in fact logically distinct.

The first step has led to the proliferation of different methods of structural analysis, all of which "are nothing more than a ready way of finding a reasonable equilibrium solution that works in practice" [1]. This situation has been discussed elsewhere [2] with particular reference to the use of plastic theory for finding the basic equilibrium solution. (It may be noted here that the whole of the discussion in the present paper is confined to the case where deflexions are not the primary design criterion. That is, member sizes are determined on the basis of the strengths of the various portions of the structure, and it is assumed that any necessary deflexion checks will be made as a secondary matter).

The use of plastic theory as a design tool implies the use of load factors to give the required margin of safety to the actual structure. Thus the working values of the loads acting on a frame are hypothetically increased to certain factored values, and the frame is then designed to resist the action of those factored loads. The actual value of the factor used in the calculations depends on the type of structure and loading being considered, and is different in different countries and at different dates. In England it is usual to apply a factor of 1•75 to both dead and superimposed loading in the design of simple factory buildings. However, a recent report [3] on the design of braced multi-story frames recommends the use of the factor 1•5, providing the design is carried out by the methods proposed in the report. This particular recommendation is examined in more detail below.

The function of the load factor is essentially two-fold. In the first place it provides a margin of safety against imperfections in the structure itself, which can be introduced at any stage in the processes of design, fabrication, and erection. Secondly, there is always some uncertainty in the actual values of the loads; that is, the real loading on a structure can only be assessed on a probabilistic basis. Thus the use of a load factor of 1•5, for example, implies that the probability of a 50% overload occurring is acceptably small.

However, a load factor need not be used only in conjunction with a plastic method of design. As an immediate example, the limit state of a column in a multi-storey frame may be governed by elastic instability; in this case, the designer would wish to check that the column remains stable under the factored loading. Again, this particular aspect is discussed more fully below.

THE DESIGN OF BRACED FRAMES There is some measure of agreement about the way in which braced multi-storey frames should be designed, even if considerable differences of detail are apparent between different proposals. Thus the report of a Joint Committee [3] referred to above outlines certain steps that will lead to a satisfactory design, and these steps are reflected, for example, in recent work in the US [4]. The two key moves in the Joint Committee's proposals are (a) the plastic design of the beams, and (b) the use of a limited substitute frame for the stability check of the columns.

Plastic design of beams is usually direct; that is, the determination of suitable equilibrium bending moment diagrams and the actual design of the beams proceed simultaneously. On relatively infrequent occasions it may be necessary to make iterative calculations, for example when a column section proves on later examination to be inadequate to carry the required full plastic moment of the beam. Leaving aside such anomalies, all the beams in a braced multi-storey frame may be designed virtually span by span just to carry the factored dead and superimposed loading.

By contrast all the methods so far developed for column design involve two distinct processes for (a) the determination of column bending moments and

(b) the actual proportioning of a particular column. The Joint Committee propose a limited substitute frame that should be considered for the design of each column length; in Fig.1, the column being investigated is regarded as connected to the adjacent members but there is no further "spread" into the structure as a whole. This substitute frame can be traced back to the work of the Steel Structures Research Committee [5], and the use of the frame greatly simplifies the work. The analysis must proceed by trial and error, since a column size has to be assumed in order to determine the elastic bending moments in the substitute frame. The stiffnesses of the members are calculated, and out-of-balance bending moments are then distributed either by the Hardy Cross method or by a one-step formula given by the Joint Committee.

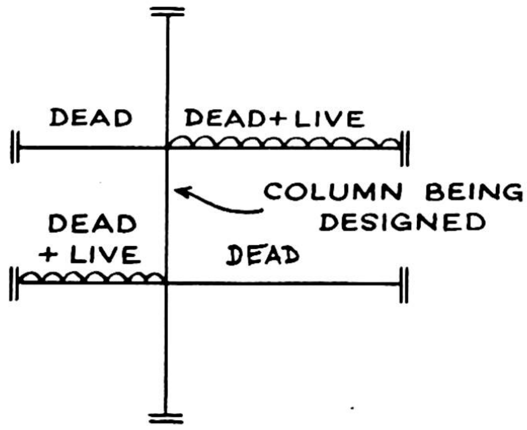
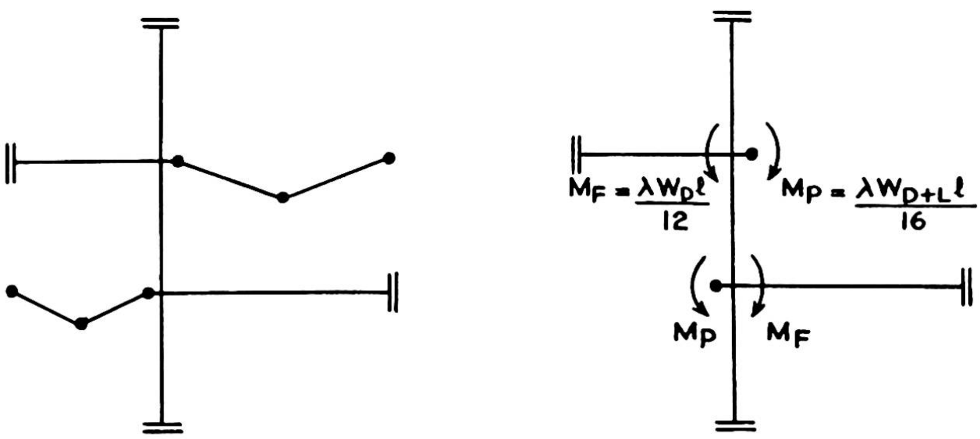


Fig. 1

The Joint Committee requires the calculations to be carried out using factored values of the loads; a typical beam loading pattern is shown in Fig.1, and the load factor 1·5 is supposed to be applied to both dead and live (superimposed) loading. Now, under full factored dead plus live loading, a beam will be on the point of collapse, and the state of the substitute frame of Fig. 1 will be as shown in Fig.2(a). The collapsing beams cannot absorb any further



(a) (b)

Fig. 2

bending moment, and, in any moment distribution process, their stiffnesses must be assumed to be zero. The reduced substitute frame of Fig.2(b) is therefore used for checking the columns under these conditions; the collapsing beams have been replaced by "dead" moments of value M_p .

The out-of-balance bending moments in Fig.2(b) are distributed, and lead to values of terminal bending moments in the central column length; these values, together with that of the (factored) axial load, can then be used to check the suitability of the chosen column according to any required criterion (e.g. stability or the condition that the column shall just remain elastic).

There seem to be two anomalies in the method just outlined, the discussion of one of which is straightforward. In the first place, it is clear that more severe conditions would arise for the central column length in Fig.2(b) if the dead load moments (M_F), opposing the full plastic moments (M_p), were not factored. The use of an overall load factor (of value 1·5) on both dead and live loading allows the dead load to partially relieve the bending moments. In such cases, it might be appropriate to use a load factor of unity on the dead load, or of value 0·9 in accordance, for example, with current recommendations [6] on limit state design or with the French regulations for steelwork [7]. Thus the value of M_F in Fig.2(b) should be calculated with the factor λ set equal to unity or 0·9.

Retaining, however, the notion of an overall load factor of 1·5 to be applied to both dead and live loading, there is another sense in which the frames of Fig.2 may be criticized. Had the calculations of column moments been made under working values of the loads for the pattern shown in Fig.1, then there would have been no question of any of the beams collapsing. The fixed-end moments at the ends of the loaded beams would then have values $W_{D+L}\ell/12$, as shown in Fig.3, instead of the values $\lambda W_{D+L}\ell/16$ of Fig.3. In addition, the stiffnesses of all the beams should have their full values in the distribution process.

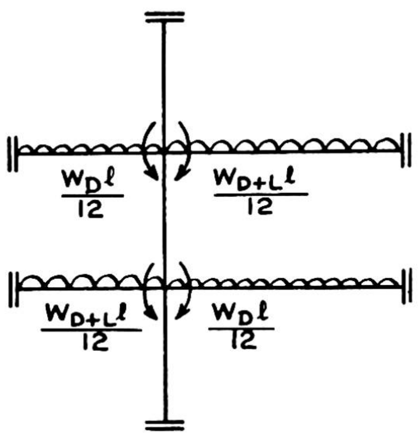


Fig. 3

A numerical example (below) confirms that the column moments resulting from Fig.3, when post-multiplied by the load factor, can exceed the moments resulting from Fig.2(b), in which the loading is pre-multiplied by the load factor. It becomes essential at this point to be clear about the nature of the check that is made to confirm the suitability of the columns.

The column is a potentially unstable structural element, and, if the designer is to be completely assured of the safety of an entire structure, he must be satisfied that there is no danger of premature failure due to instability. Thus there must be an adéquate margin of safety between the values of end moments and axial thrust computed for a particular column length and the corresponding values that would just cause failure of the column. If the calculations are made for the nominal working values of the loads, Fig.3, the column can be computed to have a certain margin of safety, and this margin may well be less than that given by the apparently more "real" configuration of Fig.2(b).

<u>SAMPLE COLUMN CALCULATIONS</u> The design of a large laboratory block has been reported [8], and some of the calculations afford a convenient basis for comparison. Fig.4 shows the three substitute frames for the calculations for a

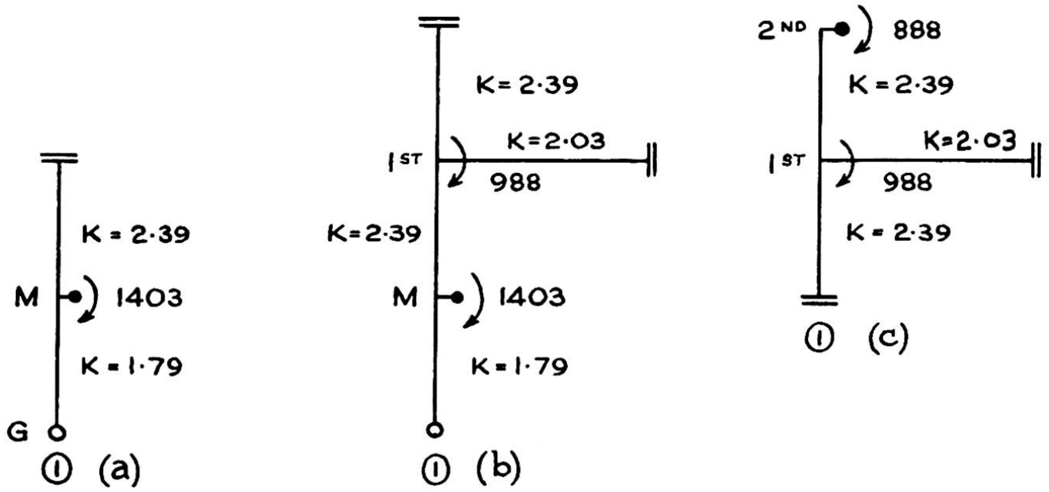


Fig. 4

typical external column; the column length under consideration is the lowest, centre and upper storey in Figs.4(a), (b) and (c) respectively. Note the introduction of the plastic hinges in accordance with Fig.2. The resulting bending moments in the individual column lengths are shown in Fig.5. A load factor of 1•5 has been used in these calculations, and the axial loads marked in Fig.5 are factored values.

The calculations made for unit load factor proceed using the substitute frames shown in Fig.6. The resulting bending moments are shown in Fig.7, to be compared with the values marked in Fig. 5. Comparing these two figures, it will

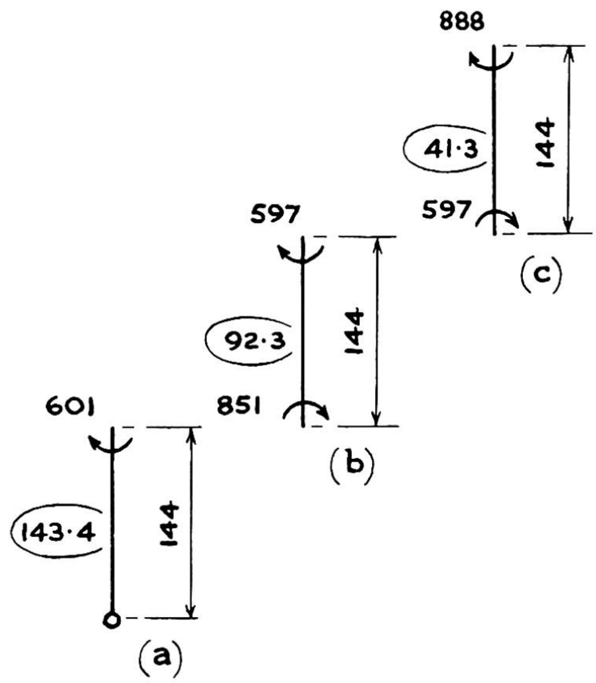


Fig. 5

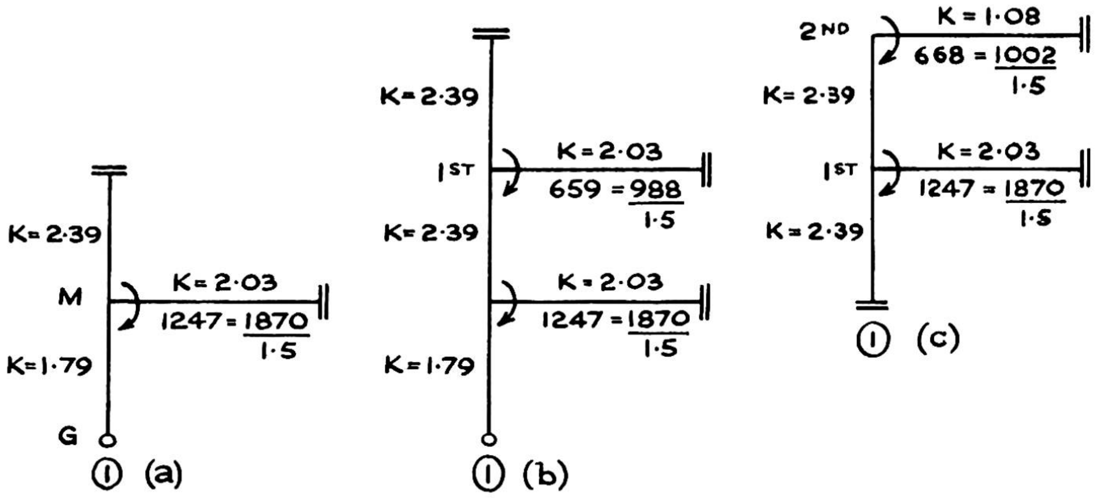


Fig. 6

be seen that for this case of an external column, the original design approach, using factored loads, leads to the more severe design condition. This result is typical for external columns, but the reverse is true for internal columns.

Fig.8 reproduces the design conditions for an internal column [8], and Figs. 8(b) and (c) show the factored design conditions for single and double curvature bending. The alternative calculations using working loads and a completely elastic frame, are displayed in Fig. 9. It will be seen that the resulting bending moments in the column when post-multiplied by the load factor

1•5, are some 50% higher than the corresponding bending moments marked in Fig. 8. Thus the apparent margin of safety would be less if the calculations were performed according to the substitute frame of Fig. 9 rather than that of Fig. 8.

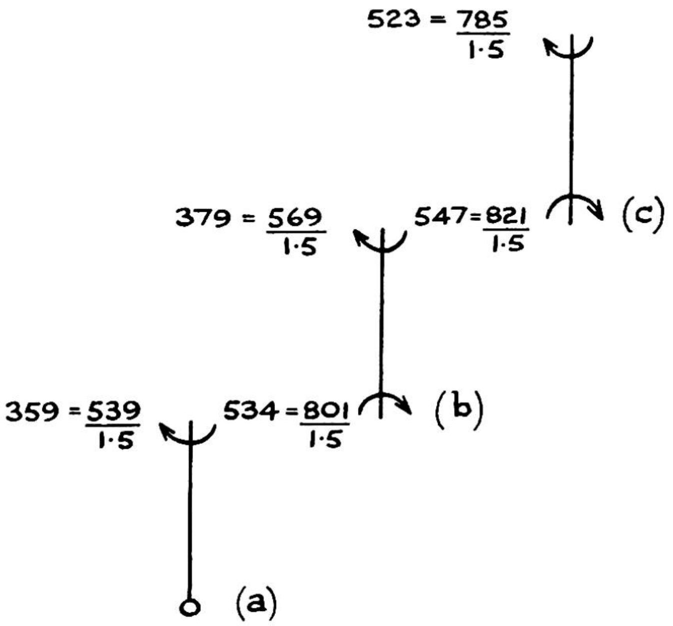
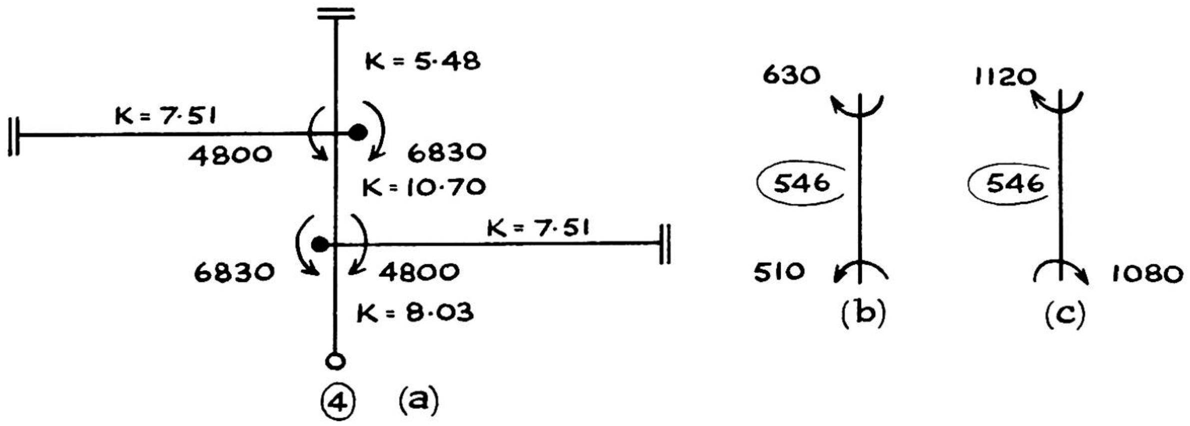


Fig. 7



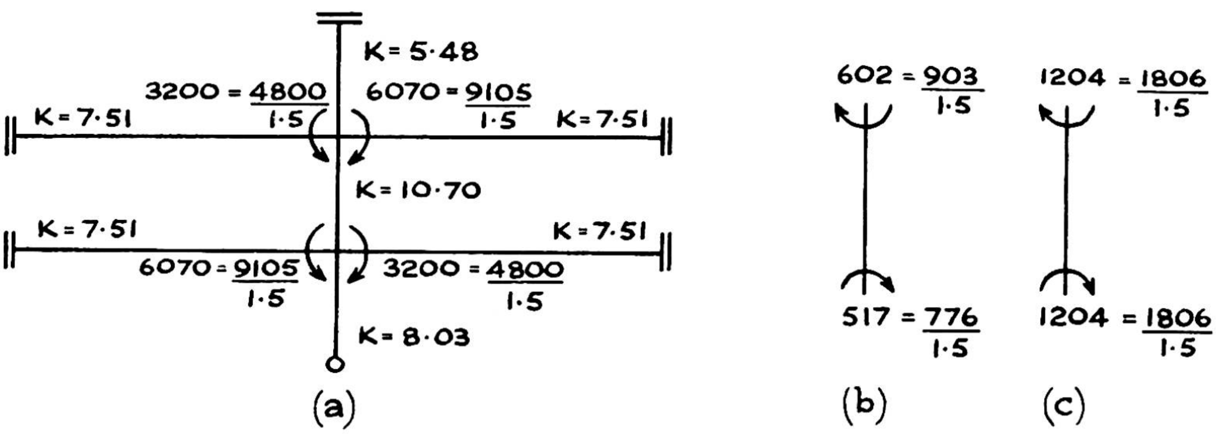


Fig. 9

DISCUSSION In a sense, this Paper is nothing more than an attempt to consider the concept of load factor which is so important in the design process. The idea of working loads gradually increased in proportion leads on the one hand to the idea of plastic collapse of a steel frame (which can be observed readily and accurately both in the laboratory and in the field), and on the other to the mathematical development of master theorems of structural design, concerned, for example, with the overall safety of a frame. The fact remains that the concept of a collapse load factor is reflected only in an insignificantly small probability of an actual overload of a real structure in practice. It is the nominal working loads that are of interest, and, in this sense, plastic theory is an easy and economical way of designing a frame under working loads.

There are no difficulties in the calculations by simple plastic theory of the beams in a multi-storey frame; The ratio collapse load to working load can be calculated uniquely for each beam. However, the determination of elastic bending moments in the columns is sensitive to the development of plastic hinges in the beams. Thus the apparent margin of safety of a column will depend on whether the calculations are made for the working values of the loads or for their fully-factored collapse values. If the second approach is adopted, and an attempt made to allow for the "real" behaviour of the frame by the insertion of plastic hinges in the beams, then less severe conditions may arise for the columns than if the frame were assumed to remain completely elastic.

REFERENCES

1. Arup, Ove and Jenkins R.S., The design of a reinforced - concrete factory at Brynmawr, South Wales. Proc. Instn. civ. Engrs., Part III, December 1953.
2. Baker, Sir John and Heyman J., Plastic design in Britain. Publications, International Association for Bridge and Structural Engineering, vol. 26, p.31, 1966.
3. Report of a Joint Committee: Fully rigid multi-storey welded steel frames. The Institution of Structural Engineers; The Institute of Welding, December 1964.
4. Tall S., Beedle L.S., Galambos T.V. et al, Structural steel design, New York, 1964.
5. Steel Structures Research Committee, First Report 1931, Second Report 1934, Final Report 1936, London, H.M.S.O.

6. British Standards Institution, Draft Code: New unified code for structural concrete (To be published).

7. Société de diffusion des techniques du bâtiment et des travaux publics, Règles de calcul des constructions en acier, Paris, December 1966.

8. Heyman J., Finlinson J.C.H., and Johnson R.P., Inglis A: A fully rigid multi-storey welded steel frame. The Structural Engineer, vol. 44, p.435, December 1966.

SUMMARY

A convenient way of making the design of a multi-storey braced frame is to allow a plastic method for the beams, and to ensure that the columns remain elastic and stable under all loading conditions. The plastic design of beams is straightforward, but the determination of the worst loading conditions for the columns is more difficult; a limited substitute frame can greatly shorten the work. The use of a load factor requires some care, or designs can result which are less safe than those intended by the designer.

RÉSUMÉ

Une façon pratique de projeter un portique à plusieurs étages renforcé consiste à dépasser la limite d'élasticité seulement pour les poutres, en garantissant que les colonnes restent élastiques et stables dans tous les cas de charge. Le calcul plastique des poutres est simple, mais la détermination du cas de charge déterminant pour les colonnes est plus difficile; un portique-modèle simplifié peut raccourcir le travail considérablement. Des précautions sont requises lors de l'utilisation d'un facteur de charge, sinon il pourrait résulter des constructions d'une moins grande sécurité que projetée.

ZUSAMMENFASSUNG

Ein gangbarer Weg, die Bemessung unverschieblicher Stockwerkrahmen vorzunehmen, besteht darin, für die Riegel plastische Rechnung zu erlauben, während für die Stützen angenommen wird, dass diese elastisch und für alle Lasten stabil bleiben. Einfach ist die plastische Bemessung für die Riegel, hingegen bereitet die Bestimmung der schlimmsten Laststellung für die Stützen Schwierigkeiten; ein begrenzter Ersatzrahmen kann die Rechnung erheblich kürzen. Die Verwendung der Lastfaktoren erfordert Sorgfalt, sonst kann die Bemessung unsicherer als diejenige des Verfassers sein.